Specific Sliding of Trochoidal Gearing Profile in the Gerotor Pumps

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The present paper considers the kinematic parameters of the trochoidal gearing in gerotor pumps. The equations for the rolling and sliding velocity at the contact point of the meshing gear profiles were first determined and then the equations which define specific sliding of the trochoidal profile were derived. After that, the conditions for the phenomenon of the singular points of the specific sliding distribution were analyzed. Based on the graphic interpretation of kinematic parameters of trochoidal gear pair profiles in contact, conclusions can be drawn about the influence of geometrical parameters on the sliding size, and accordingly on the wear intensity of the tooth profiles too.

Keywords: *trochoidal gearing, sliding velocity, specific sliding.*

1. INTRODUCTION

The gerotor pumps belong to the group of the planetary rotating machines whose kinematics is based on the principle of planetary mechanism with internal gearing. The number of teeth of external gear is always higher by one than the teeth number of internal gear. In this kind of gearing, a moving circle is rolling without sliding along a fixed circle, with the chosen point drawing the tooth profile – trochoid [1]. A fixed circle is, taken conditionally, a pitch circle of gear. The meshing profile can be represented as the envelope of successive positions of the basic profile at its relative moving. Accordingly, the meshing profiles are presenting the envelope curves, whose geometry is in accordance with the fundamental law of gearing. In the general case, the meshing envelope has the cusps which are the unwanted phenomenon because they cause intensive wear, and to avoid that phenomenon, modification of the basic trochoid is introduced. Trochoidal curves are modified by the increase of the constant value of r_c, which is drifting along the normal of the given curve. In that way the curve is obtained that is parallel to the given curve, and because their distance, measured along the common normal, remains constant the obtained curve is also called equidistant, and constant increase of r_c can be defined as the radius of equidistant.

On the basis of geometrical and kinematic models, developed in papers [2], [3], [4], the formulae for calculation of specific sliding and its distribution on the trochoidal gearing profile will be defined in the present paper. To this aim, the following coordinate systems are introduced: generating, attached to generating point, coordinate system of trochoide, coordinate system of envelope, and fixed coordinate system. The moving of the profile point is observed in relation to different coordinate systems and with the application of coordinates transformation it is possible to realize the most simple equations in matrix form. For kinematical analysis of two profiles in contact, the moving of the contact point of these profiles is considered.

2. GEOMETRICAL AND KINEMATIC RELATIONS OF TROCHOIDAL GEARING

The basic geometrical relations by generating unmodified and modified epitrochoid are illustrated in the Fig.1. The epitrochoid is generated by a point D on the plane attached outside to a circle of radius r_a which is rolling on the outside of an enclosed circle of radius r_t. According to Fig. 1, the equations of the epitrochoid are defined in the coordinate system of the epitrochoid $O_t\, x_t\, y_t$. In Fig 1. it is shown that during the relative moving of pitch circles, when the point D is generating epitrochoid, the point P is generating equidistant.

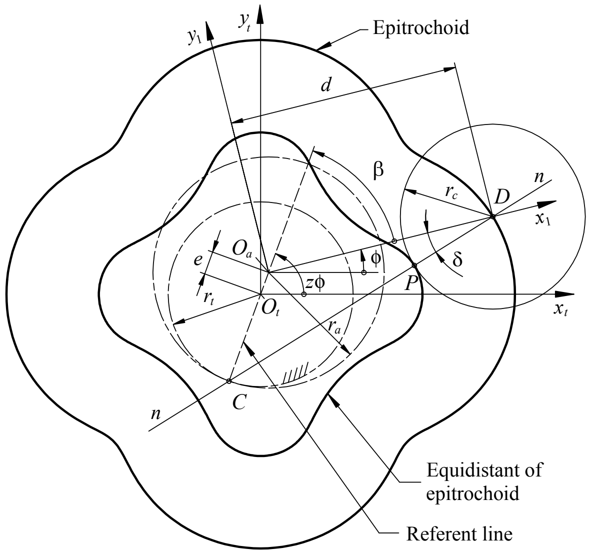

Figure 1: Generating of unmodified and modified epitrochoid

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The angle signified as δ is an angle between the normal *n-n* and radius vector of the point *D*, and can be defined as leaning angle [5]. Coordinates of the contact point *P* in the coordinate system of epitrochoid can be written as:

$$\vec{r}_t = \begin{bmatrix} x_t \\ y_t \\ 1 \end{bmatrix} = \begin{bmatrix} e\left(\cos z\phi + \lambda z \cos\phi\right) - r_c \cos\left(\phi + \delta\right) \\ e\left(\sin z\phi + \lambda z \sin\phi\right) - r_c \sin\left(\phi + \delta\right) \\ 1 \end{bmatrix}, \quad (1)$$

where λ is coefficient of trochoid by means of which relations are defined between the values of the trochoid radius and the radius of moving circle and $\lambda = d/ez$, [6].

Based on geometrical relations from the Fig. 1, the formula for determination of angle δ can be obtained:

$$\delta = \arctan \frac{\sin\left(z-1\right)\ \phi}{\lambda + \cos\left(z-1\right)\ \phi}. \quad (2)$$

For kinematic analysis of the meshing profiles, the moving of the point P_t on the profile of the internal gear and of the point P_a on the profile of the external gear (Fig. 2) is considered.

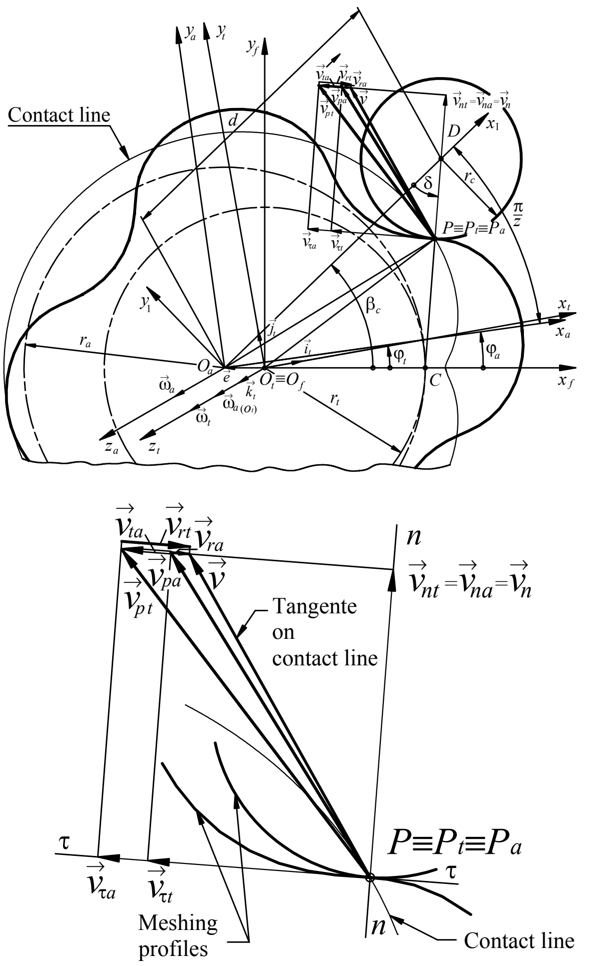


Figure 2: Kinematic parameters of the trochoidal gear pair

During the meshing the profiles of the trochoidal gearing are simultaneously rolling and sliding relative to each other. Rolling of the profile can be presented as its rotation around the contact point, the rolling velocity being equal to the relative velocity of the contact point $\vec{v}_{rt}$ of the internal and $\vec{v}_{ra}$ of the external gear, respectively. Sliding of the profiles at the contact point is a consequence of the difference in the relative velocities intensity of the points on the profiles of the internal and external gear, respectively.

It is known from the gearing theory that only pitch circles can realize rolling without sliding. Based on this, it results that the sliding of profiles is inevitable because they are formed by the curves which are differentiating in relation to pitch circles. In this case the sliding velocity of the meshing profiles at the observed contact point is velocity of the contact point by the relative moving of the profiles.

Fig. 2 shows the disposition of the velocity at the contact point of the two meshing profiles, where [7]:

- $\vec{v}$ is vector of the absolute velocity of the meshing profiles at the contact point *P*;
- $\vec{v}_{pt}$, $\vec{v}_{pa}$ are vectors of the transfer velocities of the contact point P_t, P_a;
- $\vec{v}_{nt}$, $\vec{v}_{na}$ are projections of the transfer velocity on the common normal and $\vec{v}_{\tau t}$, $\vec{v}_{\tau a}$ on the tangent at the contact point P_t, P_a;
- $\vec{v}_{rt}$, $\vec{v}_{ra}$ are vectors of the relative velocities of the contact point P_t, P_a but their intensity is equal:

$$v_{rt} = \left\{ ez\left(1+\lambda^2 + 2\lambda\cos\beta\right)^{\frac{1}{2}} - r_c\left(1+\delta'\right) \right\}\omega_r, \quad (3)$$

$$v_{ra} = r_c \delta' \omega_r, \quad (4)$$

where:

$$\delta' = \frac{d\delta}{d\phi} = \frac{\left(z-1\right)\left[1+\lambda\cos\left(z-1\right)\phi\right]}{1+\lambda^2 + 2\lambda\cos\left(z-1\right)\phi}; \quad (5)$$

- $\omega_r = \omega_t - \omega_a$ is angular frequency of the epitrochoid in relation to the envelope;
- $\vec{v}_{ta}$ is vector of the sliding velocity of the internal gear profiles in relation to external gear profiles;
- $\vec{v}_{at}$ is vector of the sliding velocity of the external gear profiles in relation to internal gear profiles.

Intensity of the sliding velocity at the contact point of the profile is:

$$v_r = \left|\vec{v}_{ta}\right| = \left\{ ez\left(1+\lambda^2 + 2\lambda\cos\beta\right)^{\frac{1}{2}} - r_c \right\}\omega_r. \quad (6)$$

Apart from the sliding velocity, summary rolling velocity is important for the analysis of the phenomenon of wear. Intensity of summary rolling velocity can be written in the form:

$$v_{\Sigma} = \left\{ ez\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{1}{2}} - r_c\left(1+2\delta'\right) \right\}\omega_r . \quad (7)$$

On the basis of realized equations, the formulae for determination of the specific sliding of the meshing profiles at the contact point can be defined.

3. SPECIFIC PROFILE SLIDING

The sliding velocity of meshing profiles during the motion changes; from maximal value on the top of the tooth it decreases abruptly to minimal value at the bottom of the tooth and then increases again. The presence of sliding in the meshing profiles process causes their wear, the sliding velocities defining the wear forces direction and intensity which act on the meshing profiles of the gears. The wear force is in the direction opposite to the relative motion velocity at the contact point. So the direction of the sliding velocity $\vec{v}_{ta}$ is in agreement with the direction of the wear force which acts on the profile of the external gear, and the direction $\vec{v}_{at}$ with the direction of the wear force on the trochoidal profile (Fig. 3).

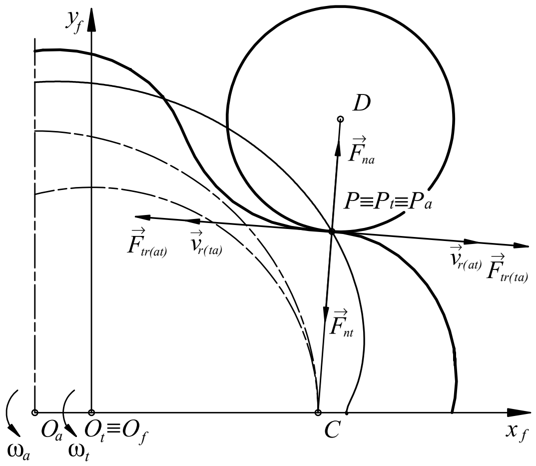


Figure 3: Sliding velocities, wear forces and normal forces in the contact point of the meshing profiles

For the analysis of sliding of the meshing profiles, it is necessary to know, apart from the sliding velocity at the contact point, its distribution in relation to corresponding relative velocity of the contact point. Relation between the sliding velocity and relative velocity of the contact point of the meshing profiles is specific sliding and is given by the formula:

$$\xi_t = \frac{v_{ta}}{v_{rt}} = \frac{v_r}{v_{rt}} , \quad (8)$$

for internal, and for external gear:

$$\xi_a = \frac{v_{at}}{v_{ra}} = \frac{v_r}{v_{ra}} . \quad (9)$$

After the substitution of corresponding formulae for velocities, the following relation is obtained for specific sliding on the gear profile of the internal gear:

$$\xi_t = \frac{z\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{1}{2}} - c}{z\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{1}{2}} - c\left(1+\delta'\right)} \quad (10)$$

and analogically for the external gear:

$$\xi_a = \frac{z\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{1}{2}} - c}{c\delta'} , \quad (11)$$

where:

$$c = \frac{r_c}{e} . \quad (12)$$

At the profile point, where the directions of sliding and relative velocities are in agreement, specific sliding is positive, and where they are not in agreement it is negative.

Based on the formulae (10) and (11), it can be concluded that the values of specific sliding become infinitely great when the values of relative velocities are equal to zero. These points are singular for the distribution of specific sliding of the meshing profiles.

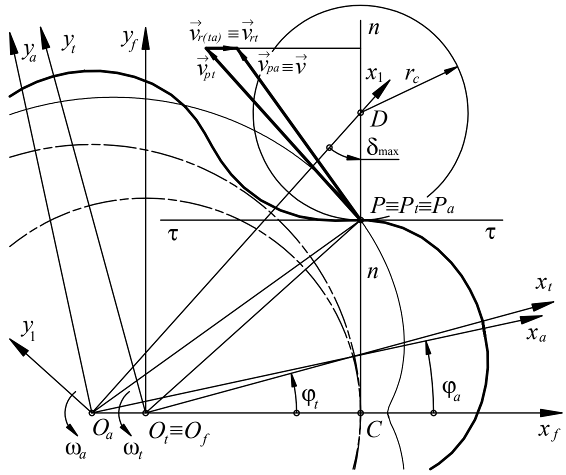


Figure 4: Polygon of velocities in singular contact point

Analysis is first done of the conditions when $\xi_t \to \infty$ and when $v_{rt} = 0$, respectively. Starting from the relation (3) and making it equal to zero we obtain:

$$c = \frac{z\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{3}{2}}}{z+\lambda^2+\lambda\left(z+1\right)\cos\beta} . \quad (13)$$

The obtained equation shows that ξ_t is not defined when the value of equidistant radius is equal to the curve radius of the base epitrochoid [2]. As the value of equidistant radius is chosen to be less than minimum value of curve radius of the epitrochoid, it means that the phenomenon of singularity for ξ_t is excluded. However, to avoid the extremely great values of specific sliding,, as shown in Fig. 5 (a), it is recommendable the choice of the equidistant radius value considerably less than limited.

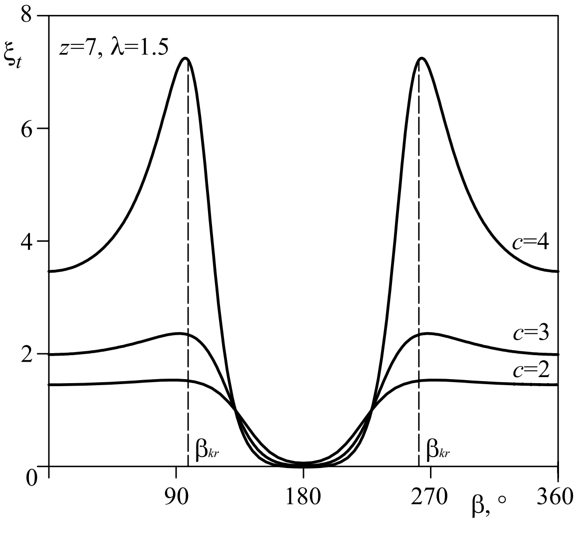
On the profile of external gear there exists the point with infinitely great specific sliding. The position of this singular point is defined by critical angle β_0 , the common normal of the meshing profiles at the contact point being in agreement with the contact line on the kinematic circles (Figure 4.) At this point there occurs the change of sign of relative velocity and $v_{ra} = 0$, therefore, based on relations (4) and (5), the critical value of angle can be determined in reference to:

$$\beta_0 = \arccos\left(-\frac{1}{\lambda}\right), \tag{14}$$

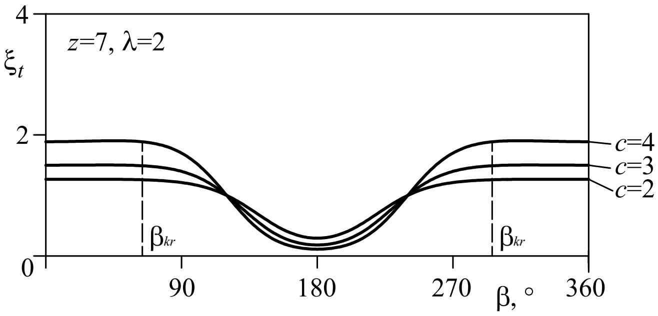
and it is corresponding to the last point of the active part of profile of the meshing envelope, to the point with the maximum of the leaning angle $\delta_{\max}$.

4. MATHEMATICAL MODEL TESTING OF SPECIFIC SLIDING

On the basis of developed mathematical model computer programs were designed in a standard program language Auto LISP for automatic drawing of the specific sliding diagrams. The programs were tested for the chosen values of the input parameters, and the results are given in the diagram form of specific sliding, depending of the referent rotation angle β.



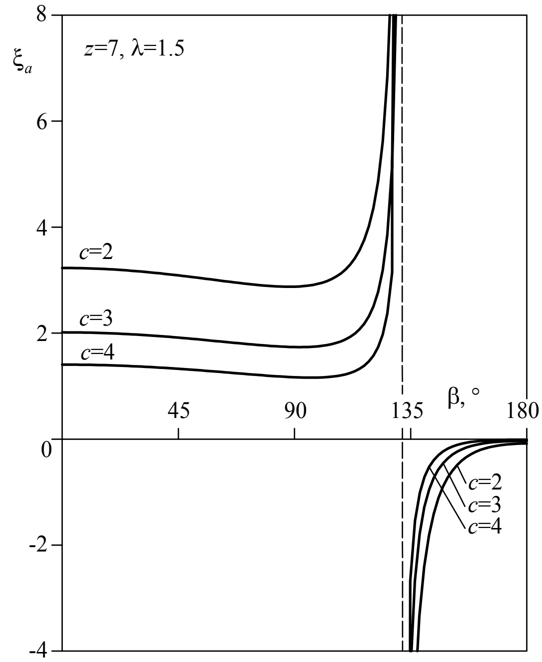
a)



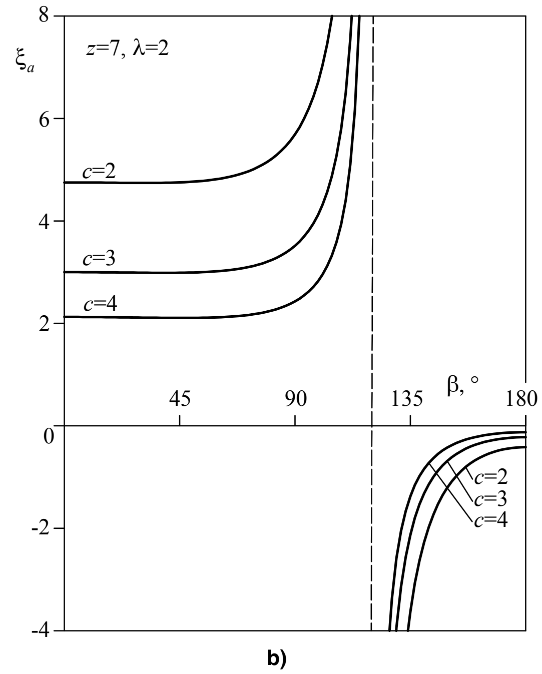
b)

Figure 5: Specific sliding of the contact point of the modified epitrochoidal profile for the tooth number $z = 7$, at the different values of coefficient c and trochoid coefficients: a) $\lambda = 1.5$ and b) $\lambda = 2$

From the Figure 5 it results that on the profile of the internal gear the specific sliding does not have singular points. Also, it can be shown that ξ_t grows abruptly on the convex part of profile and reaches the maximum value around the point with the greatest curve of the profile (β_{kr}). Based on the Figure 5, it comes out that with the same teeth number, increase of the coefficient λ leads to decrease of ξ_t on the convex part of profile, but on the concave part of profile there comes to its insignificant increase. The increase of the values of equidistant radius has the opposite effect.



a)

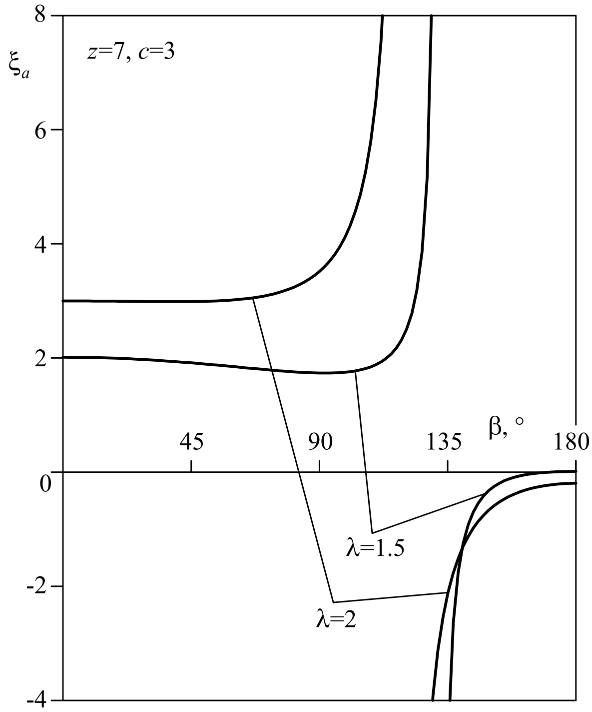


b)

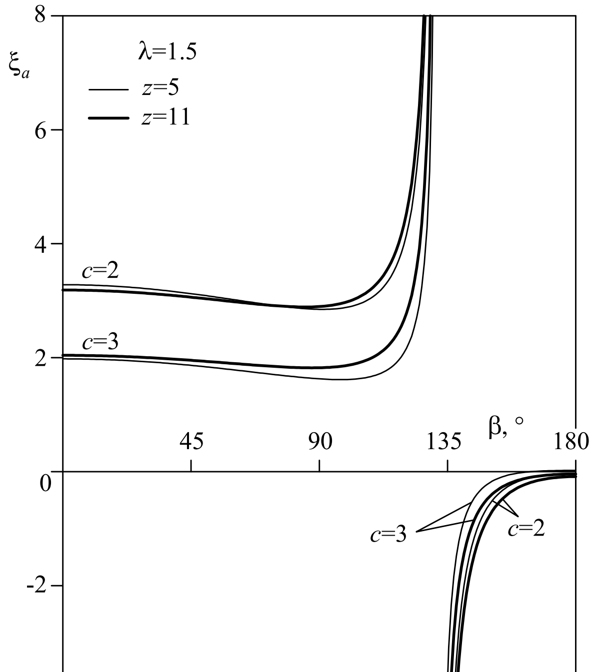
Figure 6: Specific sliding of the contact point of circular profile for the tooth number $z = 7$, at the different values of coefficient c and trochoid coefficients: a) $\lambda = 1.5$ and b) $\lambda = 2$

From the Figure 6 it comes out that with the same teeth number on the top part of the circular profile, increase of the coefficient λ leads to the growth of the value of the equidistant radius and has opposite effect.

From the Figure 7(b) it can be concluded that with the same values of the other parameters, teeth number has not significant influence on the change of values ξ_a.



a)



b)

Figure 7. Specific sliding of the contact point of the circular profile: a) for the teeth number $z = 7$, $c = 3c$ and different values of coefficient λ; b) for the different teeth number, $\lambda = 1.5$ and different values of coefficient c

In general, based on the graphical interpretation of the obtained results, shown in the Figures 5-7, it can be concluded as follows:

- Teeth number has not significant influence on the specific sliding;
- With the increase of coefficient of trochoid λ the specific sliding on trochoidal profile grows, and on the circular profile it falls;
- Increase of the equidistant radius leads to the decrease of ξ_a, and also to the increase of ξ_t, however, on the convex part of profile this influence is not significant.

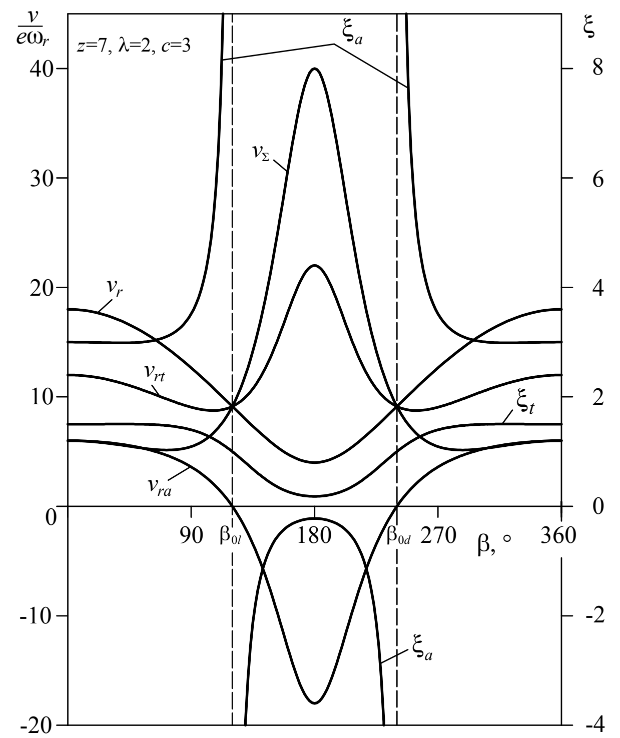


Figure 8: Diagram of kinematic parameters for one of the chosen gear pairs

For one of the chosen gear pairs the parameters of the meshing profile were determined and presented in the Figure 8, on the basis of which it can be concluded as follows:

- Rolling velocity of the contact point of the trochoidal profile v_{rt} has the trend of monotonous decrease to the value corresponding to the critical angle β_0, and then it abruptly increases to the value corresponding to the bottom of the profile;
- Rolling velocity of the contact point of the circular profile v_{ra} is characterized by attaining zero value at the point determined by the critical angle β_0 when it comes to the change of the velocity sign;
- Sliding velocity v_r decreases monotonously from the maximal (on the top) to the minimal value (at the bottom of the profile);
- Summary rolling velocity v_Σ increases abruptly from the point corresponding to angle β_0 ,where it becomes equal to sliding velocity and rolling velocity of the

contact point of trochoidal profile, respectively $v_\Sigma(\beta_0) = v_r(\beta_0) = v_{rt}(\beta_0)$;

- Specific sliding ξ_t has a trend of monotonous increase to the attaining of local extremum (around the point with the minimum radius of the curve profile), and then to the abrupt fall, approaching zero value at the bottom part of the tooth;
- Specific sliding ξ_a decreases monotonously from the value corresponding to the moment of contact of two top profiles to the attaining of local extremum (around the point with the minimum radius of the curve profile), and then asymptotically increases to $+\infty$, when a singular point appears. During further meshing, ξ_a changes the sign, and from $-\infty$ continually increases to the value corresponding to the moment of contact of the top and bottom part of tooth.

5. CONCLUSION

The paper gives a detailed analysis of specific sliding at the contact points of the meshing profiles, as well as the relations for its determination. Specific sliding is one of the more important limiting factors in the choice of the geometrical parameters of the conditions for appearing friction and wear of the contact gear surface. Based on mutual relations of the sliding and rolling velocities values, conclusions can be made about the changing of the friction conditions during the meshing of profiles.

On the profile part for $\beta_{0l} < \beta < \beta_{0d}$ the development of the fatigue pitting is expected. On the other part of the profile the sliding velocity is greater or equal to the summary rolling velocity, so that the decrease of the lubricant layer thickness, increase of temperature in contact and risk of the scoring appearance are expected on this part.

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NOMENCLATURE

$O_a x_1 y_1$	coordinate system attached to the generating point
$O_t\, x_t\, y_t$	coordinate system attached to the internal gear
$O_a\, x_a\, y_a$	coordinate system attached to the external gear
$O_f\, x_f\, y_f$	fixed coordinate system
O_t, O_a	center of the internal gear and the external gear, respectively
C	pitch point
D	generating point
z	teeth number of the external gear
$z-1$	teeth number of the internal gear
e	center distance between the internal and external gear (eccentricity)
r_t	radius of pitch circle of the internal gear $r_t = e(z-1)$
r_a	radius of pitch circle of the external gear $r_a = ez$
r_c	radius of equidistant
c	equidistant coefficient
d	distance joining the generating point D
P_t, P_a	contact point on the profile of the internal gear and the contact point on the profile of the external gear, respectively
v	absolute velocity of the meshing profiles at the contact point
v_{rt}, v_{ra}	relative velocities of the contact point P_t and P_a, respectively
v_{pt}, v_{pa}	transfer velocities of the contact point P_t and P_a, respectively
v_r	sliding velocity

v_Σ	summary rolling velocity
F_n	contact force
F_{tr}	wear force

Greek symbols

ϕ	generating rotation angle
β	referent rotation angle
λ	trochoid coefficient
δ	leaning angle
φ_t	rotation angle of the internal gear about its own axis
φ_a	rotation angle of the external gear about its own axis
ω_t	angular frequency of the internal gear about its own axis
ω_a	angular frequency of the external gear about its own axis
ω_r	relative angular frequency
ξ_t	specific sliding on the tooth profile of the internal gear
ξ_a	specific sliding on the tooth profile of the external gear

СПЕЦИФИЧНО КЛИЗАЊЕ ПРОФИЛА ТРОХОИДНОГ ОЗУБЉЕЊА КОД ГЕРОТОРСКИХ ПУМПИ

Ивановић Лозица, Јосифовић Даница

У раду су разматрани кинематски параметри трохоидног озубљења код героторских пумпи. Најпре су одређени изрази за брзину котрљања и брзину клизања у тачки додира спрегнутих профила, а затим су изведени изрази који дефинишу специфично клизање профила. Анализирани су услови за појаву сингуларних тачака расподеле специфичног клизања. На основу графичке интерпретације кинематских параметара спрегнутих профила трохоидног зупчастог пара могу се извести закључци о утицају геометријских параметара на величину клизања и, према томе, на интензитет хабања профила зубаца.