A Fundamental Theorem in Synthesis of Active Balanced RC Networks and a New Realization Procedure

Dragan Kandić
Associate Professor

University of Belgrade
Faculty of Mechanical Engineering

In the paper is presented a new proof of the fundamental theorem in network synthesis of arbitrary, square (stable) admittance matrix of real, rational functons in complex frequency s (RRF matrix), by active, multiport, transformerless, balanced RC network. Since the theorem establishes only the neccessary condition for network existence, it is proved firstly that this condition is also sufficient if active elements used in realization are balanced voltage-controlled voltage sources (VCVS) and then is given a formulation of a new realization procedure of RRF admittance matrices, by using theoretically minimum number of balanced VCVS. The proposed procedure relies on a new theorem on representation of regular, polynomial matrices (λ-matrices) in s (having the specified degree), as product of two λ–matrices. The obtained results are the most general in nature and are easily applicable in active, transformerless, multiport, RC network synthesis

Keywords: *Active RC synthesis, balanced networks, transformerless synthesis, factorization of polynomial matrices.*

1. INTRODUCTION

It is well-known that the immitance matrices of passive, multiport, transformerless *RLC* networks are *paramount* [1, 2] $\forall\ s \in [0, \infty)$ (s is, in general, the complex frequency). A symmetric, P-th order real matrix is said to be paramount if each of its r-th order *main minors* (r=1, 2, ... P) is not less than the absolute value of any other minor established from the same rows (columns). Since the paramountcy is a necessary condition, Telegen has tried (and succeeded) to prove that it is also sufficient for existence of two- and three-port, purely *resistive networks*. If $P > 3$ the paramountcy does not assure the resistive P-port existence and the synthesis problem appeares to be equivalent to synthesis of resistive network with internal nodes. Unfortunately, this problem is not solved yet, except for ladder one- and two-ports. The procedures proposed for realization of immitance matrices during past few decades reduce to active network synthesis of pertinent *admittance matrices* by networks with no internal nodes [3-5]. In these realizations the passive, transformerless, *RC* subnetworks are either balanced ($PBRCT_0$) [3, 4], or common-ground ($PGRCT_0$) [5], depending on whether the coefficient matrices of the *second Foster's expansion* of pertinent admittance matrix are dominant (DMCSFE) [2-4], or hyperdominant (HDMCSFE), respectively [5-9].

Multiport network synthesis problem is encountered yet in case of *minimal synthesis* of *RRF*, $w(s)=P(s)/Q(s)$ [$P(s)$ and $Q(s)$ are mutually prime polynomials], since it appeares to be equivalent to synthesis of passive or active resistive network with δw+1 ports {δw is the degree of $w(s)$ in Duffin-Hazony's sense [10]}. It has been proved that δw=max $\{P^0, Q^0\}$, where P^0 and Q^0 are the *algebraic degrees* of $P(s)$ and $Q(s)$, respectively [10, 11].

It has also been known [4] that arbitrary stable *RRF* admittance matrix of order P can always be realized by active $BRCT_0$ network with P negative immitance converters (*NICs*), or by common-ground ($GRCT_0$) network with 2P *VCVS* [5]. In all cases [3-5] the realization network and the complete set of equivalent realizations are strongly dependent on a special factorization of the P-th order, *generally regular*, polynomial matrix (=λ-matrix, λM; "lambda matrix") in s. A λ-matrix is said to be generally regular if it is not singular $\forall s$. In section 2 we shall first prove our prerequisite,

Theorem 1: *The sufficient condition for factorization of P-th order, generally regular λM* $\boldsymbol{P}(s)$*, having degree L, whose determinant has K distinct zeros, in form* $\boldsymbol{P}(s)=\boldsymbol{P}_1(s)\cdot\boldsymbol{P}_2(s)$*, where* $p_2=\boldsymbol{P}_2^0$ *reads*: $K > (P-1)\cdot L+p_2-1$. *The coefficients matrices of s in* $\boldsymbol{P}_1(s)$ *and* $\boldsymbol{P}_2(s)$ *are real or complex if the zeros of* det $\boldsymbol{P}(s)$ *are real or complex, respectively,*

and in sections 3 and 4 we give proof of our main result,

Theorem 2: *For realization of arbitrary P-th order matrix of RRFs in complex frequency s, as the admittance matrix of active* $BRCT_0$ *P-port network*:
(*a*) *P controlled sources* (*CS*) *with real-valued controlling coefficients are necessary,*

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Correspondence to:
Dragan Kandić, Faculty of Mechanical Engineering,
27. marta 80, 11000 Belgrade, Yugoslavia
E-mail: dbkandic@alfa.mas.bg.ac.yu

(b) *In general, the realization can not be acchieved by active P-port with less than P CS,*

(c) *P balanced VCVS with real-valued and by modulii greater than unity controlling coefficients* (*i e. voltage* "*amplifications*") *are necessary and sufficient in general case.*

2. FACTORIZATION OF REGULAR λ–MATRIX

In this section we shall prove Theorem 1. Let us first suppose that $\boldsymbol{P}=[p_{ij}(s)]$ is a P-th order, generally regular λM, such that $L=\max\,\{p_{ij}^0(s)\}$ $(i, j=1, 2, \dots P)$. Assume that the coefficient matrix of s^L in $\boldsymbol{P}(s)$ is regular. Then:

$$\det \boldsymbol{P}(s) = \sum_{k=0}^{P\cdot L} a_k \cdot s^k \,. \qquad (1)$$

Suppose det $\boldsymbol{P}(s)$ has K distinct zeros s_k $(k=1, 2, \dots K)$. For each $s=s_k$ the columns $\boldsymbol{P}^{(j)}(s_k)$ $(j= 1, 2, \dots P)$ of $\boldsymbol{P}(s_k)$ are linearly dependent, i. e. there must exist P numbers $q_{1k}, q_{2k}, \dots , q_{Pk}$ — provided that at least one of them is different from zero, which assure the following:

$$\sum_{j=1}^{P} q_{jk} \cdot \boldsymbol{P}^{(j)}(s_k) = \boldsymbol{0}_{P,1} \,, \qquad (2)$$

where $\boldsymbol{0}_{P,1}$ is P-dimensional column-vector. Let the polynomial $C_k(s)$ be the greatest common-divisor of all cofactors $D_{ik}(s)$ $(i=1, 2, \dots P)$ of $\boldsymbol{P}(s)$. Laplace expansion of det $\boldsymbol{P}(s)$ with respect to entries of the k-th column of $\boldsymbol{P}(s)$ reads:

$$\det \boldsymbol{P}(s) = \sum_{i=1}^{P} p_{ik} \cdot D_{ik}(s) = C_k(s) \sum_{i=1}^{P} p_{ik}\, D_{ik}{}'(s) \,. \quad (3)$$

If $s=s_h$ is a zero of $C_k(s)$, then rank $\boldsymbol{P}(s_h) \le P$-1. If polynomial det $\boldsymbol{P}(s)/C_k(s)$ has at least one zero $s=s_k$ which is different from zeros of $C_k(s)$ and furthermore rank $\boldsymbol{P}(s_k)$ $=P$-1, then the folowing systems of simultaneous linear algebraic equations in q_{jk} $(j=1, 2, \dots , P)$ have only the trivial solution and a non-trivial one, respectively:

$$\sum_{\substack{j=1\\ j\neq k}}^{P} q_{jk} \cdot \boldsymbol{P}^{(j)}(s_k) = \boldsymbol{0}_{P,1} \,; \;\; \sum_{j=1}^{P} q_{jk} \cdot \boldsymbol{P}^{(j)}(s_k) = \boldsymbol{0}_{P,1} \,. \quad (4)$$

Since $\boldsymbol{P}(s)$ is generally regular, we can put down:

$$\sum_{j=1}^{P} q_{jk} \cdot \boldsymbol{P}^{(j)}(s) = (s-s_k)\cdot \boldsymbol{M}_1^{(k)}(s) \,, \qquad (5)$$

where $\boldsymbol{M}_1^{(k)}(s)$ is a P-dimensional λ-column-vector with degree not greater than L-1. Now, let us consider only those zeros $s=s_k$ of det $\boldsymbol{P}(s)$ which provide rank $\boldsymbol{P}(s_k)=P$- -1. Then, at least one set of different indices (i. e. rows) $\{i_1, i_2, \dots, i_{P\text{-}1}\} \in \{1, 2,\dots, P\}$ must exist for which holds:

$$\sum_{\substack{j=1\\ j\neq k}}^{P} q_{jk} \cdot \boldsymbol{P}^{(j)}_{i_1, i_2, \dots\, i_{P-1}}(s_k) = -q_{kk} \cdot \boldsymbol{P}^{(j)}_{i_1, i_2, \dots\, i_{P-1}}(s_k) \,. \quad (6)$$

Since the second system of (4) has a non-trivial solution, we can first select $q_{kk} \neq 0$ arbitrarily and then we may calculate q_{jk} $(j \neq k, j=1, 2, \dots , P)$ uniquely from (6).

From assumption that rank $\boldsymbol{P}(s_k)=P$-1 it follows that a non-zero minor of $\boldsymbol{P}(s_k)$ reads:

$$\det\left[\boldsymbol{P}^{1,\,2,\,\dots\,,\,(k-1),\,(k+1),\,\dots\,,\,P}_{i_1,\,i_2,\,\dots\,,\,i_{P-1}}(s_k)\right], \qquad (7)$$

and that it provides the existence of the constant, regular matrices Q_k $(k=1, 2, \dots , P)$,

$$\boldsymbol{Q}_k = \left[q_{ij}\right]_{P,P} = \begin{bmatrix} 1 & 0 & \dots & 0 & q_{1k} & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 & q_{2k} & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & q_{k-1,k} & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & q_{kk} & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & q_{k+1,k} & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & q_{Pk} & 0 & \dots & 1 \end{bmatrix}, \quad (8)$$

which enable the following representation of $\boldsymbol{P}(s)$,

$$\boldsymbol{P}(s)\cdot\boldsymbol{Q}_k = \Big[\boldsymbol{P}^{(1)}(s) \;\vdots\; \dots \;\vdots\; \boldsymbol{P}^{(k-1)}(s) \;\vdots\; (s-s_k)\cdot \boldsymbol{M}_1^{(k)}(s) \;\vdots\; \boldsymbol{P}^{(k+1)}(s) \;\vdots\; \dots \;\vdots\; \boldsymbol{P}^{(P)}(s)\Big]. \qquad (9)$$

Since $C_k^0 \le (P\text{-}1)\cdot L$, number of distinct zeros of $C_k(s)$ is not greater than $(P\text{-}1)\cdot L$. The sufficient condition for any factorization of $\boldsymbol{P}(s)$ reads $K > (P\text{-}1)\cdot L$, since only then can exist at least one zero of polynomial det $\boldsymbol{P}(s)/C_k(s)$, which is different from zeros of $C_k(s)$. Let this condition be satisfied and let there exists at least P distinct zeros $s_1, s_2, \dots , s_P$ of det $\boldsymbol{P}(s)$. If $L\text{-}1 \ge 1/(P\text{-}1)$ the existence of these zeros is automatically assured if $K > (P\text{-}1)\cdot L$. Then the following factorization of $\boldsymbol{P}(s)$ holds:

$$\begin{aligned} \boldsymbol{P}(s) &= \boldsymbol{P}(s)\cdot\boldsymbol{Q}_1\cdot\boldsymbol{Q}_1^{-1} = \Big[(s-s_1)\cdot\boldsymbol{M}_1^{(1)}(s) \;\vdots\; \boldsymbol{P}^{(2)}(s) \;\vdots\; \dots \\ &\dots \;\vdots\; \boldsymbol{P}^{(P)}(s)\Big]\cdot\boldsymbol{Q}_1^{-1} = \Big[(s-s_1)\cdot\boldsymbol{M}_1^{(1)}(s) \;\vdots\; (s-s_2)\cdot \\ &\cdot\boldsymbol{M}_1^{(2)}(s) \;\vdots\; \boldsymbol{P}^{(3)}(s) \;\vdots\; \dots \;\vdots\; \boldsymbol{P}^{(P)}(s)\Big]\cdot\boldsymbol{Q}_2^{-1}\cdot\boldsymbol{Q}_1^{-1} = \dots = \\ &= \Big[(s-s_1)\,\boldsymbol{M}_1^{(1)} \;\dots\; (s-s_P)\,\boldsymbol{M}_1^{(P)}\Big](\boldsymbol{Q}_1\cdot\boldsymbol{Q}_2\cdot\;\dots\;\cdot\boldsymbol{Q}_P)^{-1} \\ &= \Big[\boldsymbol{M}_1^{(1)}(s) \;\vdots\; \dots \;\vdots\; \boldsymbol{M}_1^{(P)}(s)\Big]\cdot\boldsymbol{D}_1(s)\cdot\boldsymbol{Q}_{(1)}^{-1} \end{aligned} \qquad (10)$$

where $\boldsymbol{Q}_{(1)} = \boldsymbol{Q}_1\cdot\boldsymbol{Q}_2\cdot\;\dots\;\cdot\boldsymbol{Q}_P$, and

$$\boldsymbol{D}_1(s) = \mathrm{diag}\,\Big[(s-s_1) \;\vdots\; (s-s_2) \;\vdots\; \dots \;\vdots\; (s-s_P)\Big].$$

$\boldsymbol{Q}_{(1)}$ is a regular, constant, P-th order matrix. In this way, the generally regular matrix $\boldsymbol{P}(s)$ is represented as product of two regular, P-th order λ-matrices: $\boldsymbol{N}_{L\text{-}1}=[\boldsymbol{M}_1^{(1)} \mid \dots \mid \boldsymbol{M}_1^{(P\text{-}1)}]$ with polynomial degree L-1 and $\boldsymbol{T}_1(s)=\boldsymbol{D}_1(s)\cdot \boldsymbol{Q}_{(1)}^{-1}$ with unit polynomial degree. By extraction of a linear matrix factor $\boldsymbol{T}_1(s)$, whose determinant is a P-th order polynomial is s, we have acchieved that the number of distinct zeros of det $\boldsymbol{N}_{L\text{-}1}$ becomes equal to $K_1=K\text{-}P$.

Let us suppose that $s=s_k$ is zero of det $\boldsymbol{P}(s)$ having order $t_k > 1$ and, in addition, that the rank $\boldsymbol{P}(s_k)=r < P$-1. It is convenient that first r columns of $\boldsymbol{P}(s_k)$ be linearly independent. Since this is not the case, in general, we must apply the right-hand multiplication of $\boldsymbol{P}(s_k)$, by a transposition matrix $\boldsymbol{R}=\boldsymbol{R}_1\cdot\boldsymbol{R}_2\cdot\;\dots\;\cdot\boldsymbol{R}_r$ having r regular, column-permutation *matrix factors*. Permutation matrix of the i-th and j-th column of $\boldsymbol{P}(s_k)$ is regular, right-hand matrix multiplicant of $\boldsymbol{P}(s_k)$, whose entries are:

$$r_{mn} = \delta_{mn},\; (m,n) \neq (i,j);\; r_{ii} = r_{jj} = 0,\; r_{ij} = r_{ji} = 1 \,, \quad (11)$$

where δ_{mn} is Kronecker's δ-symbol. In this way, we can always represent $\boldsymbol{P}(s)$ as $\boldsymbol{P}(s)\cdot\boldsymbol{R}\cdot\boldsymbol{R}^{-1}$=[$\boldsymbol{P}'^{(1)}$ | $\boldsymbol{P}'^{(2)}$ | ...... | $\boldsymbol{P}'^{(P)}$]$\cdot\boldsymbol{R}^{-1}$, where the first r columns of [$\boldsymbol{P}'^{(1)}$ | $\boldsymbol{P}'^{(2)}$ | ... | $\boldsymbol{P}'^{(P)}$] are linearly independent. Bearing this in mind, we infer that the system of linear equations in q_{jk}:

$$\sum_{j=1}^{r} q_{jk}\cdot\boldsymbol{P}'^{(j)}(s_k)=\boldsymbol{0}_{P,1}\,, \tag{12}$$

has only the trivial solution since determinant of system,

$$\sum_{j=1}^{r} q_{jk}\cdot\boldsymbol{P}'^{(j)}_{i_1,i_2,\dots i_r}(s_k)=\boldsymbol{0}_{r,1}\,, \tag{13}$$

with maximum order r ($r<P$) is different from zero.

Let us now construct the following set of constant, regular matrices $\{\boldsymbol{Q}^{(1)}, \boldsymbol{Q}^{(2)}, \dots, \boldsymbol{Q}^{(P-r)}\}$ so as to be:

$$\boldsymbol{P}(s)\cdot\boldsymbol{R}\cdot\boldsymbol{Q}^{(1)}\dots\boldsymbol{Q}^{(P-r)}=\Big[\boldsymbol{P}'^{(1)}\ \vdots\ \dots\ \vdots\ \boldsymbol{P}'^{(r)}\ \vdots$$

$$\sum_{i=1}^{r+1} q_{i,r+1}^{(1)}\cdot\boldsymbol{P}'^{(i)}\ \vdots\ \dots\ \vdots\ \sum_{i=1}^{P} q_{i,P}^{(P-r)}\cdot\boldsymbol{P}'^{(i)}\Big]; \tag{14}$$

$$\boldsymbol{Q}^{(j)}=\left[q_{mn}^{(j)}\right]_{P,P}\quad [j=1, 2, \dots (P\text{-}r)]$$

where the matrices $\boldsymbol{Q}^{(j)}$ have a following structure:

$$\mathrm{Q}^{(j)}=\begin{bmatrix} 1 & 0 & \dots & 0 & q_{1,r+j}^{(j)} & 0 & \dots & 0\\ 0 & 1 & \dots & 0 & q_{2,r+j}^{(j)} & 0 & \dots & 0\\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots\\ 0 & \dots & 0 & 0 & q_{r,r+j}^{(j)} & 0 & \dots & 0\\ 0 & \dots & 0 & 0 & \dots & 0 & \dots & 0\\ 0 & \dots & \dots & 1 & \dots & 0 & \dots & 0\\ 0 & \dots & \dots & 0 & q_{r+j,r+j}^{(j)} & 0 & \dots & 0\\ 0 & \dots & \dots & 0 & 0 & 1 & \dots & 0\\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots\\ 0 & \dots & \dots & 0 & 0 & 0 & \dots & 1 \end{bmatrix}. \tag{15}$$

Matrices $\boldsymbol{Q}^{(j)}$ are regular if and only if for $\forall j$ =1, 2, ... , (P-r), $q_{r+j,r+j}^{(j)}\neq 0$. These entries are to be assumed arbitrarily, provided that the previous condition is satisfied.

Since rank ($\boldsymbol{P}\cdot\boldsymbol{R}$)=$r$, then for s=s_k the following systems of linear equations in $q_{i,r+j}^{(j)}$ [i=1, 2, ... , (r+j-1)] have unique solutions for $\forall j$=1, 2, ... , (P-r) [recall that $q_{r+j,r+j}^{(j)}\neq 0$ have already been arbitrarily assumed]:

$$\sum_{i=1}^{r+j} q_{i,r+j}^{(j)}\cdot\boldsymbol{P}'^{(i)}(s)=(s-s_k)\cdot\boldsymbol{M}^{(r+j)}(s)\,. \tag{16}$$

Once the matrices $\boldsymbol{Q}^{(j)}$ are known, then from (16) matrices $\boldsymbol{M}^{(r+j)}(s)$ can easily be determined. Thereof we get:

$$\boldsymbol{P}(s)=\{[\boldsymbol{P}(s)\cdot\boldsymbol{R}]\cdot\boldsymbol{Q}\}\cdot\boldsymbol{Q}^{-1}\cdot\boldsymbol{R}^{-1}=\Big[\boldsymbol{P}'^{(1)}\ \vdots\ \dots\ \vdots\ \boldsymbol{P}'^{(r)}$$

$$\vdots\ (s-s_k)\,\boldsymbol{M}^{(r+1)}\ \vdots\ \dots\ \vdots\ (s-s_k)\,\boldsymbol{M}^{(P)}\Big]\boldsymbol{Q}^{-1}\,\boldsymbol{R}^{-1}, \tag{17}$$

where $\boldsymbol{Q}$=$\boldsymbol{Q}^{(1)}\cdot\boldsymbol{Q}^{(2)}\cdot\ \dots\ \cdot\boldsymbol{Q}^{(P-r)}$. In this way we have proved that if rank $\boldsymbol{P}(s_k)$=$r<P$-1 matrix $\boldsymbol{P}(s)$ can be represented as product of two matrices regular for $s\neq s_k$. The first of them, $\boldsymbol{N}_{L\text{-}1}$=[$\boldsymbol{P}'^{(1)}$ | $\boldsymbol{P}'^{(2)}$ |...| $\boldsymbol{P}'^{(r)}$ | $\boldsymbol{M}_1^{(r+1)}$ | ... | $\boldsymbol{M}_1^{(P)}$] has degree L or L-1, whereas the second one, $\boldsymbol{T}_1(s)$=$\boldsymbol{D}_1(s)\cdot\boldsymbol{R}^{-1}\cdot\boldsymbol{Q}^{-1}$ has unit-degree, where $\boldsymbol{D}_1(s)$ is diagonal matrix $\boldsymbol{D}_1(s)$=diag [1 1 ... 1 (s-s_k) ... (s-s_k)] ($\leftarrow r$ unities).

Up to now we have proved that the condition $K>(P\text{-}1)\cdot L$ assures extraction of linear λ-matrix-factor from $\boldsymbol{P}(s)$. In the sequel we shall derive a sufficient condition for factorization of a regular λ-matrix $\boldsymbol{P}(s)$=$\boldsymbol{P}_1(s)\cdot\boldsymbol{P}_2(s)$, where both $\boldsymbol{P}_1(s)$ and $\boldsymbol{P}_2(s)$ are regular and, in addition, $\boldsymbol{P}_2^{\,0}$=p_2 and $\boldsymbol{P}_1^{\,0}$=$p_1\in[L\text{-}p_2, L]$ — depending on the rank of $\boldsymbol{P}(s_k)$, $\boldsymbol{N}_{L\text{-}1}(s_{k+1})$ etc. If $P_{\min}$=$L_{\min}$=2 [$\Rightarrow$ (P-1)$\cdot L\geq P$] and $K>(P\text{-}1)\cdot L$, factorization (10), or (17) (i. e. extraction of linear matrix factor) is always possible. In next step, i. e. for $K_1>(P\text{-}1)\cdot(L\text{-}1)$, we have analogously:

$$\boldsymbol{N}_{L-1}=\boldsymbol{N}_{L-2}\cdot\boldsymbol{T}_2,\ \boldsymbol{N}_{L-2}=\boldsymbol{N}_{L-3}\cdot\boldsymbol{T}_3,\ \dots, \tag{18}$$

where number of distinct zeros of det $\boldsymbol{N}_{L\text{-}2}$ is not smaller than K_2=K_1-P=K-2P, of det $\boldsymbol{N}_{L\text{-}3}$ is not smaller than K_3= =K_2-P=K-3P, and so on. Let us now suppose that in the (h-1)-st step we have represented $\boldsymbol{P}(s)$ as product of two λ-matrices $\boldsymbol{N}_{L\text{-}(h\text{-}1)}$ and $\boldsymbol{T}_{h\text{-}1}$ having degrees $\in[L\text{-}h\text{+}1, L]$ and h-1, respectively. For extraction of a linear matrix factor from $\boldsymbol{N}_{L\text{-}(h\text{-}1)}$ it is sufficient that number $K_{h\text{-}1}$ of distinct zeros of det $\boldsymbol{N}_{L\text{-}(h\text{-}1)}$ is greater than (P-1)$\cdot$(L-h+1). Since $K_{h\text{-}1}$ is not smaller than K-(h-1)$\cdot P$, and taking into account the condition $L\geq h$+1, which is neccesary and sufficient for $\boldsymbol{N}_{L\text{-}h}$ to be at least monome in s, we obtain:

$$K-(h-1)\cdot P>(P-1)\cdot[L-(h-1)]\ \wedge\ L\geq h+1\,. \tag{19}$$

This is the *worst case* sufficient condition for extraction of a linear matrix factor. Therefrom, for h=p_2 we get,

$$K>(P-1)\cdot L+p_2\text{ -1}\ \wedge\ L\geq p_2+1\,. \tag{20}$$

This is the sufficient condition for representation of $\boldsymbol{P}(s)$ as product of two generally regular λ-matrices $\boldsymbol{P}_1(s)$ and $\boldsymbol{P}_2(s)$ with degrees $p_1\in[L\text{-}p_2, L]$ and $p_2\in[1, L\text{-}1]$, respectively. For selected $p_2\in[1, L\text{-}1]\Rightarrow p_1\in[L\text{-}p_2, L]$ — depending on ranks of matrices obtained during application of the proposed procedure. It is obvious that coefficient-matrices of s in $\boldsymbol{P}_1(s)$ and $\boldsymbol{P}_2(s)$ are real or complex if the zeros of det $\boldsymbol{P}(s)$ are real or complex, respectively.

This completes the proof of Theorem 1.

Comment: In proving theorem, the P-th order coefficient matrix of s^L in $\boldsymbol{P}(s)$ is supposed to be regular. On contrary, if it is singular and has the rank $r<P$, it can be shown that sufficient condition for extraction of a linear matrix factor from $\boldsymbol{P}(s)$ becomes slightly weaker than the previous condition $K>(P\text{-}1)\cdot L$ and it now reads:

$$K>(P-1)\cdot L-(P-r)\,. \tag{21}$$

The proof of assertion (21) is straighforward and is left to the reader.

3. GENERAL ACTIVE NETWORK STRUCTURE

In this section we shall give proof of items (a) and (b) of Theorem 2, while in section 4 we prove item (c), by formulation of new, general synthesis procedure. At the outset consider general linear, active BT, or BT_0 P-port network N=$N_A\cup N_P$ depicted in Fig. 1. N_A is linear active subnetwork, while N_P is linear $BRLCT$ or $BRCT_0$ multiport subnetwork with arbitrary number of passive elements. N_A contains arbitrary number of controlled sources (CS) of all four possible types. Let $\boldsymbol{E}$=[E_1 E_2 ... E_P]$^{\mathrm{T}}$ and $\boldsymbol{I}$=[I_1 I_2 ... I_P]$^{\mathrm{T}}$ ("T" transposition of matrix) be the P-dimensional column-vectors of Laplace transforms of voltages and currents at accessible ports, respectively.

Let $\boldsymbol{A}=[I_1'...I_h'\ E_1'...E_m']^{\mathrm{T}}$ be $(h+m)$-dimensional column–vector of Laplace transforms of controlled currents and voltages of *CS* and let $\boldsymbol{B}=[I_1''...\ I_j''\ E_1''...\ E_k'']^{\mathrm{T}}$ be $(j+k)$-dimensional column–vector of Laplace transforms of controlling currents and voltages. Denote by $\boldsymbol{C}$, $(h+m) \times (j+k)$ *RRF* matrix whose entries are controlled coefficients of *generalized CS* imbeded into subnetwork N_A. Certainly, it must hold $\boldsymbol{A}=\boldsymbol{C}\cdot\boldsymbol{B}$. Assuming $\boldsymbol{E}$ and $\boldsymbol{A}$ to be the network excitations, then by force of superposition principle, which holds in *linear* electrical networks, we may write:

$$\boldsymbol{I} = \boldsymbol{Y}_0 \cdot \boldsymbol{E} + \boldsymbol{D} \cdot \boldsymbol{A} \quad \wedge \quad \boldsymbol{B} = \boldsymbol{F} \cdot \boldsymbol{E} + \boldsymbol{G} \cdot \boldsymbol{A}\,, \qquad (22)$$

where: (i) $\boldsymbol{Y}_0$ is the admittance-matrix of *P*-port network produced by elimination of *CS* ($\Leftrightarrow \boldsymbol{A}=\boldsymbol{0}$), (ii) $\boldsymbol{D}$ is $P \times (h+m)$ matrix, whose entries are current-transmittances and admittances, (iii) $\boldsymbol{F}$ is $(j+k) \times P$ matrix, whose entries are voltage-tranmittances and admittances and (iv) $\boldsymbol{G}$ is $(j+k) \times (h+m)$ matrix having as its entries the transfer-immitances, or current- and voltage-transmittances.

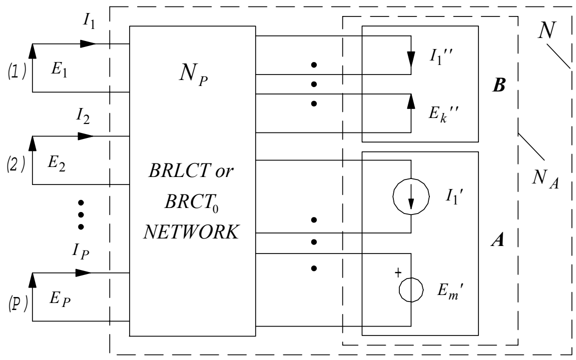

Figure 1. General active realization network

Let $\boldsymbol{U}_{h+m}$ be the $(h+m)$-th order identity matrix. If matrix $\boldsymbol{U}_{h+m}$-$\boldsymbol{C}\cdot\boldsymbol{G}$ is regular, then from (22) we obtain the admittance matrix $\boldsymbol{Y}=\boldsymbol{Y}(s)$ $[\boldsymbol{I}=\boldsymbol{Y}(s)\cdot\boldsymbol{E}]$ of the overall *P*-port, active, *BRLCT* or $BRCT_0$ network (Fig. 1), which in general may have all four types of generalized *CS*:

$$\boldsymbol{Y}(s) = \boldsymbol{Y}_0 + \boldsymbol{D} \cdot (\boldsymbol{U}_{h+m} - \boldsymbol{C} \cdot \boldsymbol{G})^{-1} \cdot \boldsymbol{C} \cdot \boldsymbol{F}\,. \qquad (23)$$

The minimum number of *CS* which is necessary for realization of matrix $\boldsymbol{Y}$ is obtained by investigating the rank of $\boldsymbol{Y}$-$\boldsymbol{Y}_0$, whose entries are *RRF*s having poles s_1, ..., s_q, with multiplicities, say, n_1, ... , n_q, respectively. Since Laurent's complex expansion of $\boldsymbol{Y}(s)$-$\boldsymbol{Y}_0(s)$ reads:

$$\boldsymbol{Y}(s) - \boldsymbol{Y}_0(s) = \sum_{k=0}^{p} \boldsymbol{A}_k \cdot s^k + \sum_{i=1}^{q} \sum_{k=1}^{n_i} \boldsymbol{B}_{(-k)}^{(i)} \cdot (s - s_i)^{-k}\,, \quad (24)$$

and since the rank of product of two matrices never exceeds the ranks of matrix factors, we can write,

$$\begin{gathered} \operatorname{rank}\left(\boldsymbol{Y} - \boldsymbol{Y}_0\right) \le \operatorname{rank} \boldsymbol{C} \le \min\,(j+k,\ h+m)\,, \\ s \to \infty\text{: rang } \boldsymbol{A}_p \le \text{rang } \boldsymbol{C}\,, \qquad (25) \\ s \to s_i\text{: rank } \boldsymbol{B}_{(-n_i)}^{(i)} \le \operatorname{rank} \boldsymbol{C},\ (i = 1,\ 2,\ \ldots,\ q)\,. \end{gathered}$$

The matrix $\boldsymbol{Y}_0$ must be *positive real* [2, 10], since it relates to the *passive* (*B*)*RLCT*, or $(B)RCT_0]$ network N_P. If network is purely resistive, $\boldsymbol{Y}_0$ must be positive semidefinite. In the most general case the matrix $\boldsymbol{Y}(s)$ can have the following two properties with respect to its poles:

(1) They may lie in right-half complex plane and/or on imaginary axis having multiplicities greater than unity.
(2) They may be in the left-half complex plane and are different from poles of $\boldsymbol{Y}_0$; or they are the same as poles of $\boldsymbol{Y}_0$, but have the greater multiplicities than poles of $\boldsymbol{Y}_0$.
In both cases (1) and (2) matrix $\boldsymbol{Y}_0$ does not influence the values of entries in coefficient matrices $\boldsymbol{A}_p$ $(p \ge 2)$ and $\boldsymbol{B}_{(-n_i)}^{(i)}$, $n_i > 1$ $(i = 1,\ 2,\ \ldots,\ q)$. If one of them has rank *P*, then from (25) follows rank $\boldsymbol{C} \ge P$. This conclusion means that, in the most general case, for existence of active, balanced *P*-port (*B*)*RLCT* or $(B)RCT_0$ network which realizes the arbitrary, *P*-th order, *RRF* matrix as admittance matrix of network, it is necessary that the active subnetwork N_A (Fig. 1) has *at least P CS* and the same number of controlling ports. The sufficiency of the proposed conditions for existence of network realization by using negative immitance converters (*NICs*) as active elements is proved elsewhere [4]. That completes proof of items (*a*) and (*b*) of Theorem 2.

Unfortunately, *NICs* as active elements are potentially unstable. This gave us impetus to develop our realization structure and our general procedure for synthesis and realization of arbitrary admittance matrices (possibly stable). In new procedure we are going to present, the minimum number (=*P*) of balanced voltage-controlled voltage sources (*VCVS*) is implemented. Herewith, we prove that *P balanced CS* is, in general, sufficient for realization of any *P*-th order admittance *RRF* matrix. The preferable matrix realizations from engineering point of view are stable ones. So, we shall apply algorithm only to those cases, bearing in mind that unstable admittance matrices can also be realizad by using the same procedure. Passive subnetwork N_P in realization structure proposed is balanced, transformerles and *RC* ($BRCT_0$). Realization networks with no inductors (*L*) and transformers (*T*) are preferable from the practical point of view.

4. GENERAL NETWORK STRUCTURE AND A NEW REALIZATION PROCEDURE

The proposed new general active network structure is depicted in Figure 2. Observe that subnetwork N_P is $BRCT_0$ and that balanced subnetwork N_A contains only *VCVS* ("voltage amplifiers"). Let us now suppose that all these amplifiers are approximately ideal. This means that their input-impedances and output-admittances are infinite and that their voltage-gains A_1, ... , A_P are real and either finite, or infinite. Almost every operatitional amplifier in common use today satisfy previously stated three conditions (its open-loop gain is "infinite"). Let:

$$\begin{gathered} \boldsymbol{E} = [E_1 \ldots E_P]^{\mathrm{T}},\ \boldsymbol{I} = [I_1 \ldots I_P]^{\mathrm{T}},\ \boldsymbol{A} = \operatorname{diag}\,(A_1 \ldots A_P), \\ \boldsymbol{E}_a = [E_{P+1} \ldots E_{2P}]^{\mathrm{T}},\ \boldsymbol{I}_a = [I_{P+1} \ldots I_{2P}]^{\mathrm{T}} = \boldsymbol{0}_{P,1}\,, \quad (26) \\ \boldsymbol{E}_b = [E_{2P+1} \ldots E_{3P}]^{\mathrm{T}},\ \boldsymbol{I}_b = [I_{2P+1} \ldots I_{3P}]^{\mathrm{T}},\ \boldsymbol{E}_b = \boldsymbol{A} \cdot \boldsymbol{E}_a\,. \end{gathered}$$

At this instance we recall that the admitance matrix $\boldsymbol{Y}$= =$\boldsymbol{Y}(s)$ to be realized must satisfy relation $\boldsymbol{I}=\boldsymbol{Y}\cdot\boldsymbol{E}$. On the other hand, the admittance matrix $\boldsymbol{Y}^*(s)$ of *balanced*, *RC transformerless, reciprocal* subnetwork N_P (Fig. 2) with augmented number of *accessible ports* (=3*P*) and no *internal nodes must* be *symmetric* and must have the following structure [2, 10] (since network N_P is passive):

$$\boldsymbol{Y}^* = \begin{bmatrix} \boldsymbol{Y}_{11} & \boldsymbol{Y}_{12} & \boldsymbol{Y}_{13} \\ \boldsymbol{Y}_{12}^{\mathrm{T}} & \boldsymbol{Y}_{22} & \boldsymbol{Y}_{23} \\ \boldsymbol{Y}_{13}^{\mathrm{T}} & \boldsymbol{Y}_{23}^{\mathrm{T}} & \boldsymbol{Y}_{33} \end{bmatrix}, \quad \begin{bmatrix} \boldsymbol{I} \\ \boldsymbol{I}_a \\ \boldsymbol{I}_b \end{bmatrix} = \boldsymbol{Y}^* \cdot \begin{bmatrix} \boldsymbol{E} \\ \boldsymbol{E}_a \\ \boldsymbol{E}_b \end{bmatrix}. \quad (27)$$

Bearing in mind that that always must be:

$$\boldsymbol{I}_a = [I_{P+1} \ldots I_{2P}]^{\mathrm{T}} = \boldsymbol{0}_{P,1} \ \wedge \ \boldsymbol{E}_b = \boldsymbol{A} \cdot \boldsymbol{E}_a , \quad (28)$$

then by eliminating $\boldsymbol{I}_a$ and $\boldsymbol{I}_b$ from (27) and using $\boldsymbol{I}=\boldsymbol{Y}\cdot\boldsymbol{E}$, we obtain, provided that $\boldsymbol{Y}_{22}+\boldsymbol{Y}_{23}\cdot\boldsymbol{A}$ is regular:

$$\boldsymbol{Y} = \boldsymbol{Y}_{11} - (\boldsymbol{Y}_{12} + \boldsymbol{Y}_{13} \cdot \boldsymbol{A}) \cdot (\boldsymbol{Y}_{22} + \boldsymbol{Y}_{23} \cdot \boldsymbol{A})^{-1} \cdot \boldsymbol{Y}_{12}^{\mathrm{T}}. \quad (29)$$

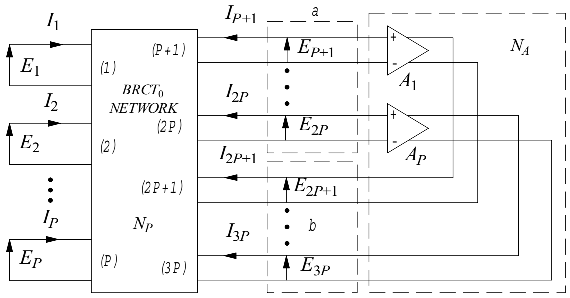

Figure 2. The proposed network structure

Our main intention in conducting the sufficiency proof of Theorem 2 is to show that for arbitrary *RRF* matrix $\boldsymbol{Y}(s)$ there always exisits at least one (and infinite many) sets of real voltage-gains A_1, A_2, … A_P for which it is possible to identify all submatrices $\boldsymbol{Y}_{ij}$ (i,j=1, 2, 3) in (27) so as to assure *dominancy* of *coefficient matrices of the second Foster's expansion* (DMCSFE) *of* $\boldsymbol{Y}^*$. As is well-known [10] the *dominancy* represents necessary and sufficient condition for $BRCT_0$-realization of $\boldsymbol{Y}^*$. Putting it methaphorically, the *VCVSs* play a role of "*Deus ex machina*" means for successful solution of all network synthesis tasks. These tasks can always be accomlished by the approprite *selection* of: (i) P-th order matrix $\boldsymbol{Y}_{11}$, which must be realizable by a $BRCT_0$ network and (ii) finite or infinite voltage-amplifier gains.

Let x_{ij}, q, N_{ij} and D (i,j=1, 2,…, P) be polynomials in s such that: {1} $\boldsymbol{Y}=[N_{ij}]_{P,P}/D$ ($\leftarrow$ $\boldsymbol{Y}$ is given by statement of problem), {2} $\boldsymbol{Y}_{11}=[x_{ij}]_{P,P}/q$ ($\leftarrow$ $\boldsymbol{Y}_{11}$ must be *appropriately selected and must be realizable by* $BRCT_0$ network) and {3} $\boldsymbol{P}=[D\cdot x_{ij}-q\cdot N_{ij}]_{P,P}$ must be a regular, factorizable λ-matrix in the sense of Theorem 1. Factorizabilty of $\boldsymbol{P}$ can always be acchieved by suitable selection of $\boldsymbol{Y}_{11}$. In addition, $\boldsymbol{Y}_{11}$ must have *strictly dominant* coefficient-matrices of its second Foster's expansion, although the *dominancy* (but not strictly) *of* $\boldsymbol{Y}_{11}$ is *necessary and sufficient* for its realization by a $BRCT_0$ network [2, 10].

In the sequel we shall distinguish between the following two cases: (i) voltage-amplifier gains are finite and (ii) these gains are infinite. For each of these cases, we shall develop the steps of general synthesis procedure.

Case 1°: Real and finite voltage-amplifier gains

Presuming the factorizabilty of $\boldsymbol{P}=\boldsymbol{P}_1\cdot\boldsymbol{P}_2$, from (29) we infer that if c_1 and c_2 are arbitrarily selected reals ($c_1\cdot c_2 \neq 0$) — a possible identification of matrices reads:

$$\boldsymbol{Y}_{12} + \boldsymbol{Y}_{13}\,\boldsymbol{A} = \frac{\boldsymbol{P}_1}{c_1\, q}\ ; \ \boldsymbol{Y}_{22} + \boldsymbol{Y}_{23}\,\boldsymbol{A} = \frac{D}{c_1\, c_2\, q}\boldsymbol{U}_P\,, \quad (30)$$

$\boldsymbol{Y}_{12} = \boldsymbol{P}_2^{\mathrm{T}} / c_2\, q$; ($\boldsymbol{U}_P$ is the P-th order identity matrix),

wherefrom it becomes clear that $\boldsymbol{Y}_{22}+\boldsymbol{Y}_{23}\cdot\boldsymbol{A}$ is generally regular matrix. Let the numerical ratio $r=c_2/c_1$ is selected arbitrarily, wherefrom it still remains to determine $c_2 \neq 0$ appropriately. By virtue of {2}, the roots of polynomial $q(s)$ must be simple and lie in the origin and/or on negative real axis [2, 6, 10]. In addition, the following must hold $\boldsymbol{P}_2(s) \leq Q$+1 ($Q$=$q^0$) [2]. We can now identify both $\boldsymbol{Y}_{12}$ and $\boldsymbol{Y}_{13}$, after the Laurent's complex expansion of $\boldsymbol{P}_2^{\mathrm{T}}/q$:

$$\boldsymbol{Y}_{12} = \frac{\boldsymbol{P}_2^{\mathrm{T}}}{c_2 \cdot q} = \frac{1}{c_2} \cdot \left(\boldsymbol{K}_\infty \cdot s + \sum_{m=0}^{Q} \boldsymbol{K}_m \cdot \frac{s}{s+s_m} \right),$$
$$\boldsymbol{Y}_{13} = \frac{1}{c_2 \cdot q} \cdot (r \cdot \boldsymbol{P}_1 - \boldsymbol{P}_2^{\mathrm{T}}) \cdot \boldsymbol{A}^{-1} \quad (31)$$

where it has been assumed $q(s)=s\cdot(s-s_1)\cdot(s-s_2)\cdot \ldots \cdot(s-s_Q)$ and $s_0= 0 < s_1 <\ldots< s_Q$. By selecting sufficiently great c_2 we can produce entries of $\boldsymbol{Y}_{12}$ and $\boldsymbol{Y}_{13}$ arbitrarily small, so as to assure the true dominancy of the first P rows and columns of *symmetric matrix* $\boldsymbol{Y}^*(s)$. Remember that $\boldsymbol{Y}^*(s)$ *must be realizable* by a $BRCT_0$ network.

All entries of matrix $\boldsymbol{Y}_{23}$ and off-diagonal entries of $\boldsymbol{Y}_{22}$ must not have poles distinct from poles of diagonal entries of $\boldsymbol{Y}_{22}$. These diagonal entries must essentially correspond to input admittances of $BRCT_0$ networks [2]. Let us identify now the other submatrices. We have:

$$\boldsymbol{Y}_{22} + \boldsymbol{Y}_{23}\,\boldsymbol{A} = \frac{D}{c_1\, c_2\, q}\boldsymbol{U}_P = \boldsymbol{F}_\infty\, s + \sum_{m=0}^{Q} \boldsymbol{F}_m \frac{s}{s+s_m}\,, \quad (32)$$

where $\boldsymbol{F}_m$=$F_m\cdot\boldsymbol{U}_P$ (m=∞, 0, 1, … , Q) are diagonal matrices. Let $\boldsymbol{G}_m$=$[g_{ij}^{(m)}]_{P,P}$ and $\boldsymbol{H}_m$=$[h_{ij}^{(m)}]_{P,P}$ be arbitrarily selected diagonal matrices whose diagonal entries are nonnegative and satisfy the conditon $g_{ii}^{(m)}\cdot h_{ii}^{(m)}$=0 ($\forall i$=1, 2, …, P; $\forall m$=∞, 0, 1, … , Q). Let $\boldsymbol{D}_m$=$[d_{ij}^{(m)}]_{P,P}$=$d^{(m)}\cdot\boldsymbol{U}_P$ (m=∞, 0, 1,…, Q) be the P-th order scalar matrix whose diagonal entries are positive and should be at first *estimated* and thereon *assumed* appropriately. If we put $\boldsymbol{F}_m$ =($\boldsymbol{G}_m$+$\boldsymbol{D}_m$)-($\boldsymbol{H}_m$+$\boldsymbol{D}_m$), then from (32) follows a possible identification of matrices $\boldsymbol{Y}_{22}$ and $\boldsymbol{Y}_{23}$:

$$\boldsymbol{Y}_{22} = (\boldsymbol{G}_\infty + \boldsymbol{D}_\infty) \cdot s + \sum_{m=0}^{Q} (\boldsymbol{G}_m + \boldsymbol{D}_m) \cdot \frac{s}{s+s_m}\,,$$
$$\boldsymbol{Y}_{23} = -[(\boldsymbol{H}_\infty + \boldsymbol{D}_\infty)\, s + \sum_{m=0}^{Q} (\boldsymbol{H}_m + \boldsymbol{D}_m) \frac{s}{s+s_m}] \boldsymbol{A}^{-1} \quad (33)$$

Let $\boldsymbol{K}_m$=$[k_{ij}^{(m)}]_{P,P}$ (m=∞, 0, 1,…, Q). From (33) we infer that the second P rows (columns) of coefficient-matrices in the second Foster's expansion of $\boldsymbol{Y}^*$ will be dominant if $\forall i$=1, 2,…, P and $\forall$ m=∞, 0, 1,…, Q — the following condition holds:

$$g_{ii}^{(m)} + d_{ii}^{(m)} \geq \sum_{\substack{j=1 \\ j\neq i}}^{P} \left[g_{ij}^{(m)} + d_{ij}^{(m)} + \left| \frac{h_{ii}^{(m)} + d_{ii}^{(m)}}{A_j} \right| \right] +$$
$$+ \frac{h_{ii}^{(m)} + d_{ii}^{(m)}}{\left|A_j\right|} + \frac{1}{\left|c_2\right|} \cdot \sum_{j=1}^{P} \left| k_{ji}^{(m)} \right| . \quad (34)$$

The previous set of conditions essentialy reduces to:

$$g_{ii}^{(m)} + d^{(m)} \geq \frac{h_{ii}^{(m)} + d^{(m)}}{|A_j|} + \frac{1}{|c_2|} \cdot \sum_{j=1}^{P} \left| k_{ji}^{(m)} \right|. \quad (35)$$

Since $g_{ii}^{(m)} \cdot h_{ii}^{(m)} = 0$ ($\forall i$=1, 2, … , P; $\forall m$=∞, 0, 1, … , Q), dominancy of the second P rows (columns) in MCSFE of $\boldsymbol{Y}^*(s)$ is acchieved if all A_i (i=1, 2, … , P) are by modulus greater than unity and if all "d"-entries of selected scalar matrices, for m=∞, 0, 1, … , Q satisfy condition:

$$d^{(m)} \geq \max_{i=1,2,\ldots,P} \left\{ \frac{h_{ii}^{(m)}}{|A_i| - 1} + \frac{|A_i|}{|A_i| - 1} \cdot \sum_{j=1}^{P} \left| \frac{k_{ji}^{(m)}}{c_2} \right| \right\}. \quad (36)$$

We can notice from (29) that $\boldsymbol{Y}$ does not depend on $\boldsymbol{Y}_{33}$. Thereof it follows that dominancy of the third P rows or columns of coefficient-matrices in the second Foster's expansion of $\boldsymbol{Y}^*$ can always be acchieved with indefinite number degrees of freedom. Among many different choices of $\boldsymbol{Y}_{33}$ the most simple one seems to be selection of $\boldsymbol{Y}_{33}$ as constant diagonal matrix with positive, sufficiently great diagonal entries.

To this end we have proved the item (c) of Theorem 2, i. e. that P balanced *VCVSs* with finite and by modulii greater than unity voltage-gains are sufficient for realization of any (preferably stable) P-th order *RRF* matrix as admittance matrix of $BRCT_0$ network. It has also been proved that there exists indefinite number of topologically and parametrically equivalent network realizations.

Case 2°: Infinite voltage-amplifier gains

According to (28), in this case it must hold $\boldsymbol{E}_a = \boldsymbol{0}_{P,1}$ (since $\boldsymbol{A} \to \infty$). Hence, the identification (30) will not be valid anymore. If we rearrange (29) as follows :

$$\boldsymbol{Y} = \boldsymbol{Y}_{11} - (\boldsymbol{Y}_{12} \cdot \boldsymbol{A}^{-1} + \boldsymbol{Y}_{13}) \cdot (\boldsymbol{Y}_{22} \cdot \boldsymbol{A}^{-1} + \boldsymbol{Y}_{23})^{-1} \cdot \boldsymbol{Y}_{12}^{\mathrm{T}}, \quad (37)$$

and let $\boldsymbol{A} \to \infty$, we finally obtain,

$$\boldsymbol{Y} = \boldsymbol{Y}_{11} - \boldsymbol{Y}_{13} \cdot \boldsymbol{Y}_{23}^{-1} \cdot \boldsymbol{Y}_{12}^{\mathrm{T}}. \quad (38)$$

By using (38) we can make the following identification of the seeked matrices:

$$\boldsymbol{Y}_{12} = \frac{\boldsymbol{P}_2^{\mathrm{T}}}{c_2 \cdot q}, \quad \boldsymbol{Y}_{13} = \frac{\boldsymbol{P}_1}{c_1 \cdot q}, \quad \boldsymbol{Y}_{23} = \frac{D}{c_1 \cdot c_2 \cdot q} \cdot \boldsymbol{U}_P. \quad (39)$$

In this case, all matrices $\boldsymbol{Y}_{ii}$ (i=1, 2, 3) are to be selected so as to assure the dominancy of coefficient-matrices in the second Foster's expansion of $\boldsymbol{Y}^*$.

In both cases **1°** and **2°** the produced matrix $\boldsymbol{Y}^*(s)$ is to be realized by a parallel connection of at most q^0+2 $BRCT_0$ networks. Eeach of them is comprised of at most $3P \cdot (3P-1)/2$ half-lattice two-poles which can be realized almost by inspection [2].

This completes the proof of item (c) and Theorem 2 altogether.

Now, we shall consider and formulate the steps of algorithm for realization of *RRF* matrix $\boldsymbol{Y}(s)=[N_{ij}]_{P,P}/D$ by an active, $BRCT_0$, P-port network.

- Firstly, we must *select* symmetric, P-th order *RRF* matrix $\boldsymbol{Y}_{11}(s)=[x_{ij}]_{P,P}/q$ with *strictly* DMCSFE. $\boldsymbol{Y}_{11}(s)$ *must be* realizable by $BRCT_0$ P-port network. Therefore, we have $x_{ij}^0 \leq x_{ii}^0 = q^0 + \varepsilon$ (i,j=1, 2, … , P; $i \neq j$), where ε=1 or 0, depending on whether the point at infinity is pole or common-point of $\boldsymbol{Y}_{11}(s)$, respectively [2, 6-8, 10].
- In addition, $\boldsymbol{Y}_{11}(s)$ must assure the factorizability of $\boldsymbol{Y}_{11}(s)$-$\boldsymbol{Y}(s)=[D \cdot x_{ij} - q \cdot N_{ij}]_{P,P}/(q \cdot D)=\boldsymbol{P}(s)/(q \cdot D)$ — according to Theorem 1.
- We shall deliberately suppose that x_{jj}^0 is independent of j (j=1, 2,…, P). Then we have L=max $[D^0$+max (x_{ij}^0), q^0+max $(N_{ij}^0)]|_{i,j=1,2,\ldots,P} = x_{ii}^0 + L_\varepsilon$, provided that L_ε=max $[D^0$, max (N_{ij}^0)-ε$]|_{i,j=1,2,\ldots,P}$.
- Strict dominancy of MCSFE in $\boldsymbol{Y}_{11}(s)$ with sufficiently great *dominancy-margin* [2, 9, 10] can always be acchieved by multiplication of its diagonal entries with *sufficiently* great, real, positive constant ρ:

$$\boldsymbol{Y}_{11}(s) = \frac{1}{q} \cdot \begin{bmatrix} \rho \cdot x_{11}' & \ldots & \ldots & x_{1P} \\ \ldots & \rho \cdot x_{22}' & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ x_{P1} & \ldots & \ldots & \rho \cdot x_{PP}' \end{bmatrix}, \quad (40)$$

so that det $\boldsymbol{P}(s)$ can be represented as follows,

$$\det \boldsymbol{P} = \det [D x_{ij} - q N_{ij}] = \rho^P \left[D^P \prod_{i=1}^{P} x_{ii}' + \frac{R}{\rho^P} \right], \quad (41)$$

$$R^0(s,\rho) \leq P \cdot x_{jj}^0 < P \cdot L \quad \wedge \quad \lim_{\rho \to \infty} \frac{R(s,\rho)}{\rho^P} = 0.$$

When $\rho \to \infty$, $P \cdot x_{jj}^0$ zeros of det $\boldsymbol{P}(s)$ approaches the zeros of product $x_{11}' \cdot x_{22}' \cdot \ldots \cdot x_{PP}'$ [← all of them should be selected to be real, simple and distinct from zeros of both polynomials $D(s)$ and $q(s)$]. Thuswith, det $\boldsymbol{P}(s)$ can be produced to have *at least* $P \cdot x_{ii}^0$ real, distinct zeros. Then, for extraction of k linear matrix factors from $\boldsymbol{P}(s)$ it is sufficient after extraction of the (k-1)-th factor to be:

$$P \cdot x_{ii}^0 - P \cdot (k-1) > (P-1) \cdot [x_{ii}^0 + L_\varepsilon - (k-1)]. \quad (42)$$

This follows straighforwardly from (19). If we *assume* now $k=L_\varepsilon$, the iniquality (42) reduces to $x_{ii}^0 > P \cdot L_\varepsilon - 1$ and can be satisfied by selecting $x_{ii}^0 = P \cdot L_\varepsilon$. It is shown thuswith that if det $\boldsymbol{P}(s)$ has at least $P \cdot x_{ii}^0 = P \cdot (q^0 + \varepsilon)$ distinct, real zeros, the regular λ-matrix $\boldsymbol{P}(s)$ can be represented as product of regular λ-matrices $\boldsymbol{P}_1(s)$ and $\boldsymbol{P}_2(s)$, having degrees $p_1 = x_{ii}^0 = P \cdot L_\varepsilon$ and $p_2 = L_\varepsilon$, respectively. We must recall that $x_{ii}^0 > P \cdot L_\varepsilon - 1$ is only a sufficient condition (and not necessary one) for factorization of $\boldsymbol{P}(s)$.

- Since $\boldsymbol{Y}_{11}(s)=[x_{ij}]_{P,P}/q$ must have *strictly* DMCSFE, it follows that is also necessary to be $x_{ii}(0) \neq 0$ (i=1, 2, … , P). Otherwise, the *strict dominancy* of MCSFE of $\boldsymbol{Y}_{11}(0)$ may be violated.

Now we formulate steps of realization algorithm:

(***a***) Select $\boldsymbol{Y}_{11}(s)=[x_{ij}]_{P,P}/q$ with *strictly dominant* MCSFE. $\boldsymbol{Y}_{11}(s)$ must be realizable by a $BRCT_0$ network. Zeros of $x_{ii}(s)$ (i=1, 2,…, P) should be selected real, simple and distinct from zeros of polynomials $D(s)$ and $q(s)$. All polynomials $x_{ii}(s)$ may have the same degree and in addition, $x_{ii}(0) \neq 0$ (i=1, 2,… , P). These diagonal entries possibly should be multiplied by sufficiently great real ρ > 0 in order to assure that the λ-matrix $\boldsymbol{P}(s)=[D \cdot x_{ij} - q \cdot N_{ij}]_{P,P}$ be regular and to have at least $P \cdot x_{ii}^0$ real, simple and distinct zeros.

(***b***) Let L_ε=max $[D^0$, max (N_{ij}^0)-ε$]|_{i,j=1,2,\ldots,P}$. Then, realize the factorization $\boldsymbol{P}(s)=\boldsymbol{P}_1(s) \cdot \boldsymbol{P}_2(s)$, where degrees of regular λ-matrices $\boldsymbol{P}_1(s)$ and $\boldsymbol{P}_2(s)$ are $p_1=x_{ii}^0=P\cdot L_\varepsilon$ and $p_2=L_\varepsilon$, respectively.

(*c*) Select the real-valued voltage-amplifier gains A_i as $|A_i|>1$ (i=1, 2,…, P). Then apply the identification procedure, either of the **Case 1⁰**, or of the **Case 2⁰** to produce necessary block-matrices of $\boldsymbol{Y}^*(s)$, which is realizable by pasive $BRCT_0$ network. Realization of $\boldsymbol{Y}^*$ is simple and straighforward [2]. And finally, by interconnection of *VCVS* and previously produced $BRCT_0$, $3P$-port network N_P the balanced, active, P-port, transformerless, *RC* network which realizes the admittance matrix $\boldsymbol{Y}(s)$ is finally constructed.

5. A NUMERICAL EXAMPLE

By using the procedure proposed let us realize 2 × 2 admittance matrix:

$$\boldsymbol{Y}(s)=\frac{[N_{ij}]}{D(s)}=\frac{1}{s+1}\begin{bmatrix}2s+1 & s\\ s & 2s-1\end{bmatrix},\ D(s)=s+1. \quad (43)$$

Although $\boldsymbol{Y}(s)$ is symmetric, it can not be realized by *passive RLCT* network, since the entry $y_{22}(s)=N_{22}s)/D(s)$ $=(2s-1)/(s+1)$ is not the input-admittance of *RC* network [2, 6]. Then, the passive network realization (*RLCT* or $RLCT_0$) of $\boldsymbol{Y}(s)$ simply does not exist. According to the algorithm, we must first arbitrarily select $\boldsymbol{Y}_{11}(s)$:

$$\boldsymbol{Y}_{11}(s)=\frac{[x_{ij}]}{q(s)}=\frac{1}{s+1}\begin{bmatrix}3s+1 & 0\\ 0 & 2s+1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}\frac{s}{s+1},\ q(s)=s+1, \quad (44)$$

which is realizable by parallel connection of two $BRCT_0$ two-ports. Nodes (1, 1') of port 1 and (2, 2') of port 2 are not mutually interconnected. Then, we obtain for $\boldsymbol{P}(s)$:

$$\boldsymbol{P}(s)=[D\,x_{ij}\text{ - }q\,N_{ij}]_{2,2}=(s+1)\begin{bmatrix}s & -s\\ -s & 2\end{bmatrix},$$
$$\det\boldsymbol{P}(s)=\text{-}s(s-2)(s+1),\ P=L=2,\ K=3. \quad (45)$$

Since the sufficient condition of Theorem 1 [$K>(P\text{-}1)\cdot L+p_2\text{-}1$] is satisfied with p_2=2, we infer that extraction of linear matrix factor from $\boldsymbol{P}(s)$ is possible. So, we get:

$$\boldsymbol{P}_1(s)=\begin{bmatrix}0 & s+1\\ -(s+1) & -(s+1)\end{bmatrix},\ \boldsymbol{P}_2(s)=\begin{bmatrix}0 & s-2\\ s & -s\end{bmatrix}, \quad (46)$$

where $\boldsymbol{P}(s)=\boldsymbol{P}_1(s)\cdot\boldsymbol{P}_2(s)$. Let us assume $c_1=c_2=c$ ($\Leftrightarrow r$=1) and $\boldsymbol{A}$=diag (-2 2) ($\Leftrightarrow A_1$= -2 $\wedge$ A_2=2). According to (30) and (31) we obtain:

$$\boldsymbol{Y}_{12}=\frac{\boldsymbol{P}_2^{\mathrm{T}}}{cq}=\frac{1}{c(s+1)}\begin{bmatrix}0 & s\\ s-2 & -s\end{bmatrix}=$$
$$=\frac{1}{c}\begin{bmatrix}0 & 0\\ -2 & 0\end{bmatrix}+\frac{1}{c}\begin{bmatrix}0 & 1\\ 3 & -1\end{bmatrix}\frac{s}{s+1}$$
$$\boldsymbol{Y}_{13}(s)=\frac{(\boldsymbol{P}_1\text{ - }\boldsymbol{P}_2^{\mathrm{T}})\boldsymbol{A}^{-1}}{cq(s)}=\frac{1}{2c(s+1)}\begin{bmatrix}0 & 1\\ 2s-1 & -1\end{bmatrix}=$$
$$=\frac{1}{2c}\begin{bmatrix}0 & 1\\ -1 & -1\end{bmatrix}+\frac{1}{2c}\begin{bmatrix}0 & -1\\ 3 & 1\end{bmatrix}\frac{s}{s+1}. \quad (47)$$

The first two rows of coefficient-matrices in $\boldsymbol{Y}^*$ are dominant if the same holds for submatrix:

$$[\boldsymbol{Y}_{11}\ \vdots\ \boldsymbol{Y}_{12}\ \vdots\ \boldsymbol{Y}_{13}]=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & \frac{1}{2c}\\ 0 & 1 & -\frac{2}{c} & 0 & -\frac{1}{2c} & -\frac{1}{2c}\end{bmatrix}+$$
$$+\begin{bmatrix}2 & 0 & 0 & \frac{1}{c} & 0 & -\frac{1}{2c}\\ 0 & 1 & \frac{3}{c} & -\frac{1}{c} & \frac{3}{2c} & \frac{1}{2c}\end{bmatrix}\cdot\frac{s}{s+1}. \quad (48)$$

Symmetric real matrix $\boldsymbol{Z}=[z_{ij}]_{P,P}$ is said to be dominant if $\forall i$=1, 2, … , P its diagonal entry z_{ii} is not less than the sum of moduli of entries in the same row (column), i. e. $z_{ii}\geq|z_{1,i}|+|z_{2,i}|+\ldots+|z_{1,i-1}|+|z_{1,i+1}|+\ldots+|z_{1,P}|$ [2]. Since each row of matrix (48) must be dominant, thereof we obtain that $c\geq 6$. According to (30) and by algorithm we have:

$$\boldsymbol{Y}_{22}+\boldsymbol{Y}_{23}\cdot\boldsymbol{A}=\frac{1}{c^2}\cdot\boldsymbol{U}_2,\ \boldsymbol{D}_0=d_0\cdot\boldsymbol{U}_2,\ \boldsymbol{D}_1=d_1\cdot\boldsymbol{U}_2, \quad (49)$$

where d_0, $d_1>0$. Then, $\boldsymbol{Y}_{22}$ and $\boldsymbol{Y}_{23}$ can be identified as:

$$\boldsymbol{Y}_{22}(s)=\begin{bmatrix}d_0+\frac{1}{c^2} & 0\\ 0 & d_0+\frac{1}{c^2}\end{bmatrix}+\begin{bmatrix}d_1 & 0\\ 0 & d_1\end{bmatrix}\cdot\frac{s}{s+1},$$
$$\boldsymbol{Y}_{23}(s)=\begin{bmatrix}\frac{d_0}{2} & 0\\ 0 & -\frac{d_0}{2}\end{bmatrix}+\begin{bmatrix}\frac{d_1}{2} & 0\\ 0 & -\frac{d_0}{2}\end{bmatrix}\cdot\frac{s}{s+1}. \quad (50)$$

The third and the forth row of coefficient-matrices in $\boldsymbol{Y}^*$ are dominant if the same holds for submatrix $[\boldsymbol{Y}_{12}^{\mathrm{T}}\mid\boldsymbol{Y}_{22}\mid\boldsymbol{Y}_{23}]$. One can easily show that the desired dominancy can be realized through selection c_0=6, d_0=2 and d_1=1. Dominancy of the fifth and the sixth row of $\boldsymbol{Y}^*(s)$ can be readily acchieved by selecting $\boldsymbol{Y}_{33}$ as diagonal matrix, as for example:

$$\boldsymbol{Y}_{33}(s)=\begin{bmatrix}13/12 & 0\\ 0 & 7/6\end{bmatrix}+\begin{bmatrix}3/4 & 0\\ 0 & 2/3\end{bmatrix}\cdot\frac{s}{s+1}. \quad (51)$$

Herewith, we completely accomplished construction of the seeked matrix $\boldsymbol{Y}^*$, which can be realized straightforwardly (almost by inspection), by a passive, balanced, transformerless, *RC*, six-port network:

$$\boldsymbol{Y}^*=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & \frac{1}{12}\\ 0 & 1 & -\frac{1}{3} & 0 & -\frac{1}{12} & -\frac{1}{12}\\ 0 & -\frac{1}{3} & \frac{73}{36} & 0 & 1 & 0\\ 0 & 0 & 0 & \frac{73}{36} & 0 & -1\\ 0 & -\frac{1}{12} & 1 & 0 & \frac{13}{12} & 0\\ \frac{1}{12} & -\frac{1}{12} & 0 & -1 & 0 & \frac{7}{6}\end{bmatrix}+$$
$$+\begin{bmatrix}2 & 0 & 0 & \frac{1}{6} & 0 & -\frac{1}{12}\\ 0 & 1 & \frac{1}{2} & -\frac{1}{6} & \frac{1}{4} & \frac{1}{12}\\ 0 & \frac{1}{2} & 1 & 0 & \frac{1}{2} & 0\\ \frac{1}{6} & -\frac{1}{6} & 0 & 1 & 0 & -\frac{1}{2}\\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{3}{4} & 0\\ -\frac{1}{12} & \frac{1}{12} & 0 & -\frac{1}{2} & 0 & \frac{2}{3}\end{bmatrix}\cdot\frac{s}{s+1} \quad (52)$$

Realization details of dominant matrices are duscussed elsewhere [2]. Network realizations of this class of matrices are established exclusively from half-lattices with only serial, or crossed arms (pair of branches), inserted between each pair of nodes (i. e. network ports). By use of (52) we find, for example, that the network fragment between ports 2 and 3 looks like as is depicted in Fig 3. Thereon are denoted the dimensionless values of *conductances* (G_n) and *capacitances* (C_n), normalized with respect to *assumed* values of angular frequency (ω_0) and the impedance level (R_0). Physical parameter values of these elements are obtained by denormalization process: the values of conductances are calculated as $G=G_n/R_0$, whereas the values of capacitances are obtained as $C= =C_n/(R_0 \cdot \omega_0)$.

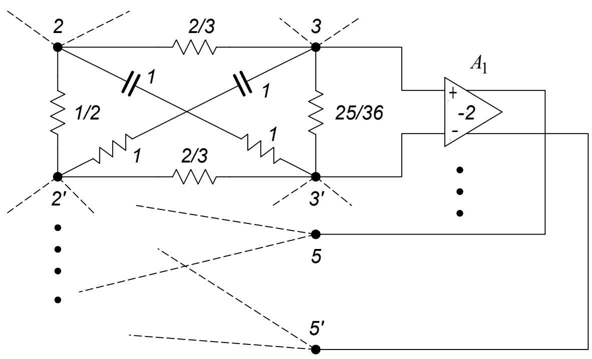

Figure 3. The fragment of active BRCT$_0$ network which realizes the admittance matrix (43)

6. CONCLUSIONS

In paper is presented a new proof of fundamental theorem in network synthesis of admittance matrices of real, rational functons in complex frequency by active, multiport, transformerless, balanced *RC* networks. It has been shown that the neccessary condition for network existence is also a sufficient one if active elements used are balanced voltage-amplifiers. A new procedure for realization of *RRF* admittance matrices is given also by use of minimum number of balanced *VCVS*. The procedure proposed relies on a new theorem on representation of regular, polynomial matrix as product of two polynomial matrices with selected degrees. The obtained results are the most general in nature and can easily be applied in transformerless, multiport, *RC* active network synthesis.

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ФУНДАМЕНТАЛНА ТЕОРЕМА У СИНТЕЗИ АКТИВНИХ БАЛАНСНИХ RC МРЕЖА И ЈЕДАН НОВ ПОСТУПАК РЕАЛИЗАЦИЈЕ

Драган Кандић

У раду се даје један нов доказ фундаменталне теореме из активне синтезе балансних *RC* мрежа са више приступа, без трансформатора. Мада та теорема даје само потребне услове за егзистенцију мреже, у раду је показано да су ти услови уједно и довољни за реализацију произвољне стабилне матрице реалних рационалних функција комплексне фреквенције, као адмитансне матрице мреже поменуте класе, у случају када се као активни елементи користе балансни напонски појачавачи, коначног или бесконачног појачања. Такође, дат је и поступак реализације мрежа са минималним бројем таквих појачавача. Поступак се заснива на примени нове теореме о факторизацији несингуларних полиномних матрица, која је у раду формулисана и доказана. Изложени резултати су генералног карактера, али се истовремено они могу користити и за практичну синтезу активних, балансних *RC* мрежа са више приступа, без трансформатора.