Non-Linear Control Technique of a Pendulum via Cable Length Manipulation: Application of Particle Swarm Optimization to Controller Design

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This paper presents a novel vibration control technique for a pendulum via cable length manipulation. To control the sway angle by using the reeling and unreeling of the hositing cable, we develop a non-linear feedback control scheme by utilising parametric resonance, in which the control input is defined as the acceleration of the cable. Because the governing equation and the control law are non-linear, it is very difficult to analytically solve the feedback gains for the stabilization of the system. Hence, the feedback gains are determined by the use of particle swarm optimization (PSO), which is an evolutionary computation technique, to reduce the sway angle to the maximum extent possible. The validity of the proposed control technique is confirmed by numerical simulations. To verify the feasibility of the present approach, experiments are also performed. From the experimental results, we demonstrate that the application of the PSO algorithm for tuning the feedback gains is valid and that the proposed non-linear feedback control scheme is effective for the vibration control of a pendulum with variable length.

Keywords: *vibration control, parametric resonance, non-linear feedback control, particle swarm optimization, experimental validation.*

1. INTRODUCTION

Crane systems have been extensively used as transport systems in many industrial fields. For the efficient and safe operation of crane systems, their sway motion must be suppressed. A considerable number of papers are therefore available on vibration control problems of crane systems. Abdel-Rahman et al. [1] published an exhaustive literature review on the modelling and control of cranes in which a number of papers on gantry, rotary, and boom cranes were reviewed. Usually, the load sway of a crane is suppressed by control forces exerted from a boom or trolley. On the other hand, when the boom or trolley is not allowed to move because of some restrictions, an effective approach for controlling the sway motion is obtained by using a hoisting mechanism that lifts a load up or down.

Abdel-Rahman and Nayfeh [2] developed an approach that used the reeling and unreeling of a hoisting cable to reduce payload pendulations due to near-resonance excitations. Significant reductions were obtained via an appropriate choice of the reeling/unreeling speed. Stilling and Szyszkowski [3] considered the planar motion of a simple, variable length pendulum consisting of an inextensible cable of negligible mass and a point mass and then attempted to control the angular oscillation by sliding the point mass toward and away from a pivot. They reported that simple rules for generating either attenuation or amplification of the oscillations by sliding a point mass can be derived by analysing the energy balance or the Coriolis forces. As for studies on stabilization control of a pendulum whose centre of gravity is variable, a control law based on Lyapunov's method was proposed by Yoshida et al. [4]. Okanouchi et al. [5] also addressed the problem of suppressing the oscillation of a plane pendulum using three actuated variables (i.e., the horizontal and vertical positions of the pivot and the length of the pendulum) under amplitude constraints. In references [4,5], the effectiveness of the proposed control techniques was verified using simulations and experimental results. Szyszkowski and Stilling [6] investigated the damping properties of a frictionless oscillating physical pendulum with a moving mass and pointed out that the attenuation rule required the mass to move with a frequency that was double the pendulum frequency. A proportional derivative control law with additional gravity compensation was developed by Gutiérrez-Frias et al. [7] for active vibration damping in a frictionless physical pendulum with moving mass. However, reports published so far indicate that engineers find it difficult to construct controllers because of complex methodologies.

In this study, we develop a novel control method for a pendulum via cable length manipulation; in this method, a metaheuristic algorithm is used to easily construct the control scheme. As is well known, when the length of a pendulum changes periodically and the

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frequency is close to twice the natural frequency, parametric resonance can occur (e.g., [8]). Thus, to control the sway angle by using the reeling and unreeling of the hoisting cable, we adopt non-linear feedback control by utilising the parametric resonance phenomenon, in which the control input is defined as the acceleration of the cable. The feedback gains are determined using particle swarm optimization (PSO) [9], which is an evolutionary computation technique, to reduce the sway angle to the maximum extent possible. An important feature of the proposed method is that the controller can be easily constructed using the PSO technique. The effectiveness of the proposed control technique for suppressing the pendulum oscillation via cable length manipulation is demonstrated through simulations and experiments.

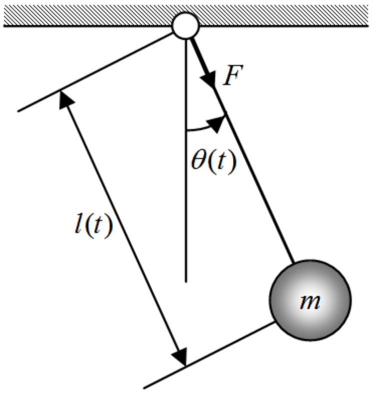

Figure 1. A variable length pendulum.

2. MATHEMATICAL MODEL

Figure 1 shows a pendulum with variable length, where l is the cable length, θ is the sway angle of the load, m is the mass of the load, and F are the control forces applied to the load. We assume that the cable is a massless rigid link and that the load is a point mass. In addition, the load is constrained to move in the vertical plane. On the basis of this assumption, the equations of motion for the system can be written as (e.g., [3])

$$m\ddot{l} - ml\dot{\theta}^2 - mg\cos\theta = F\ , \tag{1}$$

$$\ddot{\theta} + 2\frac{\dot{l}}{l}\dot{\theta} + \frac{g}{l}\sin\theta = 0\ , \tag{2}$$

where g is the acceleration of gravity. It should be noted that the dynamics of the load hoisting and swing are described by Eqs. (1) and (2), respectively. If the desired cable length l is controlled, the dynamics of the load hoisting can be negligible. The present study adopts this assumption, and hence, only Eq. (2) is used in the vibration control scheme mentioned in the next section.

3. CONTROL DESIGN

In this section, we propose a non-linear feedback control scheme for the oscillation attenuation of a pendulum via cable length manipulation. The validity of the proposed method is confirmed numerically.

3.1 Non-linear feedback control

In the present study, we attempt to attenuate the oscillation of a pendulum via cable length manipulation. Generally speaking, it is difficult to control the motion of a pendulum by vertical up-and-down motion. On the other hand, when a pendulum is subjected to a vertical harmonic excitation whose frequency is close to twice the natural frequency, a parametric resonance phenomenon can occur. Thus, we propose a control law utilising the parametric resonance.

In the proposed control scheme, the control input is defined as the acceleration of the cable for the purpose of the cable length manipulation. Under the following condition

$$\ddot{l} > l\dot{\theta}^2 + g\cos\theta\ , \tag{3}$$

the cable sags. To simply avoid this situation, we impose the following constraint on the acceleration of the cable:

$$|\ddot{l}| < 8.0\ \mathrm{m/s^2}\ . \tag{4}$$

Taking into account Eq. (4), a non-linear feedback control is defined as

$$u \equiv \ddot{l} = \begin{cases} U\ , & (|U| \le 8) \\ 8\mathrm{sign}(U), & (|U| > 8) \end{cases}, \tag{5}$$

$$U = -\left[k_1|\theta| + k_2(l - l_0) + k_3\dot{l}\right], \tag{6}$$

where u is the control input, sign(•) is the signum function, and l_0 is a target length of the cable. The term $k_1|\theta|$ is introduced to move the cable in twice the frequency of the pendulum oscillation, in which the parametric resonance is utilized to suppress the sway angle. Furthermore, it is desirable to control not only the sway angle but also the cable length. Thus, PD control for the cable length is given by the terms $k_2(l\text{-}l_0)$ and $k_3\dot{l}$.

Because the governing equation (2) and the control law (5) and (6) are non-linear, it is very difficult to analytically solve the feedback gains (k1, k2, and k3) in Eq. (6) for the stabilization of the system. Hence, in the present study, the feedback gains are determined by use of the PSO. The algorithm for the non-linear control design method using the PSO will be mentioned in the next subsection.

3.2 Control design using PSO

In the algorithm, the feedback gains (k_1, k_2, and k_3) are considered to be the optimized parameters. First of all, the optimized parameters are randomly initialized. By using the control law (5) and (6), the sway angle of the pendulum is calculated by numerically integrating (2) until time t = 10 s, where the feedback gains are set to k_1= 0, k_2= 10, and k_3= 5 in the time interval ($8 \le t \le 10$ s). In order to reduce the pendulum oscillation as much as possible, the fitness value f is defined as

$$f = |\theta|_{\max}\ , \tag{7}$$

where $|\theta|_{\max}$ is the maximum absolute value of the sway angle in the time interval ($8 \le t \le 10$ s). In addition, to realise the suppression of oscillations within a certain cable length range, we consider the following constraint

$$l_L \le l \le l_U \,. \tag{8}$$

If Eq. (8) is not satisfied, we add a large penalty to the fitness value (i.e., this operation is a penalty function method to handle constraints in the constrained optimization problem). According to the PSO algorithm, the fitness value can be minimised and then the optimal feedback gains can be obtained. The algorithm for tuning the feedback gains based on the PSO can be summarized as follows:

1) The positions and velocities of all particles are initialized randomly. The position and velocity vectors of the i-th particle are respectively defined as

$$\boldsymbol{x}_i = [x_{i,1}, x_{i,2}, \cdots, x_{i,d}]\,, \boldsymbol{v}_i = [v_{i,1}, v_{i,2}, \cdots, v_{i,d}]\,, \tag{9}$$

where the elements of the position $x_{i,j}$ represent the optimized parameters (i.e., the values of the feedback gains), and d is the dimension of the search space (i.e., the number of the feedback gains).

2) Evaluate the fitness value (7) from the numerical integration of Eq. (2) by using Eqs. (5) and (6), in which the cable length constraint (8) is checked. Next, set $\boldsymbol{pbest}_i$, which represents the previous best position vector of the i-th particle, to the fitness value. Furthermore, choose the position vector with the best fitness value among all the particles as $\boldsymbol{gbest}$.

3) The velocity and position vectors of the i-th particle are updated using the following equations:

$$\begin{aligned}\boldsymbol{v}_i^{(n+1)} = \chi[\boldsymbol{v}_i^{(n)} + c_1 r_1^{(n)}(\boldsymbol{pbest}_i - \boldsymbol{x}_i^{(n)}) \\ + c_2 r_2^{(n)}(\boldsymbol{gbest} - \boldsymbol{x}_i^{(n)})],\end{aligned} \tag{10}$$

$$\boldsymbol{x}_i^{(n+1)} = \boldsymbol{x}_i^{(n)} + \boldsymbol{v}_i^{(n+1)}\,, \tag{11}$$

where n stands for the iteration number, $r_1^{(n)}$ and $r_2^{(n)}$ are two independent uniform random numbers with values from 0 to 1. In addition, the following relation prevails among the coefficients χ, c_1, and c_2.

$$\chi = \frac{2}{|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}\,|}, \quad \varphi = c_1 + c_2, \quad \varphi > 4\,. \tag{12}$$

Typically, c_1 and c_2 are both set to 2.05.

4) Calculate the fitness value of all the particles using the same procedure as described in Step 2). For each particle, if the current fitness value is better than $\boldsymbol{pbest}_i$, then $\boldsymbol{pbest}_i$ is replaced by its position vector. If the best value in the current $\boldsymbol{pbest}_i$ is better than $\boldsymbol{gbest}$, then $\boldsymbol{gbest}$ is replaced by its position vector.

5) If n is less than the maximum iteration number, then the iteration number is updated as $n = n + 1$ and Steps 2)-4) are repeated. The value of $\boldsymbol{gbest}$ obtained finally is inferred as the optimal solution (i.e., the optimal feedback gains are determined).

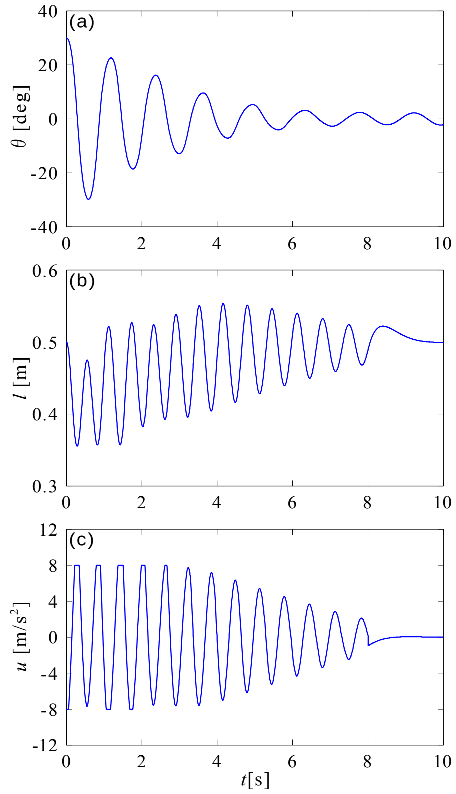

Figure 2. Simulation results: (a) sway angle, (b) cable length, and (c) control input.

3.3 Numerical results

Numerical examples are considered to evaluate the performance of the proposed cable length manipulation technique for the vibration control of the pendulum. In the following numerical simulations using the PSO algorithm, the number of particles is set to 20 and the maximum number of iterations is set to 50. The search space for the optimised parameters is considered to be as follows:

$$k_1 \in [0, 150], \;\; k_2 \in [0, 500], \;\; k_3 \in [0, 50]\,. \tag{13}$$

The initial conditions for the sway angle and cable length are taken as

$$\theta(0) = 30\ \text{deg}, \;\; \dot{\theta}(0) = 0, \;\; l(0) = 0.5\ \text{m}, \;\; \dot{l}(0) = 0\,. \tag{14}$$

The target cable length in Eq. (6) and the constraint condition in Eq. (8) are set to be

$$l_0 = 0.5\ \text{m}, \;\; l_L = 0.3\ \text{m}, \;\; l_U = 0.7\ \text{m}\,. \tag{15}$$

For the above parameters, the optimal feedback gains obtained using the PSO algorithm are as follows:

$$k_1 = 19.37, \;\; k_2 = 90.45, \;\; k_3 = 1.631\,. \tag{16}$$

Figure 2 shows time histories of the sway angle, cable length, and control input. It should be noted that the feedback gains are set to k_1= 0, k_2= 10, and k_3= 5 in the time interval ($8 \le t \le 10$ s), as mentioned in the previous subsection. It can be observed from Figure 2(a) that the pendulum oscillation decays sufficiently, thereby ensuring the validity of the proposed control technique. Comparing Figures 2(a) and 2(b), it can be seen that the frequency of the cable length is about twice that of the sway angle. This fact indicates that the effect of the term $k_1|\theta|$ in the non-linear feedback attenuates the pendulum oscillation.

4. EXPERIMENTAL VALIDATION

Experiments are performed to validate the non-linear feedback control scheme developed in the previous section.

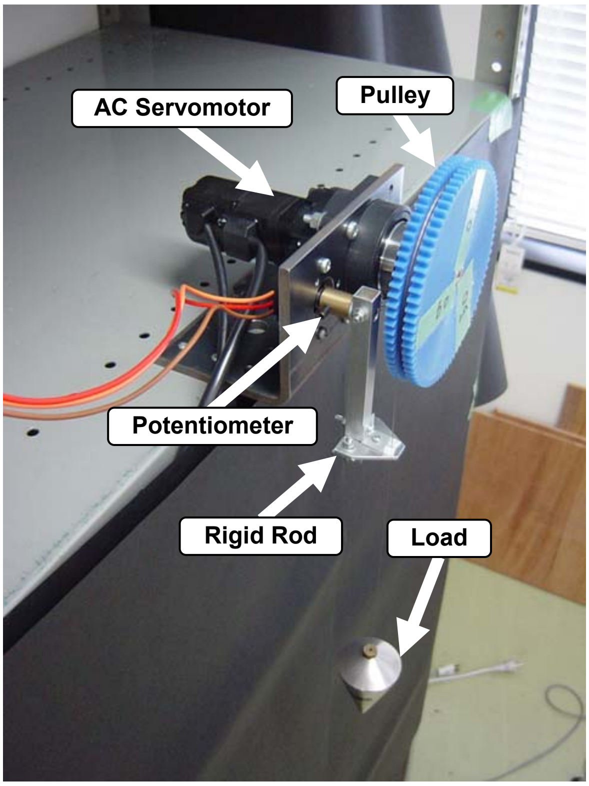

Figure 3. Photograph of the experimental setup.

4.1 Experimental setup

Figure 3 shows a photograph of the experimental setup for a pendulum with variable length. The cable length can be changed by using a pulley, which is actuated by an AC servomotor (SGMAH; Yaskawa Electric Corp.). The servomotor is operated in the speed control mode, which facilitates the control of the desired cable length using a servo drive unit (SGDM; Yaskawa Electric Corp.). A serial encoder mounted on the servomotor measures the pulley angle, and then, a measurement of the cable length is performed. As shown in Figure 3, the cable is passed through a slit of a rigid rod, whose angle is measured by a potentiometer (SP2800; B & PLUSS K. K.). The sway angle of the pendulum is acquired from the potentiometer. The measurement and control of the experimental setup are implemented on a DSP board (DS1104; dSPACE GmbH), which has a 500 Hz sampling rate.

Figure 4(a) shows the free vibration of the pendulum measured by the potentiometer when the cable length is 0.5 m. As can be seen in Figure 4(a), the data of the potentiometer signal contains some noise, which has a harmful influence on the performance of the feedback control scheme. To eliminate the noise in the potentiometer signal, we employ a nonlinear digital filter to Estimate the Smoothed and Differential values of the sensor inputs by using Sliding mode system (ESDS) [10, 11]. The system of the ESDS can be given by the following differential equations:

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -UR^2\mathrm{sign}[aR(x_1 - y) + x_2] \end{aligned}\right\}, \tag{17}$$

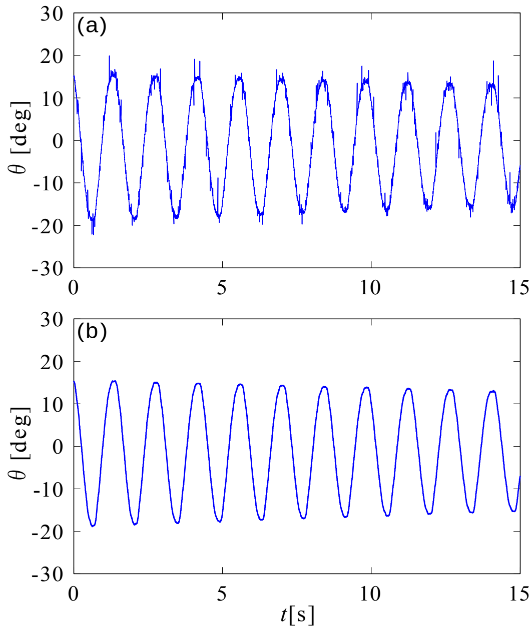

Figure 4. Free vibration of the experimental setup: (a) potentiometer signal and (b) values estimated by ESDS.

where x_1 is the noise removed signal (the estimated angle) and y is the input signal (the potentiometer signal). The parameters UR^2 and aR are chosen as 100 and 30, respectively. The sway angle estimated by the ESDS is presented in Figure 4(b). As depicted in Figure 4(b), the estimated value is smooth and the noise is perfectly removed. In the following experiments, we adopt the ESDS as the noise removal filter for the potentiometer signal, and then, the estimated value is used in the non-linear feedback control scheme.

On the other hand, it can be observed from Figure 4 that the pendulum oscillation does not decay with time. Therefore, we can recognise the experimental setup to be a frictionless pendulum.

4.2 Experimental results

In the previous section, we show from the numerical simulations that the proposed cable length manipulation technique facilitates the suppression of the pendulum oscillation. To further evaluate the feasibility of the vibration control scheme, we perform experiments.

The time histories of the experimental results obtained by using the non-linear feedback law (5) and (6) are illustrated in Figure 5. Here, the feedback gains, initial cable length, and target cable length are the same as those in Figure 2. It should be noted that Figure 5(a) displays the data obtained from the ESDS. A comparison between Figure 4(b) and 5(a) demonstrates that the proposed feedback controller, whose gains are tuned by the PSO, is quite effective in attenuating the oscillation of the pendulum. As can be seen in Figures 5(a) and 5(b), the frequency of the cable length is about twice that of the sway angle as observed in the numerical results, and hence, we reconfirm the effect of the term $k_1|\theta|$ in the non-linear feedback law. Moreover, the cable length converges to the target length with a decrease in the amplitude of the sway angle. This confirms the effectiveness of the terms $k_2(l\text{-}l_0)$ and $k_3\dot{l}$ in Eq. (6).

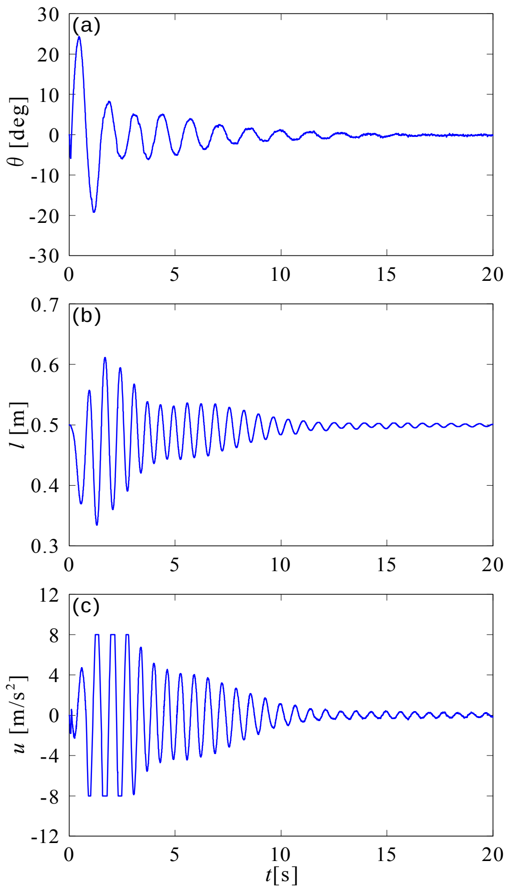

Figure 5. Experimental results: (a) sway angle, (b) cable length, and (c) control input: (l(0)=0.5 m, l_0=0.5 m, l_L=0.3 m, l_U=0.7 m, k_1=19.37, k_2=90.45, k_3=1.631).

In order to further check the feasibility of the proposed method for controlling the vibration, we change the cable length conditions and then perform the simulations and experiments. Figures 6 and 7 respectively present experimental results under the cable length conditions ($l(0)$=0.6 m, l_0=0.5 m, l_L=0.3 m, l_U=0.8 m) and ($l(0)$=0.7 m, l_0=0.7 m, l_L=0.5 m, l_U=0.9 m). It should be noted that the initial length $l(0)$ and target length l_0 are not mutually equal in Figure 6. The feedback gains are obtained as k_1=18.67, k_2=92.04, k_3=1.904 for ($l(0)$=0.6 m, l_0=0.5 m, l_L=0.3 m, l_U=0.8 m) and k_1=14.97, k_2=68.14, k_3=1.000 for ($l(0)$=0.7 m, l_0=0.7 m, l_L=0.5 m, l_U=0.9 m). It is apparent in Figures 6 and 7 that the present method can control both the sway angle and the cable length under different cable conditions. Therefore, we can say that the proposed cable length manipulation technique for a pendulum with variable length facilitates the simultaneous control of the sway angle and the cable length.

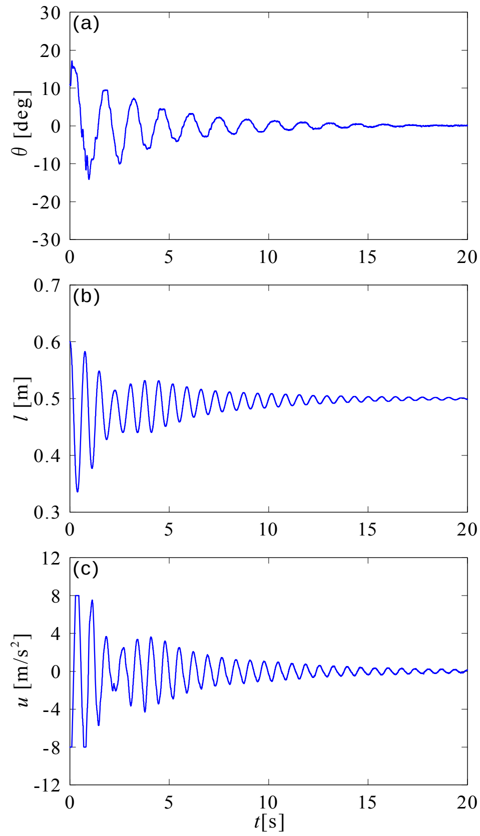

Figure 6. Experimental results: (a) sway angle, (b) cable length, and (c) control input: (l(0)=0.6 m, l_0=0.5 m, l_L=0.3 m, l_U=0.8 m, k_1=18.67, k_2=92.04, k_3=1.904).

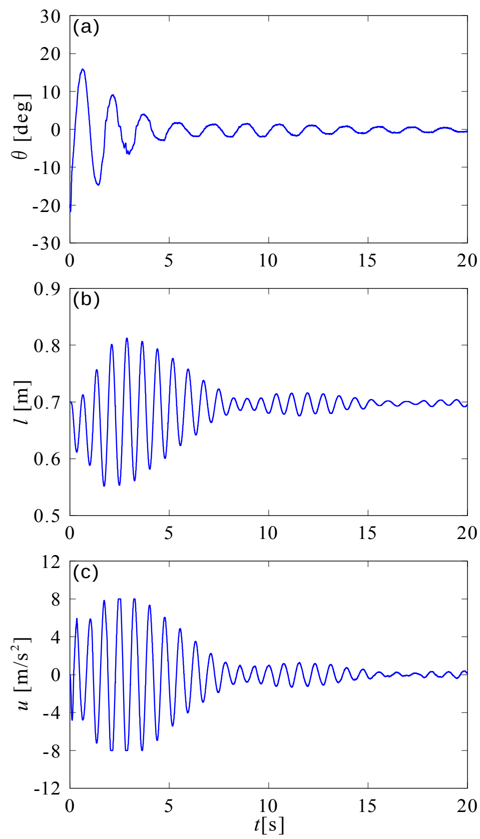

Figure 7. Experimental results: (a) sway angle, (b) cable length, and (c) control input: (l(0)=0.7 m, l_0=0.7 m, l_L=0.5 m, l_U=0.9 m, k_1=14.97, k_2=68.14, k_3=1.000).

5. CONCLUSIONS

In this study, we develop a cable length manipulation technique for the vibration control of a pendulum with variable length. In the proposed non-linear feedback control law, we introduce a term that allows us to move the cable at a frequency that is twice the frequency of the pendulum oscillation. A parametric resonance phenomenon is thus utilised to suppress the sway angle. Because the governing equation for the pendulum and the proposed control law are non-linear, it is very difficult to analytically solve the feedback gains for the stabilization of the system. Thus, we attempt to tune the feedback gains by using a PSO algorithm. The PSO is a metaheuristic algorithm, and it facilitates the easy determination of the feedback gains without any control knowledge. From the experimental results, we confirm that the controller tuned by the PSO realises not only oscillation attenuation but also cable length control. Therefore, it is concluded that the proposed control scheme is effective for the oscillation attenuation of a pendulum whose length can be variable as a control input.

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ТЕХНИКА НЕЛИНЕАРНОГ УПРАВЉАЊА КЛАТНОМ ПРОМЕНОМ ДУЖИНЕ УЖЕТА: ПРИМЕНА ОПТИМИЗАЦИЈЕ РОЈЕМ ЧЕСТИЦА КОД ПРОЈЕКТОВАЊА УПРАВЉАЧА

Akira Abe

Рад приказује нову технику управљања вибрацијама клатна променом дужине ужета. У циљу управљања углом њихања применом намотавања и одмотавања дизаличног ужета развили смо шему нелинеарног управљања у повратној спрези коришћењем параметарске резонанце код које је улаз управљања дефинисан као убрзање ужета. Како су главна једначина и закон управљања нелинеарни, веома је тешко аналитички решити утицај повратне спреге у стабилизовању система. Отуда су учинци повратне спреге одређени применом оптимизацијеројем честица (ПСО), еволутивном техником израчунавања, да би се угао њихања редуковао до максимално могућег. Валидност предложене технике управљања је потврђена нумеричким симулацијама. У циљу верификације изводљивости приказане методе извршени су експерименти. На основу резултата експеримената показали смо да је примена ПСО алгоритма валидна за усклађивање учинка повратне спреге и да је предложена шема нелинеарног управљања у повратној спрези ефикасна за управљање вибрацијама клатна са промењивом дужином ужета.