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VAWT Optimization Using Genetic Algorithm and CST Airfoil Parameterization

Darrieus type vertical axis wind turbine (VAWT) is optimized using the genetic algorithm (GA). The airfoil shape is parameterized using the Class-Shape Transformation (CST) method. The double multiple stream tube (DMST) method with the Gormont dynamic stall modification is used for the calculation of the VAWT performance parameters. Once the numerical codes are validated using availible experimental results, the airfoil parameters are varied as to achieve the optimum value of the genetic algorithm fitness function.

Keywords: *genetic-algorithm, parameterization, CST, VAWT, DMST.*

1. INTRODUCTION

The Darrieus type VAWT are a safe, inexpensive solution for off-grid power generation. Their construction and maintenance are relatively simple, they work with lower rotational speeds and blade stresses than the HAWT and can be placed closer to the ground. They can also take wind from any direction and work efficiently in turbulent wind conditions. For these reasons VAWT are better suited in terms of use for power generation by a single household or similar smaller consumer. On the negative side they operate with unsteady aerodynamics and their operating characteristics are very complex. Most often they cannot start themselves and need external drives in order to reach optimal operating speeds.

In order to maximise VAWT torque and efficiency researchers increasingly use different optimization algorithms and methods. Recently researchers have been using coupled models containing performance prediction and optimization algorithms. Carrigan et.al. [1] used computational fluid dynamics to obtain the VAWT performance and differential evolution as the optimization technique. Ferreira et.al. [2] used 2D unsteady panel method for the performance and the genetic algorithm for optimization. They have also derived an objective function intended for airfoil optimization. Bedon et.al. [3] used blade element – momentum theory for the performance and an evolutionary algorithm for optimization. Paraschivoiu et.al. [4] used DMST and GA in order to optimize the azimuthal variation of the blade. The GA was also used for wind farms layout [5] and wind farms maintenance optimization [6].

The airfoil has a significant role in the VAWT efficiency and power coefficient. The most common airfoils in use in VAWTs today are the symmetric 4-digit NACA series, especially the NACA 0012, 0015 and 0018. Their aerodynamic characteristics have been extensively tested in wind tunnels [7-10].

There are numerous techniques and software packages for numerical analysis of airfoils. One of the more popular ones is the freeware XFOIL software developed by MIT [11,12]. This software will be used for obtaining the airfoils aerodynamic characteristics in this paper.

The most common method for airfoil parameterization is the use of Bezier curves. However Kulfan's CST method provides a very good approximation for WT airfoils using small number of control points and is increasingly used by researchers.

2. NUMERICAL MODELS

2.1 CST parameterization

The CST method was developed by Boeing employee Brenda Kulfan in 2006. Since then it has been extended a few times to account for different 2D and 3D geometries [13-16]. The method is based on Bezier curves and consists of a simple analytic shape function that controls the parameters such as leading edge radius, trailing edge boattail angle and closure to a specified aft thickness and a class function that generalizes the method for a variety of geometries.

The general form that represents the typical airfoil geometry is:

$$\zeta(\psi) = \sqrt{\psi} \cdot (1-\psi) \cdot \sum_{i=0}^{N} A_i \psi^i + \psi \zeta_T . \qquad (1)$$

where: $\psi = x/c$, $\zeta = z/c$ and $\zeta_T = \Delta\zeta_{TE}/c$.

The term $\sqrt{\psi}$ provides a round nose, the term $(1-\psi)$ insures a sharp trailing edge, the term $\psi\zeta_T$ provides control of the trailing edge thickness and the term $\sum_{i=0}^{N} A_i \psi^i$ represents a general function for the shape between the round nose and the sharp aft end. The term $\psi(1-\psi)$ represents the "class function" and in general form is defined as:

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$$C_{N2}^{N1}(\psi)=\psi^{N1}\cdot(1-\psi)^{N2} \tag{2}$$

For a NACA type round nose and pointed aft end airfoil the values of N1 and N2 are 0.5 and 1.0 respectively.

The shape function is derived from the basic geometric equation and has the form:

$$S(\psi)=\sum_{i=0}^{N} A(i)\cdot S(\psi,i)\,. \tag{3}$$

where $S(\psi,i)$ is the component shape function and is represented by a Bernstein polynomial. $A(i)$ is the set of curvature coefficients used to represent a given airfoil. The component shape function is given as:

$$S(\psi,i)=K_i^N\cdot\psi^i\cdot(1-\psi)^{N-i} \tag{4}$$

where K is the binomial coefficient defined as:

$$K_i^N=\frac{n!}{i!(n-i)!} \tag{5}$$

2.2 Genetic Algorithm

The genetic algorithm is based on evolutionary process of organisms in nature. It uses a randomly selected initial "population" with predefined bound limits. The population is composed of "individuals", which represent solutions of the problem computed for particular, randomly selected values of the optimization variables. The individuals are most often represented with a 0-1 binary chromosome representation scheme.

Once the algorithm has been initialized it performs a "selection" of the fittest individuals for the next generation. The individual's fitness is defined via a "fitness function". In order to prevent premature convergence to sub-optimal solutions few selection methods, such as the "uniform", "roulette", "tournament", "remainder" etc. are in use. In this paper the "roulette" selection function is used. After the selection, the individuals are "mated" together (they exchange genetic information) via a "crossover" function. In this manner set of new solutions or "offspring" is produced. The offspring individuals inherit characteristics from both "parents".

"Mutation" is often applied to some of the individuals after the crossover as a safety net in order to recover good genetic material that might have been lost through the procedures of selection and crossover. The mutation modifies elements in the chromosomes and is randomly applied usually in the range of probability between 0.001 and 0.01.

These procedures are repeated until a convergence or a stopping criteria is satisfied.

In order to perform an optimization an "objective function" needs to be defined. The objective function is the function that is going to be minimized or maximized with the GA. In this paper the objective function is set to be the inverse of the rotor power for given conditions of the wind turbine operation:

$$f_{\min}=\frac{1}{P},\ \text{where}\ \ P=\frac{1}{2}C_p\rho\vartheta^2 A \tag{6}$$

The traditional form of the genetic algorithm as given by Beasley et al. [15] is shown in Figure 1. More detailed overview of GAs is given in Refs. [17-20].

```
BEGIN /* genetic algorithm */
    generate initial population
    compute fitness of each individual

    WHILE NOT finished DO
    BEGIN /* produce new generation */

        FOR population_size / 2 DO
        BEGIN /* reproductive cycle */
            select two individuals from old generation for
            mating
                /*biased in favour of the fitter one */
            recombine the two individuals to give two
            offspring
            compute fitness for the two offspring
            insert offspring in new generation
        END

        IF population has converged THEN
            finished := TRUE
    END
END
```

Figure 1. Traditional Genetic Algorithm [15]

2.3 DMST

The double multiple streamtube model combines the multiple streamtube model and the double actuator disk theory. It was introduced by Paraschivoiu in 1981. In this model the multiple streamtube system is divided into two parts, downwind and upwind corresponding to two actuator disks in tandem. This allows for the calculation of the influence of the upwind part to the downwind part. Namely because the blade extracts energy in the upwind domain, the energy in the downwind domain is going to be reduced.

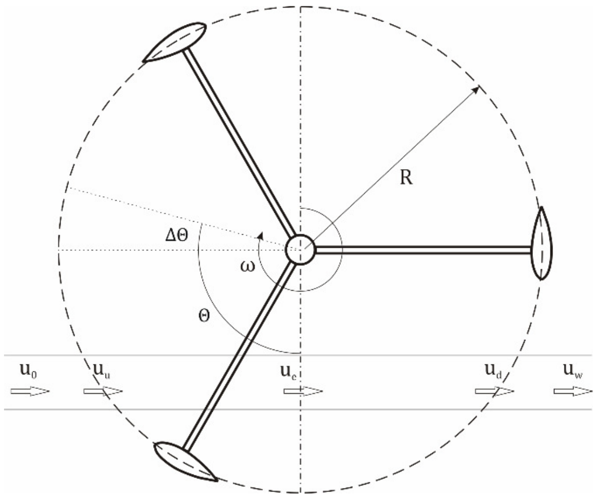

Figure 2. Double multiple streamtube model

The equations for the upwind and downwind part of the rotor are solved separately. The resultant velocity and the local angle-of-attack (hereinafter: AoA or α) can be stated as:

for the upwind domain $(\pi/2\le\theta\le 3\pi/2)$:

$$w_u=u_u\sqrt{(\lambda-\sin\theta)^2+(\cos\theta)^2} \tag{7}$$

$$\alpha = \sin^{-1}\left(\cos\theta \frac{u_u}{w_u}\right) \tag{8}$$

for the downwind domain $(3\pi/2 \le \theta \le \pi/2)$ **:**

$$(3\pi/2 \le \theta \le \pi/2) \tag{9}$$

$$\alpha = \sin^{-1}\left(\cos\theta \frac{u_d}{w_d}\right) \tag{10}$$

where $\lambda = R\omega/u_0$ is the tip speed ratio.

From here the non-dimensional thrust coefficient C_T for a single stream tube can be defined as:

$$C_T = \left(\frac{Bc}{2R}\right)\left(\frac{w}{u_0}\right)^2\left(\frac{2}{\pi}\right)\left(c_t \frac{\cos\theta}{\sin\theta} - c_n\right) \tag{11}$$

Where B is the number of blades, c is the airfoil chord, c_t and c_n are the tangential and normal forces coefficients respectively and are evaluated for each streamtube using the airfoil lift c_L and drag c_D coefficients for Reynolds number $\mathrm{Re} = \rho wc/\mu$:

$$c_t = c_L(\mathrm{Re})\cos\alpha + c_D(\mathrm{Re})\sin\alpha, \tag{12}$$

$$c_n = c_L(\mathrm{Re})\sin\alpha + c_D(\mathrm{Re})\cos\alpha. \tag{13}$$

The torque coefficient can be defined as:

$$C_Q = \left(\frac{Bc}{2R}\right)\sum_{i=1}^{N_\theta} \frac{\left(\frac{w}{u_0}\right)^2 c_t}{N_\theta}, \tag{14}$$

and the power coefficient as:

$$C_p = C_Q \cdot \lambda. \tag{15}$$

In order to account for the transient rotational regimes dynamic stall correction models were introduced. One of the most popular ones is the Gormont model. Gormont [21] proposed this model to take into account the dynamic stall of helicopter blades. He proposed a certain delay $\delta\alpha$ on the angle-of-attack based on hysteresis behaviour. Therefore the resulting reference angle of attack is:

$$\alpha_{ref} = \alpha - K\delta\alpha. \tag{16}$$

where K is a parameter defined according to empirical observations and has the values:

$$K = \begin{cases} 1 & : \dot{\alpha} \ge 0 \\ -0.5 & : \dot{\alpha} < 0 \end{cases}. \tag{17}$$

The term $\delta\alpha$ is empirical function of the airfoil's thickness and Mach number of the flow and can be:

$$\delta\alpha = \begin{cases} \gamma_1 S & : S \le S_c \\ \gamma_1 S_c + \gamma_2 (S - S_c) & : S > S_c \end{cases} \tag{18}$$

where S is is the non-dimensional rate parameter: $S = \sqrt{|c\dot{\alpha}/2w|}$, S_c is a function of the airfoil's relative chord thickness $S_c = 0.006 + 1.5(0.006 - t_c/c)$ and $\gamma_{1/2}$ are functions of the chord thickness and Mach number and have different values for the lift and the drag.

The change of the air velocity along a streamtube is defined by the factors: $a_u = u_u/u_0$ and $a_d = u_d/u_e$.

They can be solved iteratively for each streamtube using the following equation:

$$\alpha_{i+1} = f(\alpha_i) = \frac{1}{1 + g(\alpha_i)}. \tag{19}$$

where:

$$g(\alpha_i) = \frac{3c}{8\pi R|\cos\theta|} \cdot \left[c_n \cos\theta - c_t \sin\theta\right]\left(\frac{w_{u,d}}{u_{u,d}}\right). \tag{20}$$

A more comprehensive overview of the performance numerical calculation methods can be found in Refs. [22-24].

3. ANALYSIS

In order to validate the numerical models, the NACA 0015 airfoil was parameterized using the CST parameterization method. Because this airfoil is symmetric only 3 control points were used to represent the upper surface while the lower surface was just mirrored. The GA method was used in order to find the control points values that approximate the NACA 0015 airfoil the best. Initial set of variables was randomly selected and the objective function for which the minimum was searched was the metric norm between the parameterized airfoil and the NACA 0015. The resulting airfoil and its difference (deviation) from the original NACA 0015 is shown in Figure 3.

After the airfoil parameterization was done, the parameterized airfoil aerodynamic properties such as lift and drag coefficients for different angles of attack were obtained using XFOIL. Since XFOIL can only give quality results for a limited range of angles of attack the drag and lift coefficients were obtained using XFOIL for 0 to 25 degrees angle of attack after which they were extrapolated. These values were then used in the DMST method for determining the WT performance.

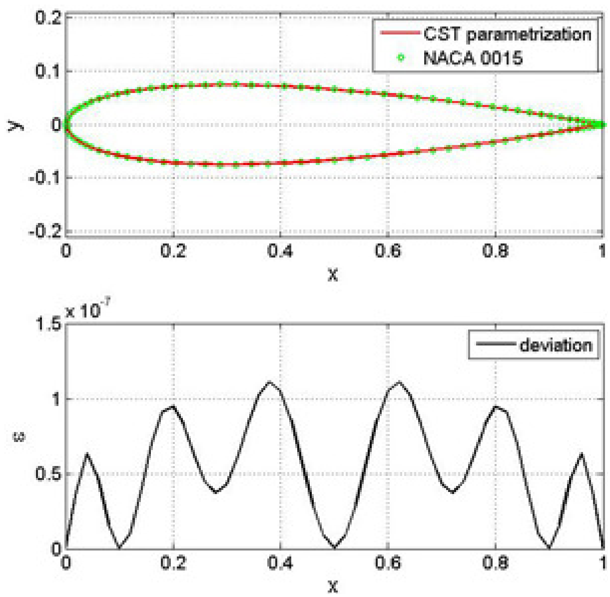

Figure 3. CST parametrization of the NACA 0015 airfoil

Validation of the numerical model was done with the experimental results obtained by Sheldahl et.al. [25] for a 5-metre 3 bladed turbine with 0.1524 chord NACA 0015 airfoil, 5-metre diameter and rotating at 150 rpm.

Comparison of the experimental and numerical results of the power coefficient C_p for different tip speed ratios λ is shown in Figure 4. It can be seen that the Gormont model gives good matching with the experimental results and therefore it is going to be used in the further analysis.

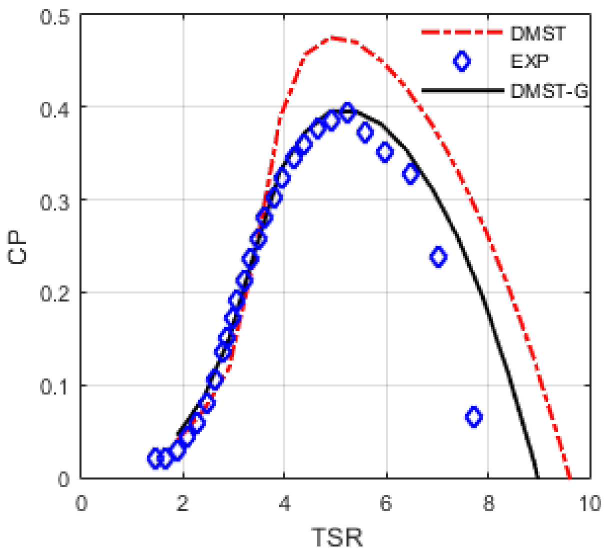

Figure 4. Power coefficient C_p as a function of tip speed ratio λ

Optimization of the airfoils geometry in order to maximize the performance of the previously mentioned 5 metre wind turbine was done. The same operating conditions and airfoil chord as in the validation case were kept. For the results to have structurally achievable and feasible meaning, the minimal relative thickness of the parameterized airfoil was limited to 7%. Also, for cutting down computational time the maximum relative thickness was limited to 30%.

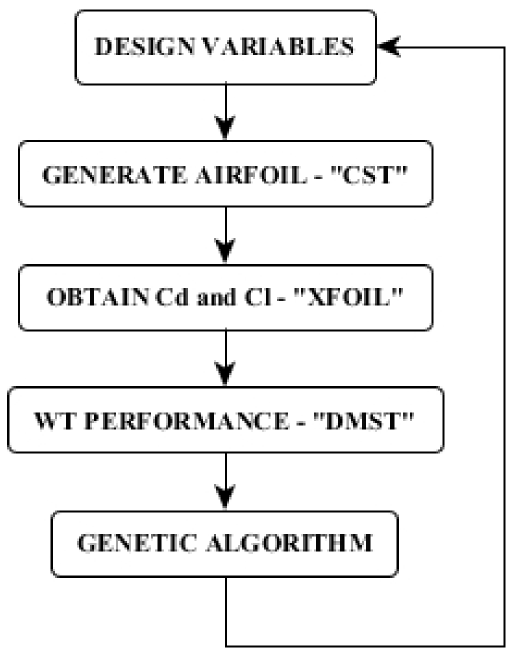

Figure 5. Automation flowchart

4. RESULTS

Two different cases were considered in the analysis. In the first case the optimization of a symmetric airfoil is done using 3 control points while in the second the airfoil was parameterized with five control points. Since all other values except the C_p in the fitness function Eq.(6) are kept the same, the power coefficient should be maximized by the optimization procedure.

The search for the optimal solution via the genetic algorithm for the 3 control points optimization is shown in Figure 6.The search is represented through the mean and best results for the fitness function during different generations of solutions. The slow convergence is due to the fact that in the first few generations the mutation rate was deliberately held high in order to avoid local optimums.

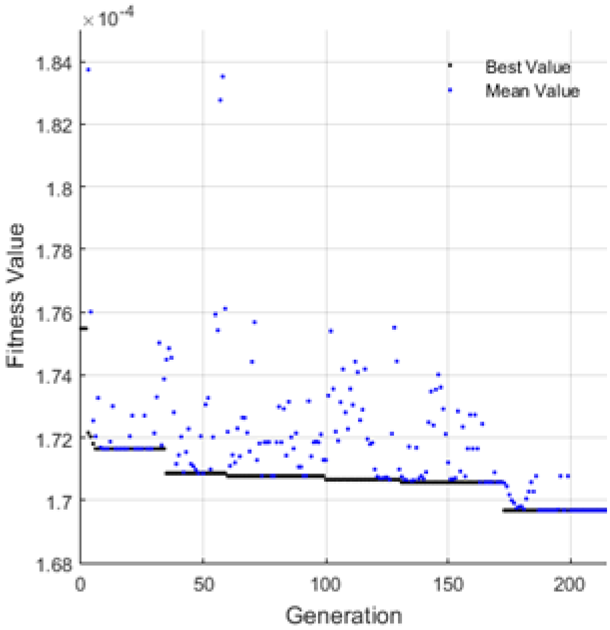

Figure 6. GA's search for the optimal solution for 3 control points

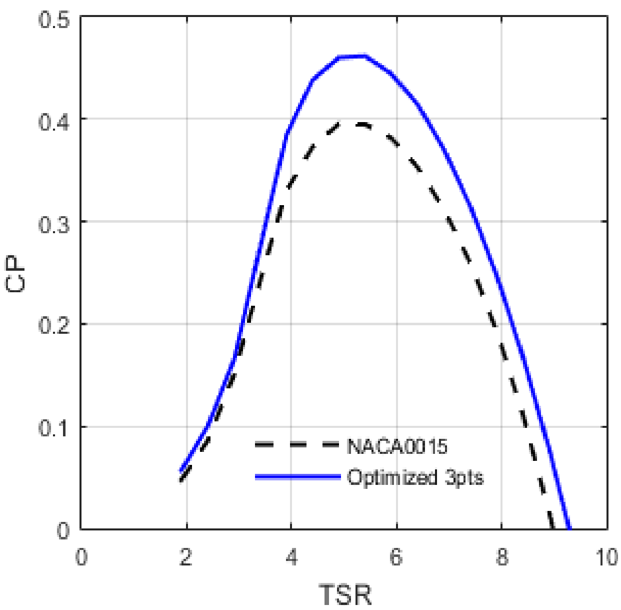

Figure 7. Power coefficient C_p as a function of tip speed ratio λ for optimized airfoil parameterized with 3 control points

The computed power coefficient C_p for the airfoil optimized using 3 control points is shown in Figure 7 while in Figure 8 a comparison between the calculated power coefficients of the wind turbine with the optimized airfoil using 3 and 5 control points and the NACA airfoil is shown.

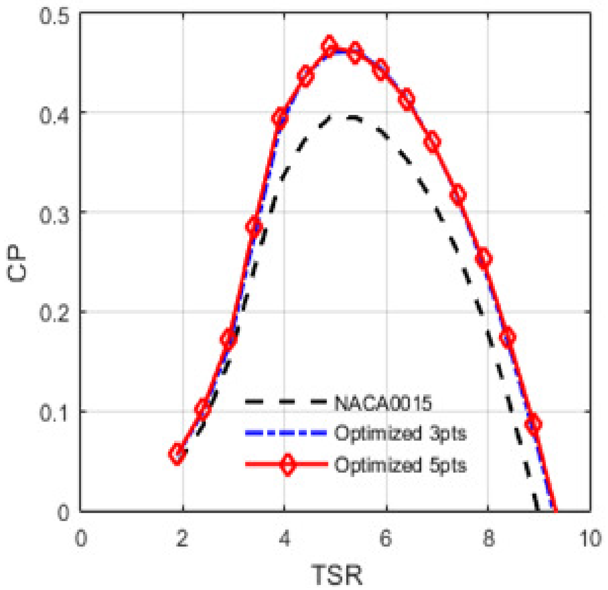

Figure 8. Power coefficient C_p as a function of tip speed ratio λ for optimized airfoil parameterized with 3 and 5 control points

There is a small difference between the power coefficients gained with 3 and 5 control points optimization as can be seen in Figure 8. The difference in the maximum C_p is around 0.97 percent.

In Figures 9 and 10 the best solutions of the optimized airfoils for airfoil parameterized with 3 control points and for airfoil parameterized with 5 control points in comparison with the NACA 0015 airfoil are shown.

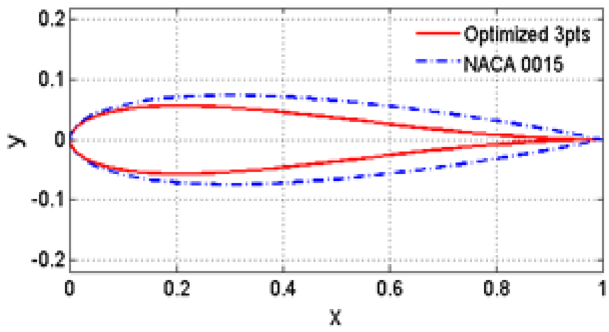

Figure 9. Comparison of the optimized airfoil using 3 control points and the NACA 0015

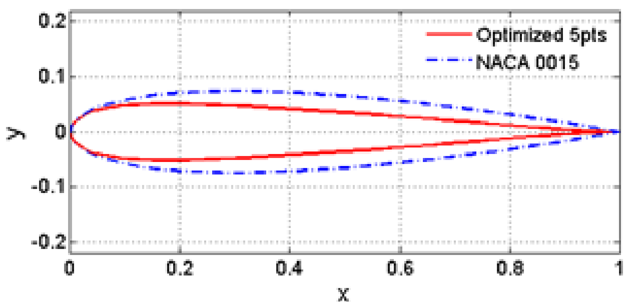

Figure 10. Comparison of the optimized airfoil using 5 control points and the NACA 0015

Table 1. Comparison of C_{pmax} for different airfoils

Airfoil	C_{pmax}
NACA 0015	0.3955
3 control points airfoil	0.4621
5 control points airfoil	0.4666

5. CONCLUSION

An optimization using genetic algorithm and CST parameterization of a symmetric airfoil was done. It was shown that using a coupled optimization technique, a Darrieus type vertical wind turbine aerodynamic characteristics can be improved with modification of the airfoils geometry.

Two different cases were considered and it was shown that there is a small difference in the power coefficients obtained using CST parameterized airfoil with 3 and 5 control points respectively. The new, optimized airfoils geometries were generated and they were shown in comparison with the original NACA 0015 airfoil.

The 5 control points airfoil proved to be slightly better than the 3 control points one which was expected. Anyhow this difference is somewhere around 1% and is therefore neglectably small.

It can be seen that a significant improvement of the VAWT power coefficient for various tip speed ratios can be achieved through CST parameterization of the airfoil using as little as 3 control points.

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ОПТИМИЗАЦИЈА ВЕТРОТУРБИНЕ СА ВЕРТИКАЛНОМ ОСОМ УПОТРЕБОМ ГЕНЕТСКИХ АЛГОРИТАМА И ЦСТ ПАРАМЕТРИЗАЦИЈОМ АЕРОПРОФИЛА

Т. Иванов, А. Симоновић, Ј. Сворцан, О. Пековић

Ветротурбина са вертикалном осом Дареиусовог типа оптимизована је применом генетских алгоритама (ГА). Облик аеропрофила параметризован је помоћу Класа-Облик трансформационог (ЦСТ) метода. Метод двојне вишеструјне цеви са Гормонт модификацијом за динамички слом узгона је коришћен за одређивање перформанси ветротурбине са вертикалном осом. Кад су нумерички кодови валидирани са доступним експерименталним резултатима, параметри аеропрофила су варирани како би се постигла оптимална вредност функције циља генетског алгоритма.