A New Method of Constructing the Kinetic Fatigue Fracture Diagrams – Crack Propagation Equation Based on Energy Approach

Mieczyslaw Szata
Associate Professor
Wroclaw University of Technology
Faculty of Mechanical Engineering

Grzegorz Lesiuk
Wroclaw University of Technology
Faculty of Mechanical Engineering

The force criterion being a local criterion is frequently applied for the description of a fatigue crack growth. The rate of a crack growth is a function of stress intensity factors. This function based on experimental data has an exponential form in relation to K_I, as it has been shown in Paris' law. The deformation criterion based on the δ_K – model is used in the case when the plastic zone before a crack tip is of the same order as a body dimension. The presented approach is effective in the case of a stable loading cycle without taking into account the "history" of material deformation and crack propagation. The stress intensity factors or the values of a crack opening become then the invariants of a fatigue damage. The energy approach makes possible to take into consideration the elements mentioned earlier. The energy criterion of fatigue crack propagation in the isotropic body has been formulated. Finally, a kinetic equation of fatigue fracture as the analytical relation between fatigue crack surface propagation rate and dissipation energy of plastic deformation in the precracked zone has been obtained. A new method of constructing kinetic fatigue fracture diagrams (KFFD) has been presented on the basis of measurement results of a hysteresis loop area for the isotropic body with an internal flat crack under a cyclic loading. For the experimental verification, the results of fatigue crack propagation studies for 18G2A and 40H steels have been utilized.

Keywords: *fatigue fracture, crack propagation, lifetime of mechanical construction.*

1. INTRODUCTION – FATIGUE FRACTURE

Fatigue of materials is a particularly menacing and dangerous kind of material destruction. For a long time the exploited construction does not indicate any symptoms of approaching materials decohesion. Although the investigations of materials fatigue date from the first half of the XIX century (1837 – W.A.J. Albert "Über Treibseile am Harz") by now any compact hypothesis of fatigue destruction has not been worked out. A problem of materials fatigue is still waiting for solving. Up to now the only well-established standing is a division of fatigue into 3 stages:

- Phase I – the ensemble of cyclic phenomena connected with the change of materials properties (cyclic weakening or hardening, etc), motion of defects on the atomic level without breaking the bonds.
- Phase II – the formation of submicrocracks propagating from the surface layer into the bulk in the area of one or several grains.
- Phase III – it contains the propagation of the main macroscopic crack initially stable (in the literature it is defined as a subcritical one) and then reaching the value of light propagation velocity in a given material.

The contribution of fatigue phases depends on a loading level. The cracks are inevitable consequences of fatigue process as well as inevitable remains after technological, assembly and working processes. Fracture mechanics becomes a very useful tool for estimating the time of crack propagation. This young enough and thunderously developing branch of science goes back to the second decade of the XX century. Its origin is closely connected with the series of damages to the ships "Liberty". During the Second World War 1289 these objects of among about 4700 ones have suffered destruction. The application of a new welding technology was conductive to a rise of cracks. A spectacular example was a half-and-half fracture of a tanker T2 "Schenectady" (Fig. 1), 16.01.1943, standing in the port during a fine weather (air temperature – 3 °C, water + 4°C).

Another well known crashes caused by fatigue fracture are as follows:

- Destruction of gas pipeline in Canada in 1958 at a few miles of its length;
- The crash of the Kings Bridge in Melbourne (1962);
- Destruction of Mianus River Bridge (28.06.1983) in Greenwich (USA) caused by

Received: July 2008, Accepted: September 2008
Correspondence to: Dr Mieczyslaw Szata
Wroclaw University of Technology,
Faculty of Mechanical Engineering,
Smoluchowskiego 25, 50-370 Wroclaw, Poland
E-mail: mieczyslaw.szata@pwr.wroc.pl

corrosion and fatigue cracks propagation; a result: 3 persons killed and 5 injured;
- Crash of Plane Douglas DC-10-10 (19.01.1989) United Airlines flight 23 Iowa State (USA); a result: 110 passengers killed and 175 injured;
- I-35W Mississippi in Minnesota State – bridge crash (01.08.2007 – bridge opening 1967!!!); a result: 6 persons killed, 100 injured, a lot of cars were to the bottom.

These examples are only the representatives. Much more information one can find in the papers [1] and [2] devoted to the analysis of these crashes.

Figure 1. Tanker "Schenectady" [2]

The examples presented above become a start point for proposing a new approach in description of fatigue fracture problem. A new method of constructing kinetic fatigue fracture diagrams (KFFD) has been presented on the basis of measurement results of a hysteresis loop area for the isotropic body with an internal flat crack under a cyclic loading. For the experimental verification, the results of fatigue crack propagation studies for three types of steels have been utilized.

2. ENERGETIC DESCRIPTION OF FATIGUE FRACTURE KINETICS

The force criterion being a local criterion is frequently applied for the description of a fatigue crack growth. The rate of a crack growth is a function of stress intensity factors. This function based on experimental data has an exponential form in relation to K_I, as it has been shown in Paris' law. In the diagram (Fig. 2), one can see 3 ranges of fracture propagation (I – low-amplitude, II – stable – middle-amplitude, III – critical – high-amplitude).

Two asymptotes (K_{th}, K_{fc}) determining all the range of stress intensity factor (SIF) are also presented.

A lifetime of mechanical construction elements is assumed to be a sum of the initiation processes and precritical growth of fatigue cracks until they lose a global stabilization. The constructional materials always contain the defects of a definite (specific) dimension that is a characteristic feature for a given material and given element technology. At present, this defect dimension is introduced as a construction parameter in order to calculate the prognostic lifetime without collapsing the construction element. Such an approach makes possible to resolve the problem of calculating a lifetime of construction element conditioned by a precritical defect growth time from the dimension of construction parameter up to a critical defect value.

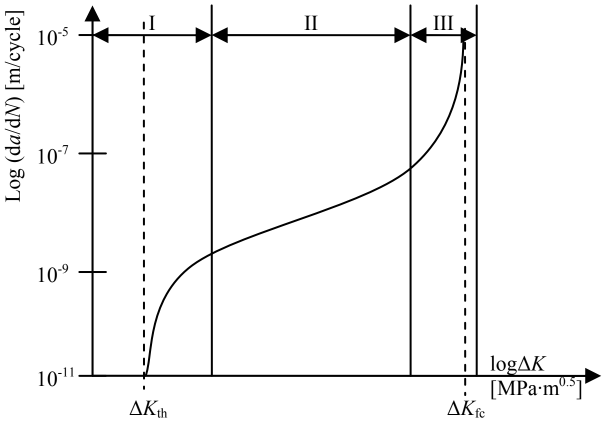

Figure 2. Classical diagram FFKD

The deformation criterion based on the δ_K – model is used in the case when the plastic zone before a crack tip is of the same order as a body dimension.

The presented approach is effective in the case of a stable loading cycle without taking into account the "history" of material deformation and crack propagation. The stress intensity factors or the values of a crack opening are the invariants of a fatigue damage. The energy approach makes possible to take into consideration the elements mentioned earlier.

The problem dealing with the determination of the period N_s of a precritical crack propagation by means of the energetic approach described in [3] needs the application of the first principle of thermodynamics. Taking the quantities A, Q, W, K_e, Γ as a linear density (divided by thickness B), namely $A^* = BA$, $Q^* = BQ$, $W^* = BW$, $K_e^* = BK_e$, $\Gamma^* = B\Gamma$, we can write a global balance in the following form

$$A + Q = W + K_e + \Gamma \tag{1}$$

where: A – the work of external stresses σ_{zw} after N cycles of loading, Q – the heat input to the body during the loading, W – a deformation energy after N cycles of loading, K_e – a kinetic energy of the body, Γ – a damage energy during the change of a crack surface after one "quantum".

After differentiating (1) over the number of cycles, assuming that a slow growth of a crack length does not go together with heat processes, and neglecting the small changes of a kinetic energy (for low frequencies of a cyclic loading) we obtain:

$$\frac{\partial A}{\partial N} = \frac{\partial W}{\partial N} + \frac{\partial \Gamma}{\partial N}, \tag{2}$$

The process of a precritical crack propagation is described by (2). The case of the values σ_{zw} for which the propagation will not occur should be also considered. The equation (2) can be written in another form:

$$\frac{\partial (A - W)}{\partial N} = \frac{\partial W_p^o}{\partial N}, \tag{3}$$

where: W_p^0 – energy dissipation of plastic deformation.

It was assumed that

$$W_p^o = W_c^o + W_s^o \tag{4}$$

where: W_c^0 – energy of cyclic plastic deformation, W_s^0 – a static component of energy, equivalent to σ_{zwmax} and changing with the crack growth.

If $W_p^0 < \Gamma$, the crack propagation process will not start, if $W_p^0 = W_p = \Gamma$, the crack will grow for ΔS value. Then

$$W_p = W_s + W_c = \Gamma \tag{5}$$

Both the components W_s and W_c depend on the external stresses σ_{zw} and their sum (under fixed Γ value), given by (5), can be reached by different combinations of σ_{zwmax} and $\Delta\sigma_{zw} = \sigma_{zwmax} - \sigma_{zwmin}$. The quantity of σ_{zwmax} determining W_s, and W_c depends on $\Delta\sigma_{zw}$.

The quantity Γ can be defined as a maximum of energy static component ($\Gamma = W_{smax}$) causing crack process without a cyclic energy ($W_c = 0$).

In the equation (5), a static component of energy W_s occurs only once. The energy of cyclic plastic deformation W_c can be written in the form

$$W_c = \frac{\partial W_c}{\partial N}\Delta N , \tag{6}$$

assuming $W_c^{(1)} = \frac{\partial W_c}{\partial N}$ as a constant in each cycle [3].

After a simple transformation the equation for the rate of crack surface growth is obtained:

$$\frac{\partial S}{\partial N} = \frac{\dfrac{\partial W_c}{\partial N}}{\dfrac{\partial(\Gamma - W)}{\partial S}} \tag{7}$$

2.1 Energetic kinetic fatigue fracture diagrams

According to [3], the crack propagation formula on the basis of Dugdale's model can be written as:

$$\frac{da}{dN} = \frac{\alpha W_c^*}{B\sigma_{plf}\varepsilon_{fc}(1 - K_{Imax}^2 / K_{fc}^2)} \tag{8}$$

where: K_{fc} – a cyclic fracture toughness, K_{Imax} – a maximum value of stress intensity factor, $\alpha = 0.5$.

It seems that for estimation of the influence of a loading cycle asymmetry R it is proper to construct experimental kinetic fatigue fracture diagrams da/dN – ΔH where the energetic parameter ΔH depending on $W_c^{(1)}$ is equal to:

$$\Delta H = \frac{W_c^*}{B(1 - K_{Imax}^2 / K_{fc}^2)} \tag{9}$$

In opposition to the $da/dN - K_{max}$ diagrams, in the $da/dN - \Delta H$ diagrams obtained for a given range of the crack propagation rate any difference in fracture kinetics has not been found. It means that in contradistinction to the force factor K_{max} the energetic parameter ΔH describes synonymously the fatigue crack propagation rate, independently of a cycle asymmetry R.

2.2 Measurement stand

A measurement setup (Fig. 3) for determining the energy dissipated in the unit volume of material is consisted of:

- hydraulic pulsator,
- extensometer,
- optical system,
- A/C converters – eg. HP E1432A and
- PC computer with a special software, eg. package HPVEE.

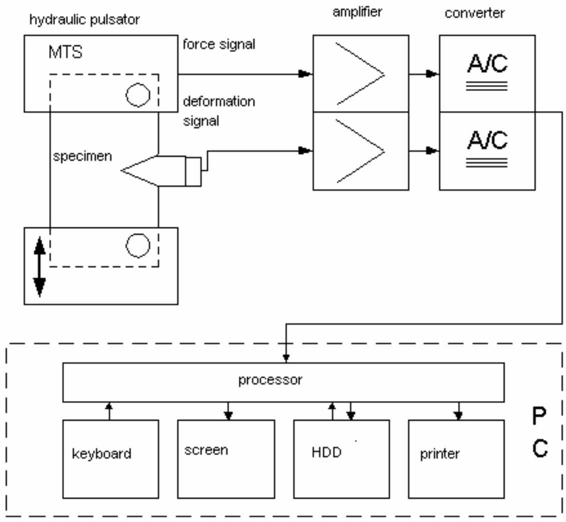

Figure 3. Measurement stand scheme [3]

The determination of the energy dissipated in the unit volume of material is faced with many difficulties, particularly in the case of a high cycle fatigue. The hysteresis loop area is then very small and the algorithms should be of a great accuracy. This work has been realized with the help of a software package using the programming language Hewlett – Packard HP VEE version 4.0 [3].

The computer screen during a testing procedure is presented in Figure 4. One can observe the loading (forces) and the answer (displacements) diagrams. The ranges of replaceable MTS panels for the force (± 25 kN) and displacement (± 0.5 mm) are registered. A sinusoid-shaped loading diagram has been presented around the mean value level (3462.5 N) indicated in the diagram as zero.

Similarly, the displacement diagrams were shown around the mean values which can be determined on the basis of a maximum (0.11 mm) and minimum (0.09 mm) displacement values. The period T is equal to 0.1 s and the phase shift angle between a force and displacement is equal to 0.67°. The hysteresis loop in axis force versus the displacement corresponding to the energy for one cycle is equal to $0.51 \cdot 10^{-3}$ J.

3. RESULTS AND DISCUSSION

In the experiment, the beam specimens having dimensions 12 × 18 × 8 with a lateral concentrator clamped at one edge were bent. The second kind of

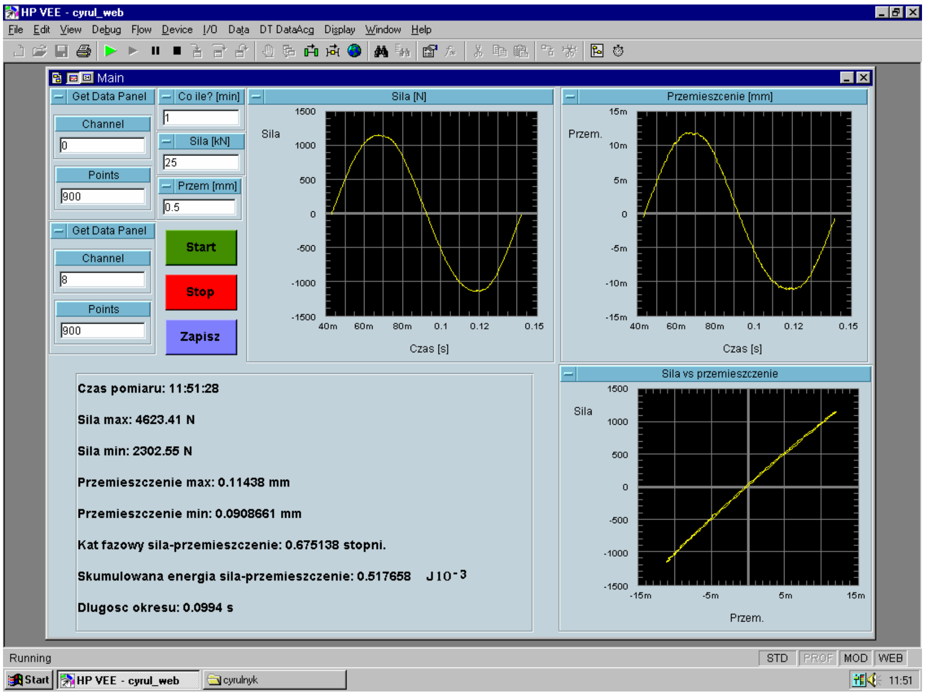

Figure 4. Computer screen during tests [3]

specimens was the compact specimens according to ASTM E399-81 [4].

Three kinds of steel 12HMF and 18G2A and 40H were tested:

- **12HMF** (0.1 % C; 1.1 % Cr; 0.26 % Mo; 0.17 % V; 0.54 % Mn; 0.019 % S; 0.015 % P),
- **18G2A** (0.2 % C; 0.26 % Mo; 0.2 % Cu; 1.3 % Mn; 0.03 % S, 0.02 % P) and
- **40H** (0.4 % C; 0.7 % Mn; 1.1 % Cr; 0.3 % Si; 0.3 % Ni; 0.03 % S; 0.02 % P).

Strength properties of tested materials are shown in Table 1.

Table 1. Strength properties of tested steels

Material	R_m [MPa]	$R_e/R_{0.2}$ [MPa]	A_5 [%]	K_{fc} [MPa·m$^{0.5}$]
12HMF	470	208	29	80
18G2A	600	350	22	105
40H	980	780	10	45, 80, 100*

* The values of critical stress intensity factors K_{fc} are 45, 80, 100 MPa·m$^{0.5}$, for the temperature 200, 450, 700 °C, respectively.

The kinetic diagrams were prepared for different values of cycle asymmetry coefficients R = 0.1; 0.5; 0.65; 0.75.

Two kinds of kinetic diagrams for the 40H steel have been constructed in Figures 5 and 6. In the first diagram one can observe the range of a stress intensity factor K characterizing the intensity of cyclic material deformation in the crack tip, in the second one – the magnitude ΔH corresponding to dissipation of deformation energy.

For a fatigue crack propagation rate (for 18G2A), the kinetic diagram da/dN – K has been presented in Figure 7. According to our expectations the diagrams show the influence of a cycle asymmetry on material resistance concerning a fatigue crack propagation.

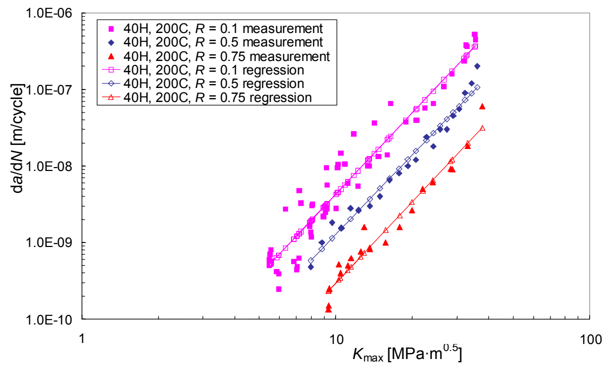

Figure 5. Crack propagation rate d*a*/d*N* versus stress intensity factor K_{max} for different asymmetry factors *R*; 40H steel, heat treatment 200 °C (classical diagram)

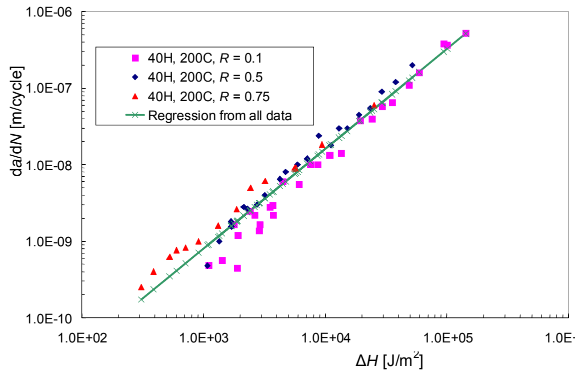

Figure 6. Crack propagation rate d*a*/d*N* versus parameter Δ*H* for different asymmetry factors *R*; 40H steel, heat treatment 200 °C (energetic diagram)

A kinetic diagram in co-ordinates da/dN – ΔH has been also constructed (Fig. 8). In contrast to the diagrams da/dN – ΔK, the differences in fracture kinetics can not be observed in the diagrams da/dN – ΔS constructed for a given range of a fracture crack propagation rate. It means that unlike the force parameter ΔK, the energetic parameter ΔS describes explicitly a fatigue crack propagation rate independently of a cycle asymmetry coefficient R. The same results are observed for 12HMF steel (Figs 9 and 10).

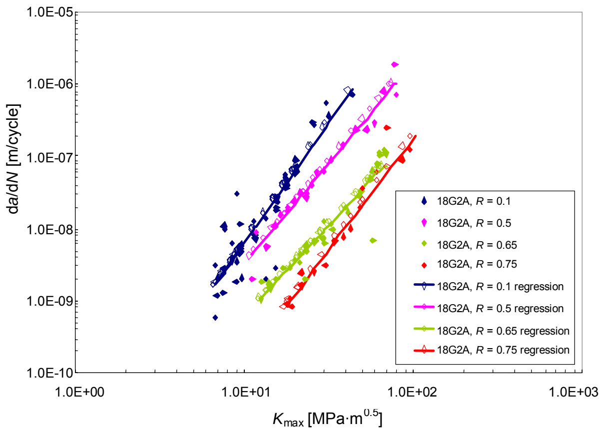

Figure 7. Crack propagation rate da/dN versus stress intensity factor K_{max} for different asymmetry factors R; 18G2A steel (classical diagram)

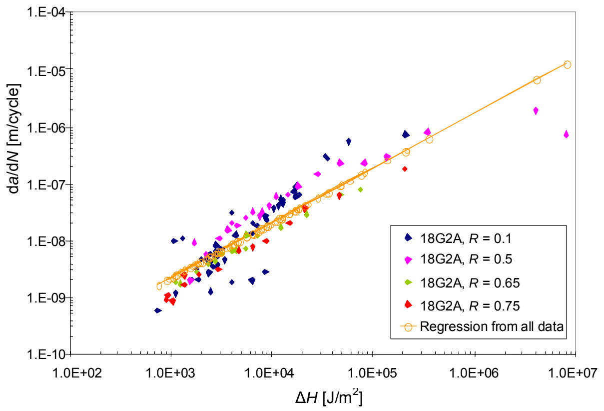

Figure 8. Crack propagation rate da/dN versus parameter ΔH for different asymmetry factors R; 18G2A steel (energetic diagram)

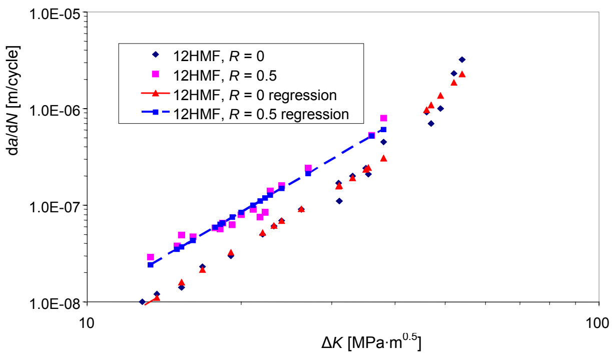

Figure 9. Crack propagation rate da/dN versus stress intensity factor ΔK for asymmetry R = 0 and 0.5 (classical diagram), for 12HMF

4. SUMMARY

In the paper, the energy criterion of fatigue crack propagation for the isotropic body has been presented. The analytical formula of a kinetic equation for fatigue crack propagation has been derived. The start point was the energy balance resulting from the first principle of thermodynamics.

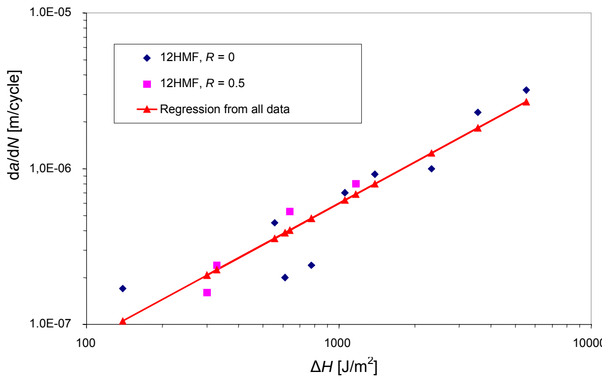

Figure 10. Crack propagation rate da/dN versus ΔH parameter (R = 0, R = 0.5) (energetic diagram), for 12HMF

A new method of constructing kinetic fatigue fracture diagrams has been developed and presented on the basis of measurement results of a hysteresis loop area for the isotropic body with an internal flat crack under a cyclic loading. For the experimental verification, the results of fatigue crack propagation studies for 18G2A and 40H steels have been utilized. The two kinds of kinetic fatigue fracture diagrams have been worked out on the basis of experimental data. In the first diagram (classical), the stress intensity factor $K_{\mathrm{I}max}$ characterizing the deformation intensity in the precracked zone before a crack tip was used for constructing the diagram. In the second one, the energetic parameter ΔH corresponding to the searched energy dissipation of deformation $W_c^{(1)}$ was used.

In opposition to the da/dN – K_{max} diagrams, in the da/dN – ΔH diagrams obtained for a given range of the crack propagation rate, any difference in fracture kinetics has not been found. It means that in contradistinction to the force factor K_{max}, the energetic parameter ΔH describes synonymously the fatigue crack propagation rate, independently of a cycle asymmetry R.

New measurement methods of hysteresis loop registration by means of Villari effect and magneto vision camera [5] provide additional possibilities for the application of the methods described in the paper.

REFERENCES

[1] Eskalul, K.A. (Ed.): *Handbook of Case Histories in Failure Analysis*, ASM International, Materials Park, 1994.

[2] Hayes, B.: Classic brittle failures in large welded structures, Engineering Failure Analysis, Vol. 3. No. 2, pp. 115-127, 1996.

[3] Szata, M.: *Modeling of fatigue crack growth using energy method*, Publishing House of Wroclaw University of Technology, Wroclaw, 2002, (in Polish).

[4] ASTM E399-90 Standard Test Method for Plane-Strain Fracture Toughness of Metallic Materials, Annual Book of ASTM Standards, Vol. 03.01, 1997.

[5] Kaleta, J.: *The experimental foundations of energetically fatigue hypothesis formulation*, Publishing House of Wroclaw University of Technology, Wroclaw, 1998, (in Polish).

НОВА МЕТОДА КОНСТРУИСАЊА ДИЈАГРАМА КИНЕТИКЕ ЗАМОРНОГ ЛОМА – ЈЕДНАЧИНЕ РАСТА ПРСЛИНЕ НА ОСНОВУ ЕНЕРГЕТСКОГ ПРИСТУПА

Mieczyslaw Szata*, *Grzegorz Lesiuk

Критеријум силе, као локални критеријум, често је коришћен за опис раста заморне прслине. Брзина раста прслине је функција фактора интензитета напона, добијена на основу експерименталних података у експоненцијалном облику, као што је дато Парисовим законом. Критеријум деформације на основу δ_K модела се користи у случају када је пластична зоне испред врха прслине истог реда величине као димензије тела. Такав приступ је ефикасан у случају стабилног цикличног оптерећења без узимања у обзир „историје“ деформације материјала и раста прслине. У том случају фактор интензитета напона или вредност отварања прслине постају инваријанте у односу на заморно оштећење. Енергетски приступ омогућава да се узме у обзир и поменути елементи. Стога је за заморни раст прслине у изотропном телу формулисан критеријум енергије. Коначно, добијена је једначина кинетике заморног лома као аналитичка релација брзине раста заморне површинске прслине и енергије дисипације пластичне деформације у зони врха прслине. Нова метода конструисања дијаграма кинетике заморног лома је уведена на основу резултата мерења површине петље хистерезиса за изотропно тело са унутрашњом равном прслином. За експерименталну верификацију коришћени су резултати испитивања раста заморне прслине код челика 18G2A и 40H.