Use of the Elasticity of Net Present Value in Risk Analysis of Engineering Investments Projects

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The hypothesis of our investigation is that for the net present value whereby the profitability of engineering-investment project is measured, it is possible to calculate the coefficient of elasticity, and the same may be used in sensitivity and break-even analysis of project risk. In this way, we come to a new, simplified concept of risk estimation. The coefficient of elasticity of net present value is mathematically defined in relation to variables, such as: investment, net annual returns, salvage value, discount rate, and years of project lasting. The check of received results of exploration has been done on the concrete example of one engineering-investment project.

Keywords: *elasticity, profitability, risk, sensitivity, break-even ability.*

1. INTRODUCTION

It is well-known that for the estimation of profitability of engineering-investment projects dynamic methods such as: net present value, internal rate of return, etc. are used. It is also well-known that for the risk project analysis and estimation - sensitivity, break-even and scenario analysis, Monte Carlo method, decision tree analysis, Hurwitz rule, etc. are most frequently used. Dynamic methods of estimation start from the point that between the moment of investment in engineering-investment project and the moment of realization of effects, based on such projects, there must be an adequate period of time.

These methods, therefore, take into consideration the factor of time during calculation of economic efficiency of investment in a project: that is why there are more efficient than static methods, so they have a dominant role in economic-engineering practice.

Their mutual characteristic is that they take interest as a basic category, whereby they reduce all future values to present worth and that all single investments of very different realization periods reduce to investments of the same realization periods, in order to get the same values and relations. In this way, a real estimation and comparison of engineering-investment projects are possible, as well as making of right decisions relating to development. Net present value and internal rate of returns, as dynamic methods are complementary methods. Taken together they represent a good and reliable indicator of project profitability and that is why they have the priority in relation to other available alternative methods. Their mutual use for estimation of economic efficiency of project is necessary, firstly because internal rate of return eliminates the basic disadvantage of the net present value method, so that a decision maker needn't choose in advance any discount rate, but may take such a rate which is unknown and to be calculated after.

However, it should be considered that examination, treatment of an estimation of economic efficiency of engineering-investment project is done in constant prices and foreseen conditions, valid in the year of its realization. Implicitly, it is supposed that all future events, which may considerably affect the realization of project, are certain. There is no doubt that during the economic period of a project, the conditions and factors of its realization are going to change. That's why we may talk about the risk of project. The presence of risk is in the way that at the time of project imagination -all the events, facts and relations to appear during its realization are still unknown. Hence, we may define the risk as a result of probability of the influence of harmful event in the future and its possible negative consequences. The risk, defined in this way, enables practical control of project risk in efficient and useful way.

It is quite normal that every engineering-investment project is not followed by the same degree of risk. Some of them are less, the others are more risky. Anyway, the risk is bigger when the bigger quantity of capital is invested, when the activity period is longer and vice versa. It is very important to identify the risk and to control it in the right way. That's why the analysis and estimation of risk are studied very carefully. For such purposes the relevant methods have been developed in engineering economic theories. In fact, new scientific discoveries and the use of computers have enabled the application of many mathematical-statistic methods for estimation i.e. measuring the degree of risk. By these methods the subtle measuring of risk is done on the basis of combining different possibilities, which enables to define more precisely the probability of realization of project goals.

Starting from this point, in this study we have especially separated and studied the relation between

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net present value method and sensitivity and break-even analysis for the purpose of project risk estimation. The use of these methods understands the application of complex economic and mathematical statistical analysis. While by the sensitivity analysis the most important variables are identified, which affect most the final result of investment, at the same time by the break-even analysis the effects of variations in results of net present value are studied. At the same time, it has been ascertained that between these two methods it is possible to make direct connection by net present value. The hypothesis of our exploration is that for net present value it is possible to calculate the coefficient of elasticity and to use the same in sensitivity and break-even analysis and in project risk estimation. In this way we have come to a very simplified measuring of project risk degree.

Hence, continued on the following pages of this study, in the order of presentation, there follow:

- calculation of coefficient of elasticity of net present value,
- calculation of coefficient of elasticity of net present value in relation to investment,
- calculation of coefficient of elasticity of net present value in relation to net annual returns,
- calculation of coefficient of elasticity of net present value in relation to salvage value of project,
- calculation of coefficient of elasticity of net present value in relation to discount rate,
- calculation of coefficient of elasticity of net present value in relation to years of project lasting,
- the use of net present value elasticity in sensitivity and break-even analysis,
- illustration of got coefficients of elasticity on the selected concrete example.

2. COEFFICIENT OF ELASTICITY OF NET PRESENT VALUE

The elasticity of net present value may be defined as a relation of relative net present value variations and relative variations of adequate variables, which affects net present worth, such as: revenues (R), operative costs (C), investments (I), salvage value (L), rate of interest (k), number of years of projects exploitation (n), etc. Net present value function of a.m. variables may be expressed as:

$$S = -I + A\frac{(1+k)^n - 1}{k(1+k)^n} + \frac{L}{(1+k)^n}. \qquad (1)$$

S - net present value of project, A -net annual returns of project, i.e. $A = R - C$.

Each of these variables, by changing of its values, affects more or less the variation of net present value. If we mark the variable to change with x, then $S = f(x)$. Let's say that the function is limited and endless in a certain interval. Let's mark with Δx the increasing certain variable, and with ΔS the increasing of net present worth in the same interval, then $\Delta x / x$ is the relative change of a considered variable, and $\Delta S / S$ is the relative change of net present worth. The elasticity of net present worth is then defined as an edge value of quotient of net present value relative changes and adequate relative changes of a considered variable (x) when their increasing inclines to zero:

$$E_{(x)} = \pm \lim_{\Delta x \to 0} \frac{\frac{\Delta S}{S}}{\frac{\Delta x}{x}} = \pm \frac{x}{S} \lim_{\Delta x \to 0} \frac{\Delta S}{\Delta x} = \pm \frac{x}{S}\frac{\partial S}{\partial x}. \qquad (2)$$

In such a case the mark (+) is taken for those variables, as independent variables for which the function is increasing and vice versa, the mark () is being taken for the variables for which the function is declining.

In further examination of this problem we shall keep to the above-mentioned five variables, which the project net present value depends on most frequently:

1. investment (I),
2. net annual returns (A),
3. salvage value (L),
4. rate of interest (discount) (k),
5. years of project lasting (n).

Now we are going to calculate the elasticity coefficient of net present value for each of these variables, taking such variable as an independent variable, and the other variables as constants.

1. Elasticity coefficient of net present value in relation to investment:

$$E_{(I)} = -\frac{I}{S}\frac{\partial S}{\partial I},$$

(function $S = f(I)$ is declining so the mark (-) is being taken). Because of $\partial S / \partial I = -1$, then

$$E_{(I)} = \frac{I}{S}. \qquad (3)$$

2. Elasticity coefficient of net present value in relation to net annual returns:

$$E_{(A)} = \frac{A}{S}\frac{\partial S}{\partial A},$$

$$\frac{\partial S}{\partial A} = \frac{(1+k)^n - 1}{k(1+k)^n} \Rightarrow E_{(A)} = \frac{A}{S}\frac{(1+k)^n - 1}{k(1+k)^n}. \qquad (4)$$

3. Elasticity coefficient of net present value in relation to salvage value:

$$E_{(L)} = \frac{L}{S}\frac{\partial S}{\partial L},$$

$$\frac{\partial S}{\partial L} = \frac{1}{(1+k)^n} \Rightarrow E_{(L)} = \frac{L}{S(1+k)^n}. \qquad (5)$$

4. Elasticity coefficient of net present value in relation to discount rate:

$$E_{(k)} = -\frac{k}{S}\frac{\partial S}{\partial k},$$

$$\frac{\partial S}{\partial k} = A\left[\frac{n}{k(1+k)^{n+1}} + \frac{1}{k^2(1+k)^n} - \frac{1}{k^2}\right] - L\frac{n}{(1+k)^{n+1}},$$

$$E_{(k)} = \frac{L \cdot k}{S}\frac{n}{(1+k)^{n+1}} - \frac{A \cdot k}{S}\left[\frac{n}{k(1+k)^{n+1}} + \right.$$

$$+\frac{1}{k^2(1+k)^n}-\frac{1}{k^2}\Bigg]. \quad (6)$$

5. Elasticity coefficient of net present value in relation to years of project lasting:

$$E_{(n)}=\frac{n}{S}\frac{\partial S}{\partial n},$$

$$\frac{\partial S}{\partial n}=\left(\frac{A}{k}-L\right)\frac{\ln(1+k)}{(1+k)^n} \Rightarrow$$

$$E_{(n)}=\frac{n(A-L\cdot k)}{S\cdot k}\cdot\frac{\ln(1+k)}{(1+k)^n}. \quad (7)$$

3. USE OF $E(x)$ IN SENSITIVITY AND BREAK-EVEN ANALYSIS

Sensitivity and break-even analysis are the two most frequently used methods in defining risk in engineering economy. Sensitivity analysis defines the influence of variables, such as: revenues, costs, salvage value, etc. which are used in estimation of cash flows on net present value of the project. It discovers how much the net present value will change, depending on such variables changing.

Sensitivity analysis starts from the "basic case" got by using the most probable values for each variable. A certain variable is changed for a certain percentage above and below the most probable value, keeping the other variables constant. Afterwards, the net present value is calculated for each of these values. For the sake of better view, it is desirable for all the results to be written tabular. In addition, a favorable and useful way to express the results of sensitivity analysis is graphic description. The incline of the line represents how much the net present worth is sensitive to the changing of each variable - the bigger the incline, the more sensitive the net present value is to the changing of certain variable. In this way we can identify the most important variables, i.e. the elements of cash flow that affect most the final results of investment.

When doing sensitivity analysis of the project, it is questionable what the final effect will really be, for example, of lower incomes and higher costs. The answer to the question is how much the sale of product within the concrete investment project may fall below the foreseen before the project starts making loss, for is calculated by break-even analysis. In other words, break-even analysis is a technique for the study of variations effects on the result of the net present value of project.

The procedure of break-even analysis starts by taking one of the variables (revenues, costs, i.e. net returns, investment, salvage value, etc.) as an unknown variable (x). Then the present value of all incoming cash flows of projects (inflows) is represented as one function of the unknown variable - $S_u = f_1(x)$, while the present value of all out coming cash flows (outflows) as other function of x - $S_i = f_2(x)$. Net present value is the difference between these two functions, and break-even value of x is got when the net present value equals zero, i.e. from the equation $f_1(x) = f_2(x)$.

Instead of this "classical" analysis of project risk by these two methods, in this study we have tried to apply the elasticity of net present value for the sake of faster and simpler getting of the results of such an analysis. By the equations (3) - (7) we calculate the elasticity coefficient of net present value in relation to certain variables. Thus calculated coefficient values afterwards are ordered according to their size and in this way we get immediately the view of sensitivity of net present value in relation to these variables. In fact, the bigger coefficient value, the more sensitive net present value is to the changing of variable represented by such coefficient.

The received elasticity coefficient value for a certain variable represents, in fact, the percentage of net present worth change, when that variable changes by 1%. Knowing that, it is easy to calculate the new net present value, as a result of value change of that variable, by multiplying of its starting value by the number, which is being got as a sum or difference (depending on variable, as explained before) of 100% and elasticity coefficient, multiplied previously by the percentage of variable change:

$$S_1 = S(100\% \pm E_{(x)}p), \quad (8)$$

where "p" is the percentage of considered variable change, S_1 is the new net present value that arose as the consequence of considered variable change by "p" percentage.

For example, let the considered variable be investment (I) and let's say that with its value of $100\,000\,\$$ \$ we got net present value $S = 50\,000\,\$$. If the calculated elasticity coefficient $E_{(I)}$, for example, is 3,4 that means that with changing of investment value by 1% net present value will change by 3,4%. However, since during the analysis we are most frequently interested in variable changes of $\pm 5\%$, $\pm 10\%$, $\pm 15\%$ etc., we will come to the new net present value putting in equation (8) for "p" values $\pm 5\%$, $\pm 10\%$, $\pm 15\%$ etc. In this concrete case if we change the value of investment by 5% ($105\,000$) then the new net present value will be: $S_1 = 50{,}000(100\% - 3{,}4 \cdot 5\%) = 41500\ \$$.

For the break-even analysis, instead of making new equations and new tables, using the explained method, we come very fast to a solution, i.e. to break-even value of considered variables in the following way: in equation (8) S_1 equals zero and look for break-even value of percentage change of a considered variable p_k, and we get the break-even value of variable x_k by multipling its starting value by ($100\% + p_k$):

$$0 = S(100\% \pm E_{(x)}p_k) \Rightarrow p_k = \mp\frac{100\%}{E_{(x)}}, \quad (9)$$

$$x_k = x(100\% + p_k). \quad (10)$$

In our example break-even value of investment would be:

$$p_k = \frac{100\%}{3{,}4} = 25{,}71\% \Rightarrow$$

$$I_k = 100\,000 \cdot 1{,}2571 = 125\,710\,\$ \,.$$

4. EXAMPLE

As an illustration of all the above said, we are going to present the use of net present value elasticity in sensitive and break-even analysis on one concrete example: on the basis of given data about one engineering investment project, it is necessary to examine the project net present value sensitivity to the affecting elements of cash flows and to ascertain the break-even value of the most affecting variable, i.e. of the variable to whose changes net present value is the most sensitive.

Based on the overall analysis, the following values of money flows elements have been estimated: the investment in a project would be $8\,000\,\$$ what would give net annual returns of $3\,000\,\$$ in the next 5 years. The salvage project value after 5 years of exploitation has been estimated to $0\,\$$, and the discount rate for the following period to 15% annually.

The first step is to calculate the net present value of the project for given variables. By using equations (1) we get:

$$S = -I + A\frac{(1+k)^n - 1}{k(1+k)^n} + \frac{L}{(1+k)^n} =$$

$$= -8\,000 + 3\,000\frac{(1+0{,}15)^5 - 1}{0{,}15(1+0{,}15)^5} = 2\,056\,\$ \,.$$

The next step is to calculate the net present value elasticity coefficient in relation to four given variables. By using equations (3) to (7), we easily come to the coefficient value. For example, herewith we are going to present only the calculation of net present value coefficient in relation to net annual returns:

$$E_{(A)} = \frac{A}{S}\frac{(1+k)^n - 1}{k(1+k)^n} = \frac{3\,000}{2\,056} \cdot \frac{(1{,}15)^5 - 1}{0{,}15(1{,}15)^5} = 4{,}89 \,.$$

The other elasticity coefficients are as follows:

$$E_{(I)} = 3{,}89 \,,$$

$$E_{(n)} = 3{,}38 \,,$$

$$E_{(k)} = 1{,}74 \,.$$

On the basis of these results, we get quite clear view of net present value sensitivity to the changes of each cash flows elements of the project. It is quite evident that the variable, whose variations affect most the net present value, is the net annual returns of the project. So, for example, the variation of net annual returns by $\pm 15\%$ would result in variation of net present value by $\pm 73{,}35\%$ ($4{,}89 \cdot 15\%$). In other words if, during project exploitation, the estimated returns of 3 000$ annually don't realize, but the same are reduced by 15% amounting to $2\,550\,\$$ annually, due to unforeseen unfavorable circumstances, then the net present value will "fall" to $\$\,548\,\$$ [$S_1 = S(100\% \pm E_{(x)}p) = 2\,056(100\% - 4{,}89 \cdot 15\%) = 548\,\$$].

Net present value is a bit less sensitive to the variations of investment values and number of years of project exploitation, while it is least sensitive to the possible variations of discount rate values.

One more fact to be established is what is the break-even value of the most affecting variable, i.e. net annul returns, to the net present value, below which the project would produce losses. By using the equations (9) and (10) we come to:

$$p_k = \mp\frac{100\%}{E_{(A)}} = -\frac{100\%}{4{,}89} = -20{,}45\% \,,$$

$$A_k = A[100\% + (-20{,}45\%)] = 2\,387 \;\; \$.$$

That means that any net annul returns of a project being below $2\,387\,\$$ would produce losses; that is why such a project would not be acceptable.

5. CONCLUSION

The results of these explorations, we have come to in this study, undoubtedly indicate that the net present value elasticity may be used in the analysis and estimation of risk in engineering investment projects. By the calculated elasticity coefficients, a direct connection is made between net present value and sensitivity and break-even analysis method during risk estimation. In this way, we come to a new, very simplified procedure for risk estimation i.e. measuring. It is enough on the basis of given equations, to calculate the values of elasticity coefficient, then to order them according to their size and in this way come to fact of net present value sensitivity in relation to variables, such as: investment, net annual returns, salvage value of project, discount rate and the years of project lasting. A higher value of elasticity coefficient indicates that the net present value is more sensitive to the variation of variable, represented by such a coefficient. Based on these facts, it is easy to calculate a new net present value by the following equation:

$$S_1 = S(100\% \pm E_{(x)}p) \,.$$

Furthermore, it should be pointed out, that in break-even analysis too, instead of setting new equations and making new tables by using net present value elasticity, we come very fast to the appropriate solution, i.e. to break-even value of a considered variable, which has been explicitly proved by equations 9 and 10.

Finally, on the basis of the data of one engineering investment project the validity of presented, exploring results has been checked. Thus, by this example on the concrete project it has been presented that the use of net present value elasticity in sensitivity and break-even analysis has a great analytic value. All a.m. facts indicate that in the adequate engineering investment analysis, besides function elasticity, production elasticity, elasticity of costs, revenues, supply and demand we may also use the net present value elasticity.

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ПРИМЕНА ЕЛАСТИЧНОСТИ НЕТО САДАШЊЕ ВРЕДНОСТИ У АНАЛИЗИ РИЗИКА ИНЖЕЊЕРСКИХ ИНВЕСТИЦИОНИХ ПРОЈЕКАТА

Драган Љ. Милановић, Радојица Дубоњић

Претпоставка нашег истраживања је да се за нето садашњу вредност, помоћу које се мери профитабилност инжењерско - инвестиционог пројекта, може израчунати коефицијенат еластичности и исти користити у сензитивној и критичној анализи ризика пројекта. Тиме се долази до новог, поједностављеног концепта оцене ризика. Коефицијенат еластичности нето садашње вредности математички је одређен у односу на променљиве, као што су: инвестиционо улагање, нето годишње зараде, ликвидациона вредност пројекта, дисконтна стопа и године трајања пројекта. Провера добијених резултата истраживања извршена је на конкретном примеру једног инжењерског инвестиционог пројекта.