On the Effect of Gas Injection/Suction Through a Porous Wall on the Flow in a Gas Lubricating Slider Bearing

Cvetko Crnojević
Associate Professor

Vladan D. Djordjević
Professor

University of Belgrade
Faculty of Mechanical Engineering

Compressible, isothermal, low Mach number, viscosity dominated flow in a gas lubricating slider bearing is considered in the paper under the assumption of continues gas injection/suction through the bearing. Two characteristic cases are encountered at that: (a) forced injection/suction at a constant rate, and (b) natural injection/suction due to the pressure difference on both sides of the bearing. In the latter case ambient pressure is supposed constant and the flow through relatively long slits isothermal and slow. It is shown that the injection of the gas, even at relatively small rates, highly increases the pressure in the bearing, thus increasing the slider bearing load too, while the suction affects the pressure distribution conversely. The results obtained can be applied in the design of externally pressurized gas bearings.

Keywords: *Isothermal flow, gas lubricating, wall porosity, slider bearing.*

1. Introduction

Due to both its theoretical attraction and practical importance compressible gas flow in micro-channels has been paid much attention in the literature recently. Since micro-channels have extremely small widths, measured in microns, only moderately high values of the Reynolds number can be attained in a micro-channel flow, with the consequence that the effect of viscosity is spread over the whole cross-section of the channel. Thus, it either competes inertia for high subsonic or supersonic flow, like in the classical boundary layer theory, or dominates over it for low Mach number subsonic flow, like in the hydrodynamic lubrication theory. Pressure-driven micro-channel flow find very useful applications in problems of integrated cooling of electronic circuits and superconducting magnets, in cryo-coolers for infra-red detectors and diode lasers, in high-frequency fluidic control elements, etc. The results obtained in [1], [2], [3] and [4] show how the effect of viscosity extended over the whole cross-section of the channel may dramatically alter the flow characteristics of a pressure-driven flow, in comparison with more conventional high Reynolds number flows. This applies not only for non-isothermal flows, but also, and a little bit surprisingly, to low Mach number isothermal flows too, which are usually (and erroneously!) treated in engineering applications as incompressible ones. Shear-driven flows in micro-channels occur in externally pressurized thrust bearings and micro-motors. Thus, both pressure-driven and shear-driven micro-channel flows represent an important constituent part of what is now called Micro- Electro - Mechanical - Systems (MEMS) technology (see two excellent review papers on this theme by Beskok et al. [5], and by Ho and Tai [6]). The results obtained in [7] show that gas injection into the slider bearing through a porous wall, even at small rates, considerably improves the load characteristics of the bearing, so that this effect can be very usefully employed in the design of these slider bearings.

Received: August 2002, accepted: November 2002.
Correspondence to:
Cvetko Crnojević, Faculty of Mechanical Engineering,
27. marta 80, 11000 Belgrade, Yugoslavia
E-mail: Crnojevic@alfa.mas.bg.ac.yu

In this paper we treat the classical problem of the shear-driven gas flow in a slider bearing, with the addition of the effect of gas injection/suction through the bearing pad. The flow is supposed to be a low Mach number, isothermal flow, and for simplicity the gas injection/suction through the pad is supposed to be continuous. At that we treat two possible cases of wall porosity, which affect the obtained results via the boundary conditions on the pad. In the first case a forced injection/suction at a constants rate is due on the pad. In the second one we suppose that the bearing pad is made of a porous material, such as sintered metal, in which the flow is subjected to the well known Darcy's law. The flow in the pad is also compressible, slow and isothermal. It is affected by the difference between the outer pressure which is assumed constant and the variable pressure inside the slider bearing, and by the pad geometry. We also consider injection/suction of the gas through a series of narrow slits in the pad, and show the existence of a full analogy between this case and the previous one, provided the friction factor for the flow through slits is inversely proportional to the local value of the Reynolds number, so that an equivalent value of the permeability coefficient can be found. Our aim is to study the effect of various kinds of the gas injection/suction through the pad upon the flow characteristics, in particular upon the pressure distribution inside the bearing.

We show that gas injection into the bearing, even with the rates much smaller than the speed of the runner, greatly improves the performance of the bearing.

2. Problem statement and the derivation of the pressure governing equation

We consider the problem depicted in Fig. 1 in which the injection/suction of the gas through the bearing pad is allowed in order to improve the performance of the bearing. The flow in the bearing will be supposed to be a steady, 2-D, isothermal, compressible flow of a perfect gas. As well known, isothermal gas flow cannot be consistent with the full system of governing equations, which includes the energy equation too. However, several gas flows in

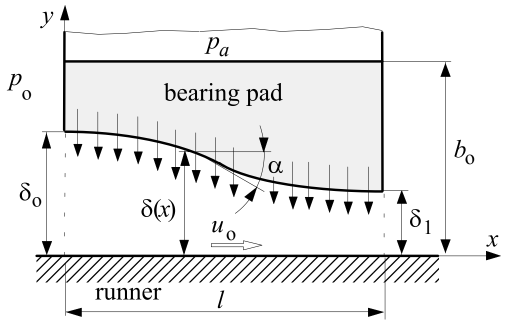

Figure 1. Slider bearing with gas injection/suction through the bearing pad.

techniques proceed with very small temperature variations, so that they can be treated as nearly isothermal. In such a case the energy equation is uncoupled from the others and serves only for the determination of heat, which is spontaneously exchanged with the environment in this case. Thus, the system of equations governing the flow in the problem considered will consist of the equation of continuity, the momentum equations in x and y direction (s. Fig. 1), and the equation of state. They will be written in nondimensional form by using the following scales (s. Fig. 1) : δ_0 for all lengths, speed of the runner u_0 for all velocities, and pressure and density at the entrance into the bearing, p_0 and ρ_0, respectively, for pressure and density.

In order to simplify this system of equations, even before we write then down, we will now make the following assumption, which can be always accepted in the theory of lubrication. Let the maximum angle of inclination of the pad contour toward the x-axis, α_{max} (s. Fig. 1), be small enough, so that it can serve as a small parameter ε : $\alpha_{max} = \varepsilon$. In this case the local thickness of the gas film $\delta(x)$ will be a slowly varying function of x, and all physical quantities, like both velocity components, pressure and density will be also slowly varying functions of x. To make these slow variations explicit, we will introduce the following slow coordinate $\xi = \varepsilon x$, instead of x. Also, since the inclination of the pad contour actually determines the ratio between velocity components u and v in x and y direction, respectively, v will be much less then u throughout the bearing, so that we can write: $v(x, y) = \varepsilon V(\xi, y)$, where $V(\xi, y)$ is an order one transverse velocity component. Further, we will assume that $\gamma M_0^2 / \mathrm{Re} = \lambda\varepsilon$, $\lambda = O(1)$, where γ is the ratio of specific heats, M_0 is the reference Mach number defined as: $M_0 = u_0 / \sqrt{\gamma\, p_0 / \rho_0}$, and Re is the reference Reynolds number: $\mathrm{Re} = \rho_0 u_0 \delta_0 / \mu$ (μ is constant viscosity).

Simplified governing equations in nondimentional form will now read (some of denotations used for dimensional quantities in Fig. 1 are retained for simplicity!):

- continuity equation in which equation of state for isothermal flow in the form: $p = \rho$ is used,

$$\frac{\partial(pu)}{\partial\xi} + \frac{\partial(pV)}{\partial y} = 0 \ , \tag{1}$$

- momentum equation in x-direction,

$$\gamma\, M_0^2\, p\,(u\frac{\partial u}{\partial\xi} + V\frac{\partial u}{\partial y}) = -\frac{\partial p}{\partial\xi} + \lambda\frac{\partial^2 u}{\partial y^2} + O(\varepsilon^2)\,, \tag{2}$$

- momentum equation in y-direction,

$$\frac{\partial p}{\partial y} = O(\varepsilon^2) \ . \tag{3}$$

Obviously, for high subsonic and supersonic flow inertia term in (2) is of the same order of magnitude as the dominant viscous term, and the problem is one of boundary layer type. However, for low subsonic Mach numbers inertia term can be neglected, and the flow is viscosity dominated. This case is particularly simple because equation (2), taking into account (3), can be easily integrated. Employment of boundary conditions: for $y = 0$, $u = 1$, and for $y = \delta(\xi)$, $u = 0$, then yields:

$$u = 1 - (1 + \frac{p'\delta^2}{2\lambda})\frac{y}{\delta} + \frac{p'\delta^2}{2\lambda}\frac{y^2}{\delta^2} \quad , \tag{4}$$

where $p' = \mathrm{d}p / \mathrm{d}\xi$ is gradient of the pressure.

Since we are primarily interested in the derivation of an equation for the pressure distribution inside the bearing, we will now circumvent the determination of V from (1). We will simply integrate (1) in y from 0 to $\delta(\xi)$, apply the boundary conditions: $y = 0$, $V = 0$, and for $y = \delta(\xi)$, $V = V_0(\xi)$, and Leibnitz's formula to get:

$$\frac{\mathrm{d}}{\mathrm{d}\xi}\int_0^{\delta} p\, u\, \mathrm{d}y = -p\, V_0(\xi) \ .$$

Finally, utilizing (4) the following equation governing the pressure is obtained:

$$\delta^3 p\, p'' + \delta^3 p'^2 + 3\delta(\delta\delta' p - 2\lambda)\, p' - 6\lambda(\delta' + 2V_0)\, p = 0 \ ,$$

with boundary conditions: for $\xi = 0$, $p = 1$, and for $\xi = L$, $p = 1$, where $L = \varepsilon\, l / \delta_0$ (s. Fig. 1). Since $v(x, y) = \varepsilon V(\xi, y)$, as stated earlier, note that the injection/suction velocity must be much smaller then the runner velocity in order for this theory to be applicable. For convenience of numerical integration of this equation we will introduce $X = \xi / L$ instead of ξ, and get:

$$\begin{aligned} &\delta^3 p\,\frac{\mathrm{d}^2 p}{\mathrm{d}X^2} + \delta^3\,\frac{\mathrm{d}p}{\mathrm{d}X} + 3\delta(\delta\,\frac{\mathrm{d}\delta}{\mathrm{d}X}\,p - 2\lambda L)\,\frac{\mathrm{d}p}{\mathrm{d}X} - \\ &- 6\lambda L\,[\,\frac{\mathrm{d}\delta}{\mathrm{d}X} + 2LV_0(X)\,]\,p = 0 \end{aligned} \tag{5}$$

with boundary conditions: for $X=0$ and $X=1$, $p=1$. Two qualitative conclusions can now be drawn from (5).

a) Even if $\delta=1$ (Couette - like flow) some non-trivial pressure distribution inside the slider bearing can be induced by injection/suction of the fluid. At that, if $V_0(X)>0$ the pressure curve is concave at the point of pressure extremum, indicating that $p<1$ inside the bearing. If $V_0(X)<0$ the pressure curve is convex at the point of pressure extremum, indicating that $p>1$ inside the bearing, so that Couette-like flow with injection can still be used for lubricating purposes. Since for $\delta=1$, $\alpha_{\max}=0$, the definition of small parameter ε should be changed. It can be redefined to be: $\varepsilon=\gamma M_0^2/\mathrm{Re}$, i.e. by chosing $\lambda=1$. For $\lambda=1$, $L=\varepsilon l/\delta_0=\mu u_0 l/p_0\delta_0^2$ and plays the role of the bearing number Λ, s. [8] ($\Lambda=6L$) in this problem.

b) If the injection/suction velocity distribution $V_0(X)$ is chosen to be: $V_0(X)=-\dfrac{1}{2L}\dfrac{\mathrm{d}\delta}{\mathrm{d}X}$, the last term in (5) disappears, and pressure extremum and inflexion points overlap, which is not feasible if both of boundary conditions for pressure have to be satisfied - the only solution of (5) for such an injection/suction velocity distribution being the trivial one: $p=1$. In what follows we will call this velocity the critical injection/suction velocity.

3. Injection/suction velocity distribution

As announced in the Introduction we will consider two possible cases of the gas injection/suction through the bearing pad, referred to in what follows as Case A and Case B.

In Case A we assume the velocity $V_0(X)$ can be chosen at will, without going into the problem of feasibility of such a velocity distribution. For convenience, in our numerical examples we will take $V_0(X)=const.$

In Case B we assume the pad is made of a porous material with given permeability coefficient α. The nondimensional pressure outside the pad is $p_a=const.$ so that the flow through the pad proceeds under the pressure difference $|p_a-p(x)|$. The flow is supposed to be, lake inside the bearing, steady, 2-D, isothermal, compressible flow of a perfect gas, and is subjected to Darcy's law. Equations governing such a flow, if written in nondimensioal form by using the same scales, used already in the normalization of equations governing the flow inside the bearing, read:

$$\frac{\partial\tilde{p}}{\partial x}=-k\tilde{u}\ ,\ \frac{\partial\tilde{p}}{\partial y}=-k\tilde{v}\ ,\ \frac{\partial(\tilde{p}\tilde{u})}{\partial x}+\frac{\partial(\tilde{p}\tilde{v})}{\partial y}=0\ . \quad (6)$$

where $\tilde{p}$, $\tilde{u}$ and $\tilde{v}$ are pressure and velocity components in the bearing pad respectively, and

$$k=\frac{\delta_0^2}{\alpha}\frac{\gamma M_0^2}{\mathrm{Re}}=\frac{\delta_0^2}{\alpha}\lambda\varepsilon\ ,$$

where α is the permeability coefficient of the bearing pad.Since both transverse velocity components in the pad and in the bearing must be of the same order of magnitude, $\tilde{v}$ can be presented as $\tilde{v}=\varepsilon\tilde{V}$, $\tilde{V}=O(1)$. Then, from the second of equations (6) it follows that k must be of the order ε^{-1}, so that the order of α/δ_0^2 is ε^2. This determines the order of permeability coefficient for which the theory presented here is valid.

Further, we will introduce the slow coordinate $\xi=\varepsilon x$ instead of x for the same reason as before, and conclude that $\tilde{u}$ is of the order ε^3 in the pad and is much smaller then $\tilde{v}$. Thus, the first order equations governing the flow in the pad are:

$$\frac{\partial\tilde{p}}{\partial y}=-k\tilde{v}\quad\text{and}\quad\frac{\partial(\tilde{p}\tilde{v})}{\partial y}=0\ ,$$

and can be easily solved with the boundary conditions (s. Fig. 1): for $y=b$, $\tilde{p}=p_a$, and $y=\delta(\xi)$, $\tilde{p}=p(\xi)$. The solutions are:

$$\tilde{p}^2=\frac{p_a^2-p^2}{b-\delta}y+\frac{b\,p^2-\delta\,p_a^2}{b-\delta}\ ,\quad \tilde{v}=\beta\frac{p^2-p_a^2}{(b-\delta)\tilde{p}}\ ,$$

where β is an $O(1)$ coefficient. From here, for $y=\delta(\xi)$ we finally get the injection/suction velocity $V_0(\xi)$:

$$V_0(\xi)=\tilde{V}(\xi,\delta(\xi))=\beta\frac{p^2-p_a^2}{(b-\delta)p}\ , \quad (7)$$

to be used in the integration of equation (5).

Practically, gas injection/suction through the pad can be maintained by a number of narrow slits, perpendicular to x - axis. If the flow in each of them is steady, 1-D, compressible, low Mach number flow, it is well known that the momentum equation for such a flow will be (in dimensional form):

$$\frac{\mathrm{d}\tilde{p}}{\mathrm{d}y}=-\frac{4\tau_w}{d}\ , \quad (8)$$

where d is diameter of the slit, τ_w is the local value of the wall shear stress: $\tau_w=f\,\tilde{\rho}\tilde{v}^2/2$, and f is the friction factor. In laminar, low Mach number flows: $f=C/\mathrm{Re}$, where C is a constant (C=16 for pipes), and $\mathrm{Re}=\tilde{\rho}\tilde{v}d/\mu$ is the local Reynolds number. If written in nondimensional form, equation (8) attains now the form of the second of equation (6), provided $\alpha=d^2/32$, which at the same time yields the estimate $d/\delta_0=O(\varepsilon)$, as a necessary condition for the validity of theory. The same holds for continuity equation for 1-D, isothermal flow, which is as well known: $\tilde{p}\tilde{v}=const.$ Thus, the two problems are fully equivalent, and there is no need to treat injection/suction through slits separately.

4. Numerical results and discussion

We assume to have the simplest pad geometry in the form: $\delta=1-(1-\delta_e)X$, where $\delta_e=\delta_1/\delta_0$ (s. Fig. 1). In this particular case $\varepsilon=\delta_0(1-\delta_e)/l$, so that $L=1-\delta_e$, and cannot be chosen arbitrarily, except in

Couette-like flow in which the definition of ε was changed. The critical velocity is constant in this case and equal to 0.5.

4.1. Case A

We first present the results of numerical calculations of in Case A, when the injection/suction velocity is: $V_0(X) = const.$.

In Fig. 2 we present the results obtained by numerical integration of (5) for a Couette-like flow. It is clearly seen that this flow with gas injection ($V_0 < 0$) can serve for lubricating purposes because large gauge pressures inside the bearing can be generated even with relatively small injection rates.

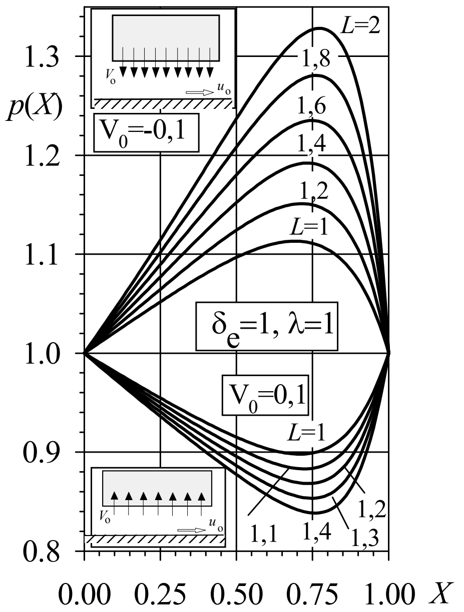

Figure 2. Couette-like flow with injection/suction, as a slider bearing.

In Fig. 3 we present the results obtained by numerical integration of (5) for the classical form of the pad and (a) $\delta_e = 0.5$, and (b) $\delta_e = 0.2$. In both cases the performance of the bearing can be greatly improved by injection of the gas through the pad. This effect is especially pronounced for relatively small exit cross section of the bearing (s. Fig. 3a). With decreasing of δ_e maximums of the pressure distribution are apparently shifted to the right, yielding very large pressure drops near the exit. It is also seen that the gauge pressure inside the bearing is maintained even if gas is withdrawn, up to the critical velocity of 0.5 for which the pressure distribution is uniform $p \equiv 1$.

In Fig. 4 the pressure extremum value is shown as a function of V_o, for $L = 1$ and $\lambda = 1$, and for different values of δ_e. It can be noticed that for a fixed value of V_o the pressure extremum decreases with δ_e, this effect being much more pronounced for the gas injection than for the gas suction.

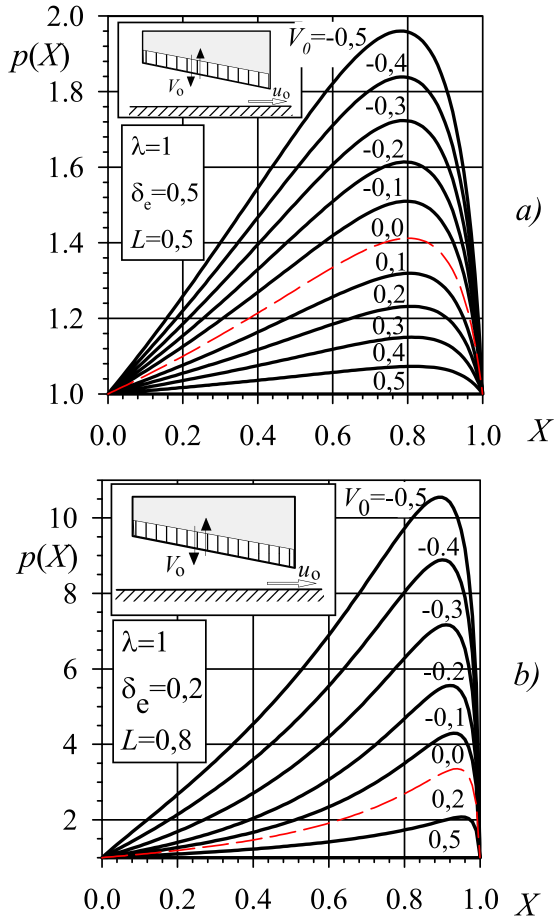

Figure 3. Pressure distribution inside a slider bearing with gas injection/suction, for (a) $\delta_e = 0.5$, and (b) $\delta_e = 0.2$

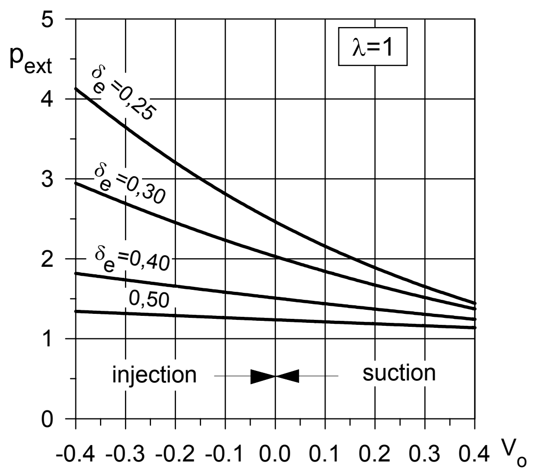

Figure 4. Pressure extremum value in the bearing as a function of V_o.

In Fig. 5 we show the development of the longitudinal velocity component, determined by (4), in a convergent part of the bearing for: $L = 1$, $\lambda = 1$, $\delta_e = 0{,}5$ and $V_o = -0{,}2$. Slow variations of the velocity field in X-direction, as well as some acceleration of the fluid particules, typical of the flow in convergent cannels, are obvious. In the case of gas injection into the slider bearing the care mast be taken about possible combinations of the governing parameters for which a back flow in the bearing may occur, because in such a case the basic assumption concerned with the order of magnitude of the velocity components is violated. Our calculations show that the back flow first occurs in the entrance cross section of channel. In order to circumvent this phenomemon the shear stress on the porous wall in this cross section must be positive, which leads: $p'(0) \le 2\lambda$.

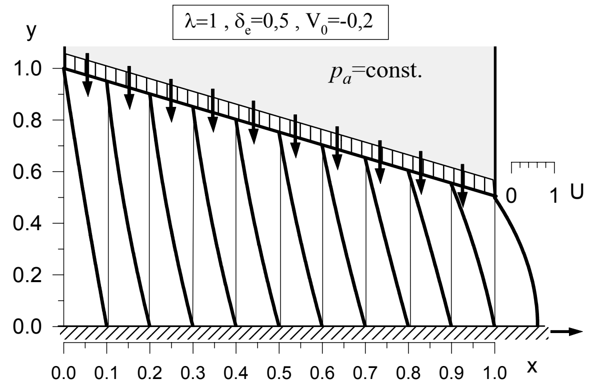

Figure 5. Longitudinal velocity field in the bearing for $\lambda = 1$, $\delta_e = 0{,}5$ **and** $V_o = -0{,}2$.

4.2. Case B

In this case the flow through the porous wall of the bearing is caused by the local pressure difference $p_a - p(x)$. In Fig. 6 and Fig. 7 pressure distribution in the bearing is presented for different values of parameters p_a / p_o, λ and δ_e. We also plot there the referential pressure distribution for $V_o = 0$. Obviously, the pressure distribution in the bearing is severely influenced by the injection/suction through the porous wall, particulary for high values of the parameter p_a / p_o, for which one can expect that gas injection into the bearing takes place. Consequently, the load characteristics of the bearing are considerably improved for these values of p_a / p_o.

In Fig. 8 we show the distribution of the injection/ /suction velocity $V_o(X)$ in the bearing for various values of the governing parameters. Naturally, for small values of p_a / p_o we have suction all over the bearing surface, while for relabively large values of p_a / p_o injection takes place, improving the load characteristics. For intermediate values for p_a / p_o, $V_o(X)$ may change sign in the bearing. For example, for $p_a / p_o = 1.10$ we have injection for $X \in (0; 0{,}30) \wedge \wedge (0{,}94; 1{,}0)$ and suction for $X \in [0{,}30; 0{,}94]$.

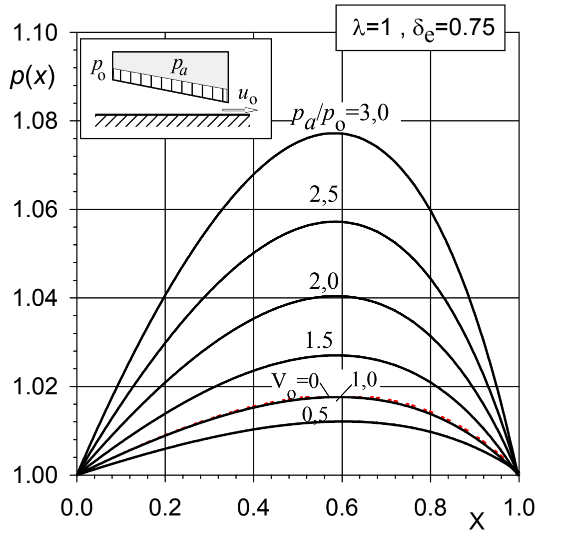

Figure 6. Pressure distribution in the porous bearing for $\delta_e = 0{,}75$

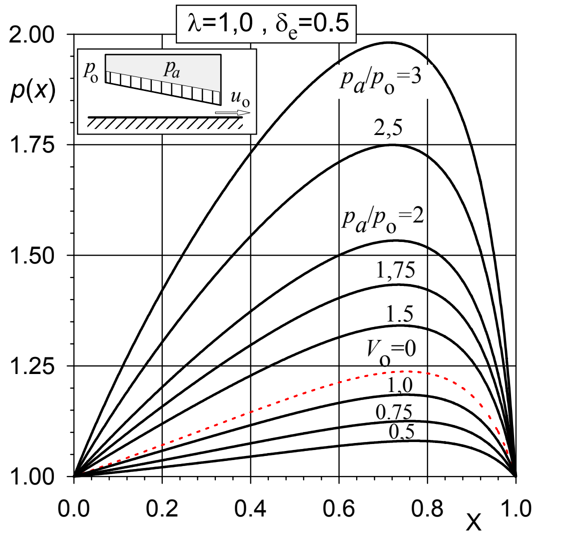

Figure 7. Pressure distribution in the porous bearing for $\delta_e = 0{,}5$.

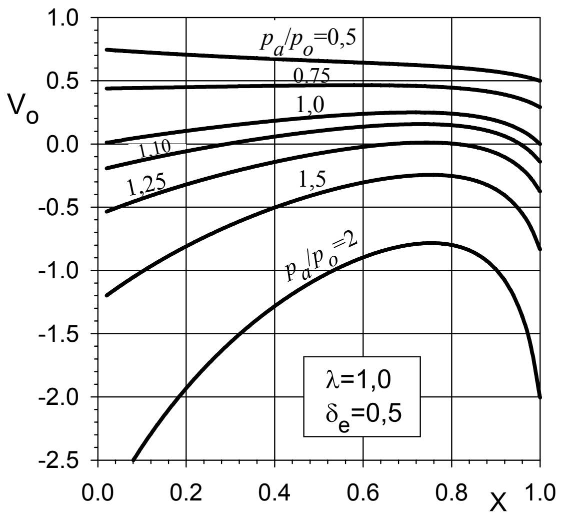

Figure 8. Velocity of injection/suction as a function of p_a / p_o.

5. CONCLUSIONS

The theory presented in this paper can be usefully applied for any form $\delta(X)$ of the bearing, within the frames of validity of the theory. Numerical examples performed for the Couette – like flow and for the case for which $\delta(X)$ us linear function show that the pressure distribution in the bearing with one porous wall is severaly affected by the gas injection/suction through the wall, with the consequence that gas injection can be usefully employed for the improvement of bearing load characteristics. At that, the governing parameters: λ, δ_e, L and p_a / p_o play the key role.

REFERENCES

[1] Schwartz, L. W., 1987: A perturbation solution for compressible viscous channel flows. Journal of Eng. Mathematics 21, pp.69-86.

[2] Crnojevic C. and Djordjevic V.D., 1997: Correlated compressible and incompressible channel flows. Journal of Fluids Engineering, Transactions of the ASME, Vol. 119, pp.911-915.

[3] Shajii, A. and Freidberg, J.P.,1996; Theory of low Mach number compressible flow in a channel. Journal Fluid Mechanics, Vol. 313, pp. 131-145.

[4] Djordjevic V.D. and Crnojevic C., 1999: Friction dominated, subsonic compressible flow in channels of slowly varing cross section. Journal Heat and Fluid Flow, Vol. 20, N° 2, pp. 166-171.

[5] Beskok, A., Karniadakis, G.E. and Trimmer, W., 1996: Rarefaction and compressibility effects in gas microflows. Trans. ASME - J. Fluids Eng, 118, No. 3, 448-456.

[6] Ho, C. M. and Tai, Y.C., 1996, Review: MEMS and its application for flow control. Trans. ASME - J. Fluids Eng., 118, No. 3, 437-447.

[7] Djordjevic V. D. and Crnojevic C., "On the gas lubrication by injection through a permeable wall". Facta Universitatis, Vol. 3, No 11, pp. 29-35, 2001.

[8] Gross, A. W., 1962: *Gas film lubrication*, John Wiley and Sons, New York.

О УТИЦАЈУ УБРИЗГАВАЊА / ИСИСАВАЊА ГАСА КРОЗ ПОРОЗНИ ЗИД НА СТРУЈАЊЕ ГАСА У КЛИЗНОМ ЛЕЖАЈУ

Ц. Црнојевић, В. Д. Ђорђевић

У раду се третира изотермско, стишљиво струјање гаса у клизном лежају при малим вредностима Маховог броја, под претпоставком да се гас убризгава/исисава кроз порозни непокретни зид лежаја. При томе се посматрају два карактеристична случаја: (*а*) принудно убризгавање/исисавање гаса константном брзином и (*б*) спонтано убризгавање/исисавање до којег долази услед разлике притисака са обе стране лежаја. У случају да је спољашњи притисак константан и да је струјање кроз дугачке процепе такође изотермско и споро, показано је да убризгавање гаса у лежај, чак и при малим брзинама, доводи до значајног повећања притиска у њему, повећавајући такође и његову носивост. Исисавање гаса делује на распоред притиска обрнуто. Добијени резултати могу бити корисно употребљени у конструкцији гасних клизних лежајева.