Multi-tier Shuttle-based Storage and Retrieval Systems

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In this paper analytical travel time model for the computation of cycle times and throughput performance of the multi-tier shuttle-based storage and retrieval system, is presented. The proposed model considers the real operating characteristics of the shuttle carrier and the elevators lifting table. Assuming uniform distributed storage locations and the probability theory, expressions for the expected cycle time of the elevators lifting table and the shuttle carrier were determined. The proposed model enables the calculation of the expected cycle time, from which the performance of the multi-tier shuttle-based storage and retrieval system can be evaluated

Keywords: *automated warehouses, shuttle-based systems, analytical modelling, performance analysis.*

1. INTRODUCTION

Automated warehouses represent important aspect in the global supply chain. Warehouses are needed for the following reasons [1]: (*i*) an imbalance in the flow and outflow of goods due to the inappropriate dynamics of production and consumption, (*ii*) taking goods from numerous producers for the production of combined shipments, (*iii*) the realization of the daily supply of goods in the production and distribution, (*iv*) the realization of additional activities, such as packaging, final assembly, etc.

Nowadays relatively large numbers of newer warehouse installations are represented with a technology called Shuttle-Based Storage and Retrieval Systems (SBS/RS). This technology already exists in the industry for nearly 10 years or more and is used in case of large demand for the throughput capacity of totes.

The SBS/RS is composed of multiple parallel aisles of Storage Racks (SR), elevator for each aisle of the SR, multi-tier shuttle carriers, Input and Output (I/O) location, buffer position in each tier and roller conveyors (see Figure 1). Compared to single-tier shuttle carriers, which can operate in one tier only, multi-tier shuttle carriers can operate and serve in multiple tiers of the SR (e.g. 5 tiers).

Advantages of the application of SBS/RS are: (*i*) efficient utilization of the warehouse space, (*ii*) reduction of damage and loss of goods, (*iii*) increased control upon storage and retrieval of goods and (*iv*) decrease in the number of warehouse workers.

On the other hand, SBS/RS require a high initial investment compared to other types of automated warehouses (mini-load AS/RS). Therefore, a careful design of SBS/RS is crucial for the SBS/RS to be successful. The performance of the SBS/RS is often evaluated by the number of totes per hour, which will be presented in our paper.

The most related paper to the studied system is completed by Carlo and Vis [2]. They studied a type of SBS/RS where there are two non-passing lifting systems mounted along the rack. They focused on scheduling problem where two (piece-wise linear) functions are introduced to evaluate candidate solutions.

Marchet et al. [3] studied main design trade-offs for SBS/RS using simulation, and completed their study for several warehouse design scenarios for 2 types of SBS/RS configurations: tier captive and tier-to-tier vehicles.

Lerher [4] studied energy regeneration and energy efficiency models for SBS/RS. The proposed models enable reduction of energy consumption and consequently the CO_2 emission, which is vital from economic and environmental point of view.

Recently, Lerher et al. [5] and Lerher et al. [6] studied the efficiency for single-tier SBS/RS. The proposed models enable calculation of performance activity for single-tier SBS/RS.

Sari at al. [7] studied experimental validation of travel time models for shuttle-based automated storage and retrieval system.

The majority of current research is based on the single-tier SBS/RS, only. Since many producers of the warehouse equipment like Dematic, Knapp, Gebhardt, Stöcklin, Savoye, etc., have begun to offer other advanced systems of SBS/RS, there is a clear necessity to study multi-tier SBS/RS. The latter was a motivation for the proposed research work.

2. TRAVEL-TIME MODEL OF THE MULTI-TIER SBS/RS

When developing proposed analytical travel time model for multi-tier SBS/RS, the following assumptions were considered:

- The multi-tier SBS/RS is divided into two sides in a picking aisle. Therefore, totes can be stored in either side in a tier.
- The input/output (I/O) location of the multi-tier SBS/RS is located at the first tier, next to the lift location (Figure 1).

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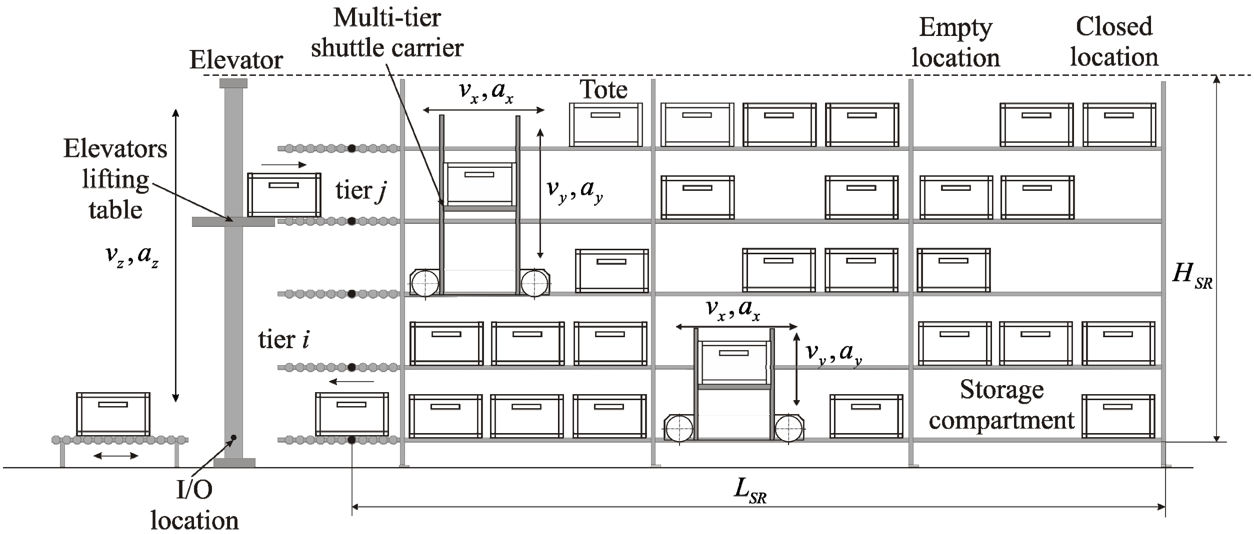

Figure 1. Multi-tier shuttle-based storage and retrieval system

- The storage rack is divided by columns and tiers. At each tier there are two buffer positions (left and right) – Figure 1.
- The elevator manipulates two lifting tables independently one of which is located at the left side and the other one is located at the right side of the elevator. Each lifting table can serve one tote at a time (Figure 2 – right side).

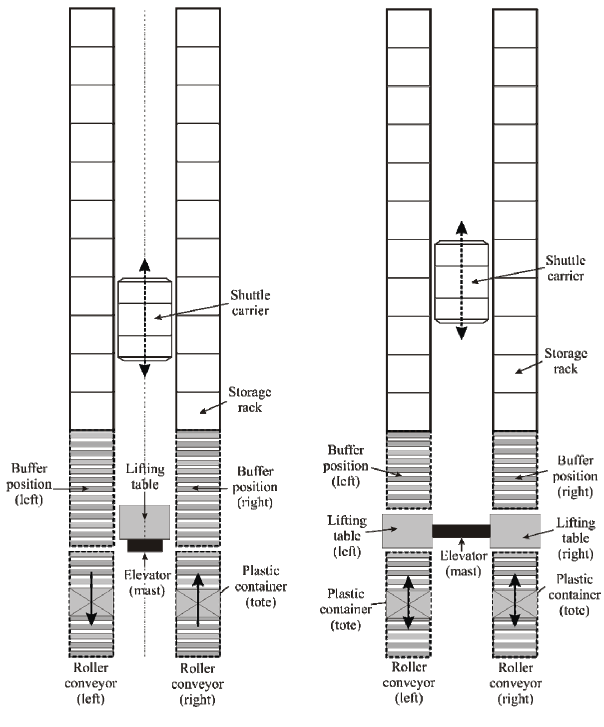

Figure 2. Elevator with one or two lifting tables (Knapp Logistik)

- The elevator and the shuttle carrier operate on the basis of Single Command (SC) cycles and Dual Command (DC) cycles.
- Drive characteristics (v_z, a_z) of the elevators lifting tables as well as the height H of the SR are known in advance.
- Drive characteristics of the shuttle carrier (v_x, a_x, v_y, a_y) as well as the length L of the SR and the height of the tier are known in advance (Figure 3).

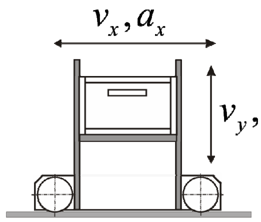

Figure 3. Multi-tier shuttle carrier (Gebhardt Foerder–technik)

- The height H and length L of the storage racks are large enough for the elevators lifting table and the shuttle carrier to reach their maximum velocity v_{max} in the vertical and in the horizontal direction.
- Randomized assignment policy is considered which means that any storage position is equally likely to be selected for storage or retrieval location to be processed.

2.1 Fundamentals of travel time

Two types of velocity profiles can be distinguished depending on whether the obtained peak velocity $v(t_p)$ is less than $v_{\max}$ (type I) or equal to $v_{\max}$ (type II).

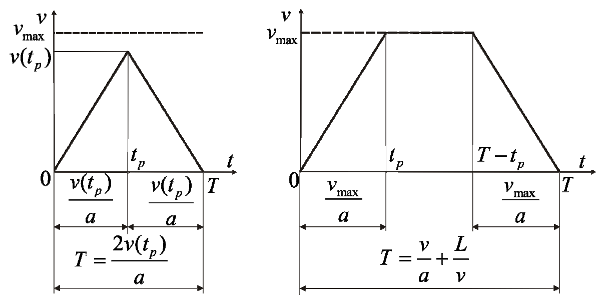

Figure 4. Velocity-time relationship

- **Travelling for type I ($T < 2v_{\max} / a$)**

The velocity in dependence of time $v(t)$, is calculated by (1):

$$v(t) = \begin{cases} at, & t \in (0, t_p) \\ -a(t-T), & t \in (t_p, T) \end{cases} \qquad (1)$$

The distance in dependence of time $d(T)$, is calculated by (2):

$$d(T)=\int_0^T v(t)dt=\frac{a\cdot T^2}{4} \quad (2)$$

Because of the acceleration and deceleration are equal in magnitude, the time necessary to reach the peak velocity equals $t_p = T / 2$.

- **Travelling for type I ($T > 2v_{max} / a$)**

The velocity in dependence of time $v(t)$, is calculated by (3):

$$v(t)=\begin{cases} at, & t\in(0,t_p) \\ v_{max}, & t\in(t_p,T-t_p) \\ -a(t-T), & t\in(T-t_p,T) \end{cases} \quad (3)$$

The distance in dependence of time $d(T)$, is calculated by (4):

$$d(T)=\int_0^T v(t)dt=v_{max}\cdot T-\frac{{v_{max}}^2}{a} \quad (4)$$

2.2 Shuttle carrier travelling in the horizontal and in the vertical direction

Travel times of the shuttle carrier are variable times. They depend from the kinematics properties of the shuttle carrier and the hoisted carriage of the shuttle carrier, the length of the SR, the height of the tier and the storage policy (random storage policy).

The travel time t_{xy} from the I/O$_{\text{tier } i}$ point to any randomly selected location in the SR is the maximal value of t_x or t_y, where t_x is the horizontal travel time and t_y is the vertical travel time. According to the condition of uniform distribution of storage locations and to the condition of the x- and y-coordinates independence, the cumulative distribution functions $F_x(t)$ and $F_y(t)$ have been accomplished.

The cumulative distributions functions $F_x(t)$ and $F_y(t)$ are distinguished according to the following condition: (*i*) shuttle carrier traveling for type I. and (*ii*) shuttle carrier traveling for type II.

- **Traveling of the shuttle carrier from the I/O$_{\text{tier } i}$ point to any randomly selected location in the storage rack**

– Shuttle carrier traveling in the horizontal direction, is calculated by (5):

$$F_x(t)=\begin{cases} \dfrac{a_x t^2}{4L} & \left(0\le t\le t_x\right) \\ \dfrac{v_x t}{L}-\dfrac{{v_x}^2}{a_x L} & \left(t_x\le t\le T_x\right) \end{cases} \quad (5)$$

– Hoisted carriage of the shuttle carrier lifting in the vertical direction, is calculated by (6):

$$F_y(t)=\begin{cases} \dfrac{a_y t^2}{4h} & \left(0\le t\le t_y\right) \\ \dfrac{v_y t}{h}-\dfrac{{v_y}^2}{a_y h} & \left(t_y\le t\le T_y\right) \end{cases} \quad (6)$$

- **Traveling of the shuttle carrier from the storage location to any randomly selected retrieval location in the SR**

– Shuttle carrier traveling in the horizontal direction, is calculated by (7):

$$F_x(t)=\begin{cases} \dfrac{a_x}{2L}t^2-\dfrac{a_x^2}{16L^2}t^4 & \left(0\le t\le t_x\right) \\ -\dfrac{v_x^2}{L^2}t^2+\left[\dfrac{2v_x^3}{a_x L^2}+\dfrac{2v_x}{L}\right]t- & \\ \dfrac{2v_x^2}{a_x L}-\dfrac{v_x^4}{a_x^2 L^2} & \left(t_x\le t\le T_x\right) \end{cases} \quad (7)$$

– Hoisted carriage of the shuttle carrier lifting in the vertical direction, is calculated by (8):

$$F_y(t)=\begin{cases} \dfrac{a_y}{2h}t^2-\dfrac{a_y^2}{16h^2}t^4 & \left(0\le t\le t_y\right) \\ -\dfrac{v_y^2}{h^2}t^2+\left[\dfrac{2v_y^3}{a_y h^2}+\dfrac{2v_y}{h}\right]t- & \\ \dfrac{2v_y^2}{a_y h}-\dfrac{v_y^4}{a_y^2 h^2} & \left(t_y\le t\le T_y\right) \end{cases} \quad (8)$$

The distribution function $F(t)$ depends on the relationships among the values of the following parameters: v_x, v_y, a_x, a_y, L, h. Therefore, $F_k(t)$ can be specified with (9):

$$F_k(t)=F_x(t)\cdot F_y(t) \quad (9)$$

The expected travel time $E_k(t)$, corresponding to the k^{th} condition is equal to (10):

$$\mathrm{E}_k\left(t\right)=\int_0^{\max.(T_x,T_y)}(1-F_k(t))dt \quad (10)$$

For a more detailed insight into the analysis of travel time by considering the real operating characteristics of material handling equipment, see paper of Hwang and Lee [7].

2.3 Lifting of the elevators lifting table in the vertical direction

Travel times of the elevators lifting table are variable times. They depend from the kinematics properties of the elevators lifting table, the height of the SR and the storage policy (random storage policy).

The travel time t_z from the I/O location to the I/O$_{\text{tier } i}$ point is the vertical travel time. According to the condition of uniform distribution of tiers and to the condition of the *y*-coordinate independence, the cumulative distribution function $F_z(t)$ has been accomplished.

The cumulative distributions function $F_z(t)$ is distinguished according to the following condition: (*i*) elevators lifting table lifting for type I. and (*ii*) elevators lifting table lifting for type II.

- **Lifting of the elevators lifting table from the I/O to the $I/O_{tier\ i}$ point, is calculated by (11):**

$$F_z(t)=\begin{cases}\dfrac{a_z t^2}{4H} & \left(0\le t\le t_z\right)\\ \dfrac{v_z t}{H}-\dfrac{v_z{}^2}{a_z H} & \left(t_z\le t\le T_z\right)\end{cases} \qquad (11)$$

- **Lifting of the elevators lifting table from the $I/O_{tier\ i}$ point to the $I/O_{tier\ j}$ point, is calculated by (12):**

$$F_z(t)=\begin{cases}\dfrac{a_z}{2H}t^2-\dfrac{a_z^2}{16H^2}t^4 & \left(0\le t\le t_z\right)\\ -\dfrac{v_z^2}{H^2}t^2+\left[\dfrac{2v_z^3}{a_z H^2}+\dfrac{2v_z}{H}\right]t- & \\ \dfrac{2v_z^2}{a_z H}-\dfrac{v_z^4}{a_z^2 H^2} & \left(t_z\le t\le T_z\right)\end{cases} \qquad (12)$$

The distribution function $F(t)$ depends on the relationships among the values of the following parameters: v_z, a_z, H. Thus, $F_k(t)$ can be specified with (13):

$$F_k(t)=F_z(t)\,. \qquad (13)$$

The expected travel time $E_k(t)$, corresponding to the k^{th} condition is equal to (14):

$$\mathrm{E}_k\left(t\right)=\int_0^{T_z}(1-F_k(t))dt \qquad (14)$$

For more detail insight for analysis of travel time by considering the real operating characteristics of material handling equipment, see paper of Hwang and Lee [7].

3. MULTI-TIER SBS/RS CASE STUDY

In this study totes with the following dimensions: length l_{tote} = 0.6 m, width w_{tote} = 0.4 m and height h_{tote} = 0.24 m have been used. With regard to the tote, the storage location has the following dimensions: length (depth) of the column l_{COM} = 0.6 m, width of the column w_{COM} = 0.5 m and height of one column (tier) h_{COM} = 0.35 m. The multi-tier shuttle carrier serves six tiers simulta–neously, which means that one level is h = 2,1 m high.

Dimensions of the SBS/RS storage rack (L and H) depends on the number of columns C in the horizontal direction and number of tiers M in the vertical direction, which give us the warehouse volume Q = 2160 totes (18 · 60 · 2). Pick up and set down times for the shuttle carrier and for the elevators lifting tabele were set to tP/S SCAR = 3,4 sec., tP/S LIFT = 1,5 sec.

Table 1. Multi-tier SBS/RS configuration

SBS/RS configuration	Multi-tier SBS/RS			
	M (/)	*C* (/)	*L* (m)	*H* (m)
1	18	60	30	6.30*

Note: multi-tier SBS/RS configuration was selected according to the references of material handling equipment producers and practical experiences of the authors. *6 tiers multiplied by 0,35 m equals 2,1 m and 2,1 m multiplied by 3 levels equals 6,3 m.

The efficiency of the multi-tier SBS/RS depends on the velocity characteristics. In this study, the proposed velocity profiles vp_i were used (Table 2).

Table 2. Velocity profile of the multi-tier SBS/RS

Multi-tier shuttle carrier velocity profiles	Multi-tier shuttle carrier			
	v_x (m/s)	a_x^+ (m/s^2)	v_y (m/s)	a_y (m/s^2)
vp_1	2	1	1.5	1.5
vp_2	2	2	1.5	1.5
vp_3	4	1	1.5	1.5
vp_4	4	2	1.5	1.5
vp_5	4	3	1.5	1.5
vp_6	4	1	3	2.25
vp_7	4	2	3	2.25
vp_8	4	3	3	2.25

Note: Velocity scenarios vp_i were selected according to the references of material handling equipment producers and practical experiences of the authors.

4. ANALYSIS

The expected dual command cycle times (in seconds) for the multi-tier SBS/RS, are given on the basis of the performed analysis. Analysis has been conducted for selected multi-tier SBS/RS (Table 1) with eight different velocity profiles (Table 2) of the multi-tier SBS/RS.

Note: Throughput performance of the lift, which is equiped to be with two lifting tables, equals appoxi–mately 1000 totes/hour. For this reason, the throughput performance of the lift was not the case of this paper.

Expected dual command cycle time $E(DC)_{SCAR}$ and throughput performance $\lambda(DC)_{SCAR}$ depend on the number of columns C and the velocity profile vp_i (Table 3). The fastest double command transactions belong to the shuttle carrier with fast drives (vp_5, vp_8), meanwhile the slowest double command transactions belong to the shuttle carrier with moderate drives (vp_1, vp_2).

According to the distribution of the $E(DC)_{SCAR}$, velocity profile vp_i has a significant impact on the expected dual command cycle time (Figure 5). A decreasing tendency of $E(DC)_{SCAR}$ are observed for the velocity profile vp_3, vp_4, vp_5 and vp_6, vp_7, vp_8 compared to vp1, vp2. This relationship shows the influence of the horizontal velocity v_x and acceleration a_x in accordance to the length of the SBS/RS. Generally, the best results are achieved with the shuttle carrier having fast drives in the horizontal travelling direction (Figure 5).

Because the throughput capacity $\lambda(DC)_{SCAR}$ is inversly dependent on the expected dual command cycle time $E(DC)_{SCAR}$, the highest throughput capacity $\lambda(DC)_{SCAR}$ belongs to the SBS/RS with the shuttle carrier with fast drives (vp_7, vp_8). On the contrary the lowest throughput capacity $\lambda(DC)_{SCAR}$ belongs to the SBS/RS with shuttle carriers with moderate drives (vp_1, vp_2).

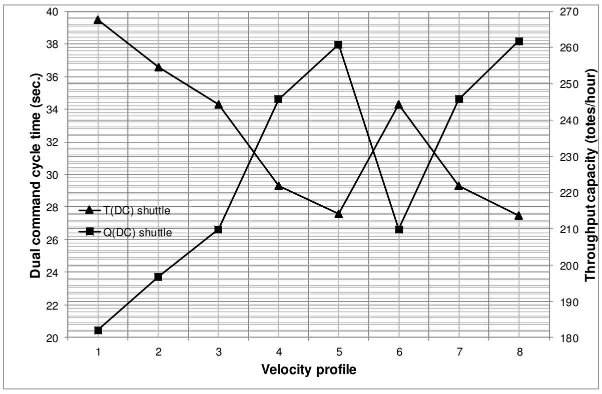

Figure 5. Cycle time and throughput performance

Table 3. Travel time and throughput performance of the multi-tier shuttle carrier

vp_i	Travel time and throughput performance analysis		
	$E(DC)_{SCAR}$ (sec.)	$\lambda(DC)_{SCAR}$ (totes/hour)	$\lambda(DC)_{SBS/RS}$ (totes/hour)
1	39.5*	182**	547***
2	36.6	197	590
3	34.3	210	630
4	29.3	246	737
5	27.6	261	783
6	34.3	210	630
7	29.3	246	737
8	27.5	262	785

The throughput performance of the SBS/RS as a whole $\lambda(DC)_{SBS/RS}$ depends on the number of multi-tier shuttle carriers in the SBS/RS (Table 3).

5. CONCLUSIONS

In this paper, analytical travel time model for multi-tier SBS/RS is presented. In the proposed model, the real operating characteristics of the multi-tier shuttle carrier and the elevators lifting table have been used. The proposed model considers the assumption of moving the hoisted carriage in the vertical direction and travelling of the shuttle carrier in the horizontal direction. Therefore, considering both independent movements of the shuttle carrier, the proposed analytical travel time model for multi-tier SBS/RS has been developed. The proposed analytical travel time model for multi-tier SBS/RS deals for the aisle-captive SBS/RS, where a single shuttle carrier serves a multiple tier simultaneously. Various elements of the multi-tier SBS/RS have been examined, such as the proposed layout of the SBS/RS and the velocity profiles of the shuttle carrier in order to investigate the efficiency of the proposed analytical travel time model for multi-tier SBS/RS.

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NOMENCLATURE

AS/RS	automated strorage and retrieval systems
DC	dual command
I/O	input and output location
SBS/RS	shuttle-based storage and retrieval systems
SC	single command
SR	storage rack
a	acceleration/deceleration
a_x	acceleration/deceleration of the shuttle carrier
a_y	acceleration/deceleration of the hoisted carriage of the shuttle carrier
a_z	acceleration/deceleration of the elevators lifting table
T	arrival time at a destination
$F(t)$	cumulative distribution function
$d(T)$	distance moved during time T
$E(DC)_{SCAR}$	expected dual command cycle time of the shuttle carrier
h_{tote}	height of the tote
h	height of the tier
h_{COM}	height of the cell (storage compartment)

H	height of the storage rack
l_{tote}	length of the tote
l_{COM}	length (depth) of the column
L	length of the storage rack
v_z	maximum velocity of the elevators lifting table in the vertical direction
v_x	maximum velocity of the shuttle carrier in the horizontal direction
v_y	maximum velocity of the hoisted carriega of the shuttle carrier in the vertical direction
$v_{\max}$	maximum velocity
C	number of columns
M	number of tiers
$v(t_p)$	peak velocity at time t_p
$t_{\text{P/S}}$	pick up and set down times
$\lambda(\text{DC})_{\text{SBS/RS}}$	SBS/RS system performance as a whole
$\lambda(\text{DC})_{\text{SCAR}}$	throughput capacity of the dual command cycle of the shuttle carrier
t_p	time necessary to reach the peak velocity
t	travel time
vp_i	velocity profile.
$v(t)$	velocity at time t
Q	warehouse volume
w_{tote}	width of the tote
w_{COM}	width of the column

СКЛАДИШТЕЊЕ У ВИШЕ НИВОА ПРИМЕНОМ ШАТЛ СИСТЕМА

Т. Лерхер

Рад приказује аналитички модел времена пута за израчунавање временског циклуса и перформанси протока код складиштења у више нивоа применом шатл система. Предложени модел узима у обзир радне карактеристике колица и подизних платформи лифта у реалном времену. Полазећи од претпоставке равномерно распоређених локација за складиштење и теорије вероватноће одређени су изрази за очекивани временски циклус рада подизне платформе лифта и колица. Предложени модел омогућава израчунавање очекиваног временског циклуса на основу кога се могу проценити перформансе складиштења у више нивоа применом шатл система.