## Determination of Cornering Stiffness Through Integration of A Mathematical Model and Real Vehicle Exploitation Parameters

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*This paper demonstrates the approach to determination of cornering stiffness as the most important parameter of motor vehicle steering and stability systems through integration of empirical models and real exploitation parameters. The approach is based on the analysis of a tyre mathematical model, identification of key real exploitation parameters and integration of a mechatronic system for tyre load monitoring. Measurement of tyre load and subsequent integration of measured values into developed software module provide a precise definition of cornering stiffness for a certain vehicle and road contact patch. This research assumes integration of a mathematical model and mechanical, electronic and information technologies with the aim of comprehensive understanding of tyre behaviour in terms of cornering stiffness determination.*

***Keywords:*** *cornering stiffness, safety, tyre, Ackermann angle.*

### 1. INTRODUCTION

If a motor vehicle is moving at high speed, lateral force acting on tyres increases the possibility for vehicle to get into a critical situation. Therefore, because of high lateral acceleration at high speed, cornering equations are different from those for moving at low speed [1]. In order to counteract lateral acceleration, the tyres must develop lateral forces, and slip angles will be present at each wheel.

During the cornering, when tyre must develop a lateral force [2], tyre will also experience lateral slip as it rolls. The angle between the direction of heading and direction of travel is known as a slip angle α (Figure 1).

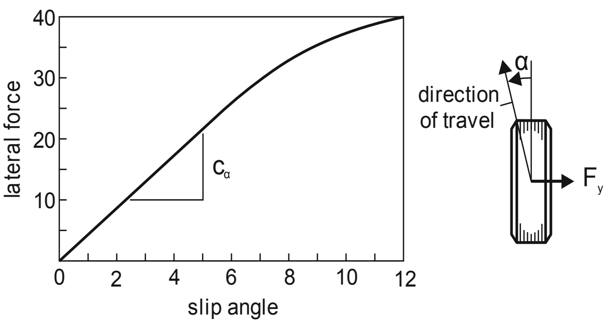


**Figure 1. Basic functional relationship of lateral force and slip angle**

The lateral force, denoted by Fy is called the cornering force when the camber angle is equal to zero. At a given tyre load, the cornering force increases with a slip angle. At low slip angles values (less than 8º), this relationship is linear, hence, the cornering force is defined as:

$$F_y = C_\alpha \cdot \alpha \, . \qquad (1)$$

The proportionality constant Cα is known as the cornering stiffness, and is defined as the slope of the curve for Fy versus α at α = 0. A positive slip angle produces a negative force (to the left) on the tyre, implying that Cα must be negative; however, SAE [1] defines cornering stiffness as the negative of the slope, such that Cα takes on a positive value.

The cornering stiffness depends on many variables - tyre size and type (radial or bias ply construction), number of plies, cord angles, tyre width and tread are significant factors. For a given tyre, the load and inflation pressure are the main factors affecting the cornering stiffness. Speed does not strongly influence the cornering forces produced by a tyre. The plots in Figures 2 to 6 illustrate the influence of many of these variables.

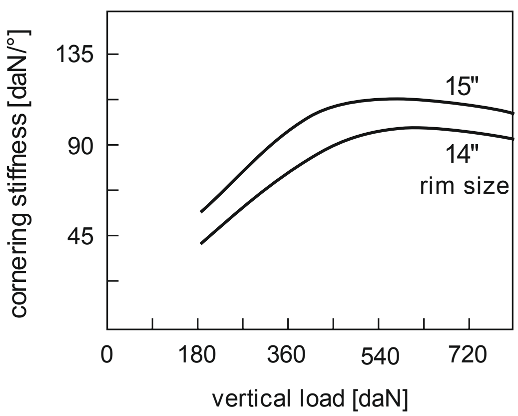


**Figure 2. Cornering stiffness versus vertical load**

Because of the strong dependence of cornering force on tyre load, tyre cornering properties may also be described using the cornering coefficient, which is

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obtained by dividing the cornering stiffness by the load. Thus, the cornering coefficient $CC_\alpha$ is defined as:

$$CC_\alpha = \frac{C_\alpha}{F_z} . \qquad (2)$$

Cornering coefficient usually has the largest value at low loads, diminishing continuously as the load reaches its rated value (Tyre & Rim Association rated load). At 100% load, the cornering coefficient is typically in the range of 0.2 (1 N cornering force per 1 N load per 1 degree of slip angle).

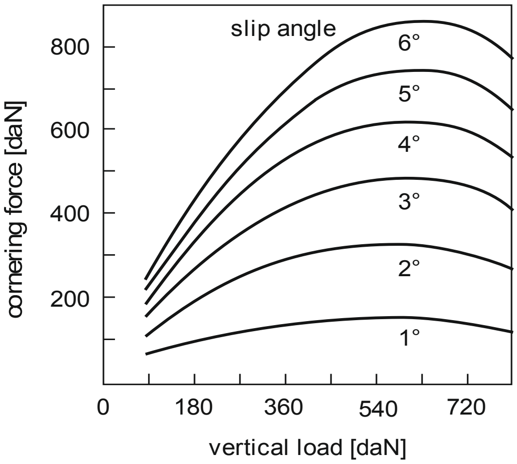


**Figure 3. Cornering force versus vertical load**

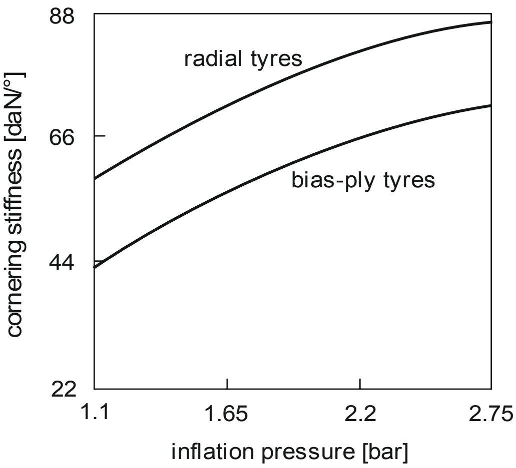


**Figure 4. Cornering stiffness versus inflation pressure**

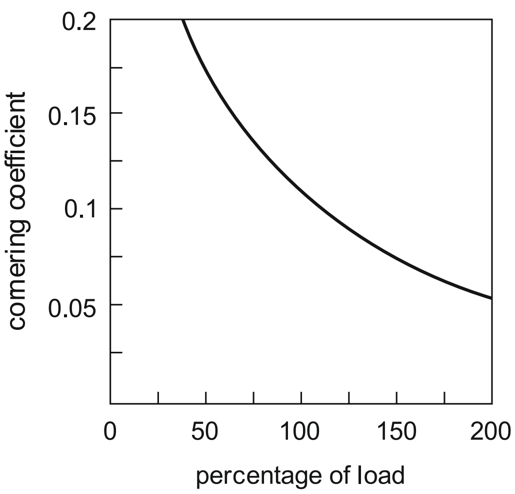


**Figure 5. Cornering coefficient versus percentage of load**

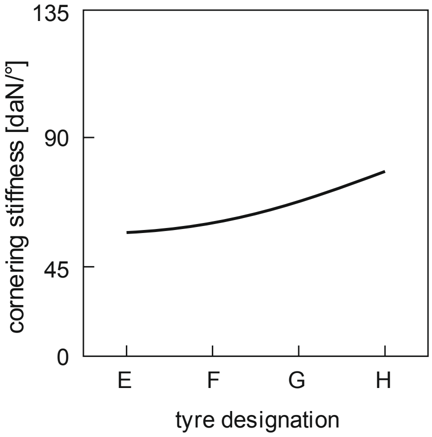


**Figure 6. Cornering stiffness for different types of tyres**

## 2. TYRE MATHEMATICAL MODEL

As it is well known, tyres are a complex composite comprising many layers of materials (Figure 7). A tyre is very anisotropic. Because of this, tyre behaviour depends not only on the material properties and structure of a tyre. Simplifications are therefore made in order to create empirical models for a tyre. There are three main models designed for better understanding of tyre forces, deflection and footprint behaviour under cornering conditions: the elastic foundation model, the string model and the beam model. Although none of these models truly addresses the complexity of a physical tyre, realistic results can be obtained by using empirical stiffness values [3].

**Figure 7. Tyre composite structure**

Each small element in the elastic foundation model is considered to act independently of the other elements (Figure 8). The fact that each element acts as a simple spring, independent of the other elements, makes this model the simplest of the three. It is interesting that the force below the lateral force distribution curve equals the lateral force measured at the axle [4]. This supports the belief that although this model may be the simplest, it can be very useful in predicting and illustrating various tyre behaviour, as is done by Dixon [5]. The elastic foundation model [6] also allows for discontinuity in distribution of displacement and in the slope of the tyre centre line. On the other hand, in the string model, lateral displacement is resisted by a tension between the elements. This model also allows for discontinuity in slope; however, discontinuity in displacement is not allowed. Similar to the string model is the beam model, where each element has an effect on the surrounding elements. In the beam model, each element creates bending moments acting on the surrounding elements. This allows no discontinuity in slope or displacement. The beam model has been found to be superior for radial ply or for belted bias ply tyres [5]. These models are often combined to gain a better model.

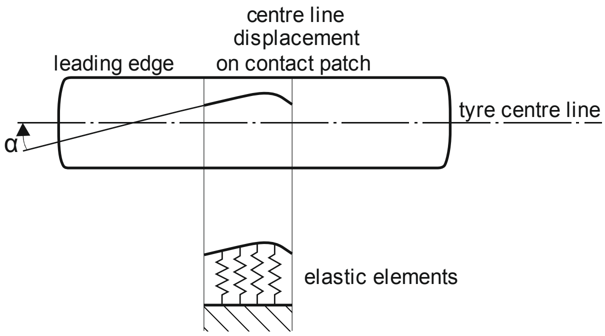


**Figure 8. Elastic foundation model of a tyre**

## 3. COEFFICIENT OF FRICTION

The coefficient of friction is defined as a unitless ratio of friction force to normal force. It is generally accepted that the resulting friction force is not proportional to the surface area of contact. However, this is far from the truth when a rubber tyre is considered. This dissimilar behaviour is due to the viscoelastic nature of rubber. Thus, as force is applied, deformation occurs both elastically and plastically in a non-linear fashion due to the mechanical behaviour of polymer chains [7]. Viscoelasticity also explains why a tyre coefficient of friction is load dependent. As a tyre is loaded, contact area grows larger, increasing total friction force and decreasing coefficient of friction [5]. Since a tyre does not follow Newton's laws of friction, it is possible to obtain a coefficient of friction above unity. For example, given a 2200 N load on a tyre, it would not be uncommon for a tyre to produce 3500 N of force giving a coefficient of friction of 1.6. Under ideal conditions, this would make the vehicle capable of pulling 1.6 *g* [8]. However, ideal conditions are rarely achieved because the coefficient of friction depends on many transients with stochastic nature [3,9].

The coefficient of friction can depend on many unknown variables such as atmospheric dust, humidity, temperature and vibration. It may also depend on the inclination of the road surface, speed and even braking duration. As the tyre is braked further, the temperature rises above the optimum value and the coefficient of friction begins to drop. Similarly, as the speed increases, temperature increases and the coefficient of friction again begins to decrease after reaching an optimum value (plotting coefficient of friction versus speed gives a curve resembling a normal distribution). A final component that should not be overlooked is a resulting molecular bonding which leads to total friction. Since contact between the road and the tyre does not require energy to create the bond, energy is dissipated when the bonds are broken. This becomes important in dry conditions. On the other hand, the tyre elastically conforming to the road becomes more important in wet conditions.

Finally, some effects of the coefficient of friction changing with speed should be discussed; when slip angle becomes large, the rear of the footprint begins to slide and thus has a lower coefficient of friction. Therefore, lateral force will reach maximum value at a modest slip angle, and then begin to decline. This phenomenon is more dramatic when braking a locked wheel. Since the wheel is locked, the local temperature rise is greater and the relative sliding speed is greater than of a rotating wheel. In either case, decreased coefficient of friction contributes to a negative self-aligning torque [5].

## 4. SLIP ANGLE

If direction of travel differs from wheel heading (if wheel's angular displacement is different from the path the tyre is following), the slip angle $\alpha$ produces a component of lateral force $F_y$. This lateral force will act in a point behind the centre of the wheel in a direction such that it attempts to re-align the tyre. It should be noted that the slip angle is not the same as the steering angle (Ackermann angle).

As can be seen from the elastic foundation model, limited value of the lateral force vs. understeer angle is expressed as [5]:

$$c \cdot d \leq \frac{\mu \cdot F_{\mathrm{V}}}{l}, \tag{3}$$

where $F_{\mathrm{V}}$ is normal force, $c$ is the stiffness [10], $d$ is the tyre centre line displacement and $l$ is the contact patch length. According to this, the lateral force is roughly proportional to the slip angle. This gives the maximum force:

$$F_{\mathrm{y}} = \frac{1}{2} \cdot c \cdot l^2 \cdot \alpha \,. \tag{4}$$

$$\frac{1}{2} \cdot c \cdot d_{\mathrm{m}} \cdot l = \frac{1}{2} \cdot c \cdot l \cdot \left( \frac{\mu \cdot F_{\mathrm{V}}}{c \cdot l} \right) = \frac{1}{2} \cdot \mu \cdot F_{\mathrm{V}}, \tag{5}$$

where $d_{\mathrm{m}}$ is maximum tyre centre line displacement.

Therefore, the lateral force is proportional to the slip angle up to the half of the maximum total friction limit [5]. This can be seen in Figure 9, which shows lateral force versus slip angle for some experimental values chosen to be similar to a typical SAE tyre. These values are as follows: 890 N normal force, coefficient of friction of 1.5, stiffness of 5 MPa and contact patch length of 216 mm.

Lateral force and cornering stiffness can be calculated through the slope of the resulting curve by using the following relationship:

$$F_{\mathrm{y}}(\alpha) = \mu \cdot F_V - \frac{\mu^2 \cdot F_V^2}{2 \cdot c \cdot l^2 \cdot \tan\alpha}. \tag{6}$$

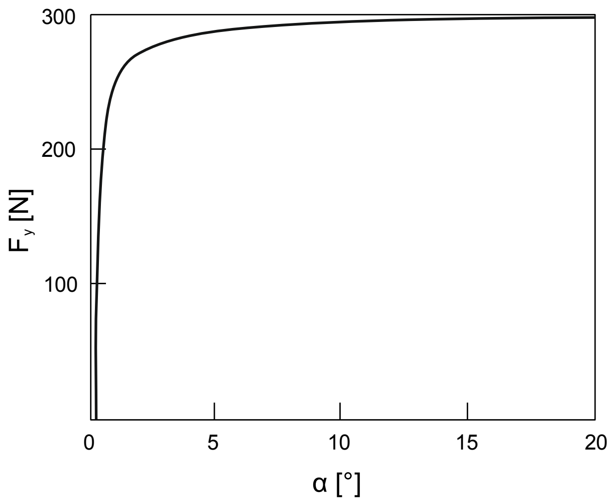


**Figure 9. Theoretical relationship of lateral force and slip angle**

As can be seen, the upper limit is reached very quickly with the maximum force occurring at 90 degrees when the wheel is fully sideways. Generally, a racing car tyre will achieve maximum lateral force at understeer angles ranging from 3 to 7 degrees [4]. In figure 9, the force at 3 degrees is 95% of maximum value. At large understeer angles, the rear of the footprint actually slides laterally along the road surface, which contributes to less capacity for lateral force and reduces the stabilizing self-aligning torque [4].

It may be important to realize that, when not completely sliding, the lateral force is not dependent on the coefficient of friction, although this provides the upper limit. Instead, it depends on the stiffness. An alternate way to look at this is to say that the lateral force is not dependent on the coefficient of friction until the tyre lose grip, indicating a large understeer angle [4]. Sometimes, it is convenient to define the lateral force versus understeer angle in terms of other coefficients as in equations:

$$F_{\mathrm{y}} = F_{\alpha} = C_{\alpha} \cdot \alpha = C_{\mathrm{S}} \cdot F_{V} \cdot \alpha \,, \tag{7}$$

and

$$C_{\alpha} = C_{\mathrm{S}} \cdot F_{V} \,. \tag{8}$$

where $C_{\mathrm{S}}$ is the cornering stiffness coefficient and $C_{\alpha}$ is the cornering stiffness. Generally, typical values for the cornering stiffness coefficient are 0.12/º for bias ply tyres and 0.16/º for radial ply tyres [5].

## 5. CORNERING STIFFNESS

The cornering stiffness can also be defined on a radian basis as shown in equation:

$$C_{\alpha} = \frac{1}{2} \cdot c \cdot l^{2} \,. \tag{9}$$

The cornering stiffness can also be seen as the initial slope of the lateral force curve. Typical values for cornering stiffness are around 867 N/º [1]. However, this value can be much higher. For example, a Formula 1 tyre may have the cornering stiffness of 3.7 kN/deg at normal load of 8 kN and 3.4 kN/º at normal load of 4.5 kN [5]. A Formula SAE tyre would have the cornering stiffness of around 734 N/º for tyre load of 1.5 kN as can be seen in Figure 10 (AVON tyres).

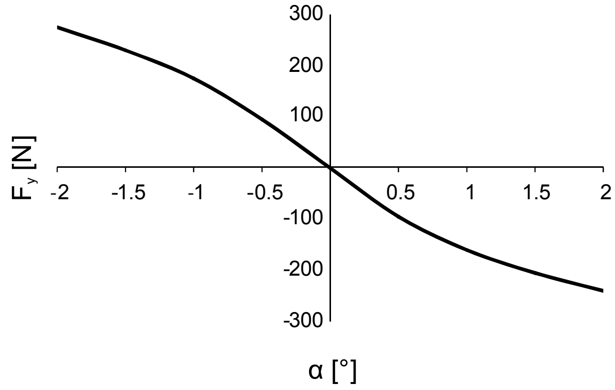


**Figure 10. Characteristic cornering stiffness values of Avon tyres at 1° camber and 1500 N normal load**

It should be noted that the cornering stiffness is sensitive to the range of understeer angle used to find the slope on the lateral force curve. This further emphasizes that one should be very careful when using cornering stiffness values in calculations, or at least realize their potential inaccuracies. As an additional point, the cornering stiffness is usually 5 to 6 times greater than camber stiffness for traditional bias tyres [4]. Since the ratio in (7) reduces with a slip angle (see also Figure 11), higher cornering stiffness is desirable. The reason for this is that a given central force will be achieved at smaller slip angles and therefore results in lower tyre drag force. This ratio is as follows [5]:

$$\frac{F_S}{F_D} = \frac{C_{\alpha} \cdot \alpha \cdot \cos\alpha}{\mu_R \cdot F_V + C_{\alpha} \cdot \sin\alpha} \,, \tag{10}$$

where $\mu_R$ is the rolling resistance, $F_D$ is the drag force in the direction of travel and $F_S$ is the force component perpendicular to travel direction. $F_D$ and $F_S$ are the components of the resultant lateral force $F_{\mathrm{y}}$.

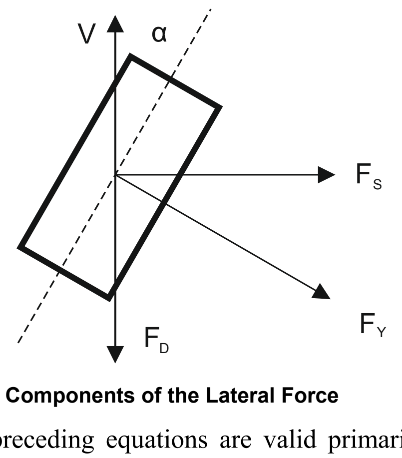


**Figure 11. Components of the Lateral Force**

The preceding equations are valid primarily in the linear part of the lateral force curve. However, when driving style is more aggressive, larger angles are deployed, as with a two-wheeled vehicle, the cornering stiffness may be reduced dramatically. When camber angle is included in the cornering stiffness, a new equation for cornering stiffness can be used:

$$C_{\alpha} = C_{\alpha 0} - k_{C\alpha\gamma} \cdot \gamma \,. \tag{11}$$

Given that:

$$k_{C\alpha\gamma} = \frac{0.005}{\circ}. \tag{12}$$

the equation $C_\alpha = C_{\alpha 0} - K_{C\alpha\gamma} \cdot \gamma$ is linear up to about 60 degrees [5]. However, for a wide racing tyre, large camber angles are rarely used as this would cause lifting of one tyre side off the ground.

During accurate determination of the cornering stiffness coefficient, it becomes evident that it is dependent on vertical load. If we look back on previous analysis, it is obvious that we need to use tyre load monitoring system in real exploitation conditions so that we can directly gather information on cornering stiffness values with the aim of improving vehicle steering and stability.

Tyre cornering stiffness can be defined directly, by using diagram in Figure 9, as the tangent of the slip angle in linear part of the curve. Determination of the cornering stiffness regarding this research can be justified only in case of dynamic determination of its values. This is possible by using specially designed software for determining cornering stiffness, based on preceding equations and dynamic monitoring of vertical tyre load values in real exploitation conditions. The software layout is shown in Figure 12.

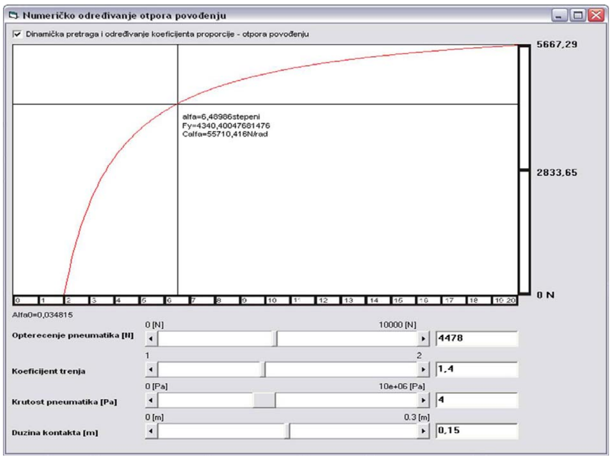


**Figure 12. Software for cornering stiffness determination**

Variables used in this software are tyre load, coefficient of friction, tyre stiffness and contact patch length. If we look back on previous text, we can see that accurate determination of cornering stiffness demands measuring tyre vertical load. Therefore, it is necessary to integrate tyre pressure sensors into the system to determine vertical load values based on pressure changes. This method is justified because of its direct implementation, while measurements could also be taken by implementation of accelerometers and direct calculations regarding geometry and mass of the set-up vehicle.

Regardless of the method chosen for determination of the tyre vertical load, information gathered directly by pressure sensors or accelerometers is transmitted to the developed software [13].

Correct estimate of cornering stiffness, being the essence of this research, can be provided based on adjusted values of coefficient of friction, tyre stiffness and contact patch length (without considering the fact that these values change on their own, which is the topic beyond the scope of this paper) and by integrating dynamic vertical load into developed software package.

## 6. CONCLUSION

Combination of cornering stiffness values, acquired by feedback systems, directly provides conditions for forming so-called ESP (Electronic Stability Program) or VSE (Vehicle Stability Enhancement) in the area of active chassis issues [11] or experimental verification of automatic vehicle controls [12].

Specifically, without considering the essence of electronic stability programme and regardless of the fact that its objective is closely related to the objective of this paper, it is also important to mention common advantages of ESP and expected research results:

- increased vehicle stability,
- increased vehicle steering by easier handling,
- reduced number of traffic accidents.

Unlike the objective of ESP, the objective of this paper is integration of a mathematical model and data acquisition and processing in real exploitation conditions. Combination of analysis and virtual model for continuously variable parameter conditions provides preliminary overview of possible vehicle scenarios, as well as of values of critical components defining vehicle in a numeric subspace.

By using scenarios and values of critical components early in the design phase, motor vehicle components can be adjusted in such way that, in combination with external systems including ESP and VSE, optimisation of stability and steering is attained.


## ACKNOWLEDGMENT

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## ИДЕНТИФИКАЦИЈА ОТПОРА ПОВОЂЕЊУ ИНТЕГРАЦИЈОМ МАТЕМАТИЧКОГ МОДЕЛА И ПАРАМЕТАРА РЕАЛНЕ ЕКСПЛОАТАЦИЈЕ ВОЗИЛА

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У раду је приказан прилаз идентификацији отпора повођењу као најзначајнијем параметру у системима управљивости и стабилности моторних возила кроз интеграцију емпиријских модела и параметара реалне експлоатације. Овај прилаз се заснива на анализи математичког модела пнеуматика, идентификацији кључних параметара из реалне експлоатације и интеграцији мехатронског система за праћење оптерећења пнеуматика. Мерењем оптерећења пнеуматика, и каснијом интеграцијом измерених вредности у развијени софтверски модул, обезбеђује се прецизно дефинисање отпора повођењу на одређеном контакту возила и пута. Истраживање подразумева интеграцију математичког модела, механичких, електронских и информационих технологија у циљу обезбеђивања свеобухватног сагледавања понашања пнеуматика у погледу дефинисања отпора повођењу.