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Calculation of the Effective Shear Modulus of Composite Sandwich Panels

Calculation Method for the effective shear modulus of composite plates with sandwich cores is presented. This elastic coefficient represents one very important material property especially in constructions subjected to pure torsion and combined bending – torsion. The structures investigated in this research are flat plates made of top and bottom composite face-sheets and hexagonal composite material sandwich core. Starting from classical Lamination theory the effective shear modulus of top and bottom plates was determined for each ply in the stack up sequence. These plies were 'lumped into a single composite layer for different fiber orientations and plies thicknesses. To verify this approach Finite element method was used to determine the displacement, stress and strain field on Composite plates with Sandwich Cores. Two types of models were compared: The initial model where all the material components, plates and the core were modeled with their intrinsic properties and "lumped" model with calculated effective elastic coefficients.

Keywords: *Honeycomb core, Composite plates, Equivalent material properties.*

1. INTRODUCTION

Sandwich composites are widely used in aerospace structural design, mainly for their ability to substantially decrease weight while maintaining mechanical performance. It has long been known, that separating two stiff materials (facings or stress skins) with a lightweight material (core) increases the structure's stiffness and strength (for more detalis see [1], [2], [3]).

The faces carry the tensile and compressive stresses in the sandwich. Faces can be manufactured from metallic material (such as Al 3003, Al 5052, Al 5056), however polymer matrix composites are being used more and more as face materials. Composites can be tailored to fulfill a range of demands like anisotropic mechanical properties, freedom of design, excellent surface finish, etc. Faces also carry local pressure. When the local pressure is high, the faces should be dimensioned for the shear forces connected to it [3].

The core's function is to support the thin skins so they do not buckle (deform) inwardly or outwardly, to keep them in relative position to each other and to carry shear stresses. To accomplish this, the core must have several important characteristics. It has to be stiff enough to keep the distance between the faces constant. It must also be so rigid in shear that the faces do not slide over each other. The shear rigidity forces the faces to cooperate with each other. If the core is weak in shear, the faces do not cooperate and the sandwich will lose its stiffness It is the sandwich structure as a whole that gives the positive effects. However, the core has to fulfill the most complex demands. Strength in different directions and low density are not the only properties the core must have. Often there are special demands for buckling, insulation, absorption of moisture, aging resistance, etc. The core can be made of a variety of materials, such as wood, aluminum (Al 3003, Al 5052, Al 5056), variety of foams (corecell foam) and polymer matrix composite materials (figure 1.) [3],[4].

To keep the faces and the core cooperating with each other, the adhesive between the faces and the core must be able to transfer the shear forces between them. The adhesive must be able to carry shear and tensile stresses. It is hard to specify the demands on the joints.

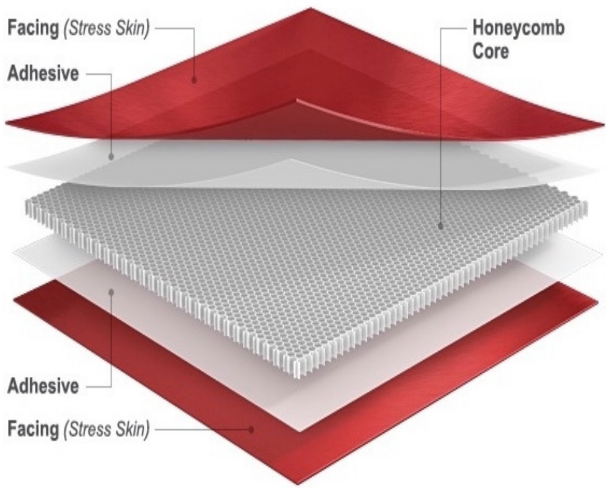

Figure1. Composite panel with honeycomb core construction

It is of great importance to predict during the design phase the properties of above mentioned construction.

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This usually means to accurately estimate all elastic coefficients (E_{ij}, G_{ij}, ν_{ij}, i,j=x,y, Young's moduli, Shear moduli and Poisson's ratios) based on sandwich panel geometry (core cell geometry, cell wall thickness, facings thicknesses e.t.c) and panel constituents material properties.

In the present work the shear moduli (in-plane G_{xy} and out-of-plane, G_{xz} and G_{zy}) of the panel with composite hexagonal core and composite faces are investigated and the model for these properties calculation is proposed [3].

2. NOMEX PRODUCTION

First, the manufacturing process of core will be analyzed. The emphasis will be given to composite material cores. Nomex is a trademarked of non-metallic paper (honeycomb core basic building block material, which is well known for its excellent mechanical and other properties relevant for aerospace applications [1].

In general, the paper is a composite that consists of two forms of polymers, the fibrids (small fibrous binder particles) and the floc (short fibres). These two components are mixed in a water-based slurry and machined to a continuous sheet. Subsequent high-temperature calendering leads to a dense and mechanically strong paper material (Figure 2.) [4].

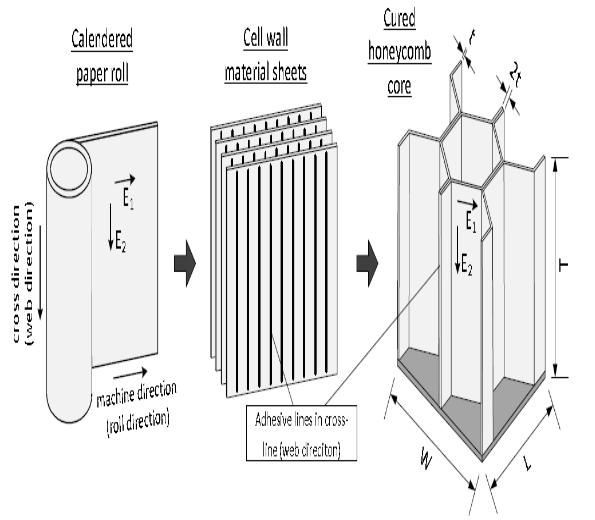

Figure 2. Honeycomb (hexagonal) core production process scheme

During this manufacturing process, the longer floc fibres may align themselves in direction of the paper coming off the machine, which leads to orthotropic mechanical structure, or the fibers may randomly distribute themselves in the paper resulting in a 2-D random composite structure (it is assumed that the paper is manufactured to be relatively thin, hence 2-D structure results). The latter case will be considered in this paper [1].

Further processing of this paper material to form hexagonal cells is commonly carried out using the adhesive bonding method, meaning that the bonded portion of two adjacent paper sheets is held together by adhesive. This method inevitably leads to double the wall thickness of the bonded cell walls if compared to the unbonded "free" cell walls (Figure 3).

After the paper sheets are bonded and shaped to hexagonal cells, the resulting honeycomb block is dipped in liquid resin (usually phenolic based) and subsequently oven cured. This dipping-curing process is repeated until the desired density of the core is achieved. The resulting resin coating of the paper leads to a layered material with an orthotropic or 2D anisotropic ductile center layer (core paper) and two isotropic very brittle outer layers (resin layers) [3].

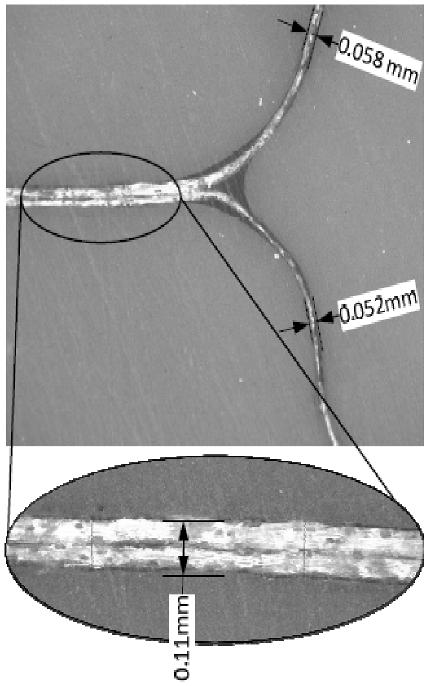

Figure 3. Double wall thickness

3. ANALYTICAL MODEL OF SHEAR MODULUS FOR THE HONEYCOMB CORE

In order to relate stress and strain in the structure the constituent material properties have to be known and elastic coefficient in the stiffness matrix have to be calculated. For the structures investigated in this paper the determination of the elastic necessary elastic coefficients is very complex due to complex geometry of the panel an furthermore complex structure of the constituent materials i.e core and facings. The analyst relies on the manufacturer data or has to perform tests if the all required values of the elastic coefficients are not supplied. Secondly numerical studies can be performed, requiring large and cumbersome finite element models if the equivalent model approach is not taken. This method consists of lumping several composite layers into a single (lumped) layer with same characteristics rendering the same stress - strain (although averaged) and displacement fields for the same boundary conditions [5, 6].

It is true that this approach still requires numerical modeling of the structure, however, the models based on equivalent material properties are much smaller in size (less D.O.F's), hence requiring less confrontational resources, computing time and yielding stress-strain and displacement fields much faster compared to detailed (micro mechanics models). This is the reason why the validation and determination of the equivalent material properties (especially when the composite panels with honeycomb core are in question).

The literature survey on the subject matter has revealed that the development of equivalent composite models in the focus of many researchers. For example Master and Evans model for equivalent Young's moduli in fiber and cross fiber directions, Ashby's model for equivalent in-plane shear modulus are only few of several existing models. However, all this models assume that the starting material is isotropic. For example, in Master and Evans model one of the required input variables is E_f which represents the Young's modulus of the paper. This is directly applicable for honeycomb cores where the basic building material is isotropic (for example: hexagonal aluminum cores), in cases where the cores are manufactured from composite materials (section 2), the equivalent properties of the building core material have to be determined first before using on of the already established and proven material equivalent material properties [3, 4].

3.1 Honeycomb core with 2-D random fibers

If the paper material is manufactured in a such a way that the resulting material is anisotropic. It consists of polymer matrix and randomly distributed fibers (in a 2D plane). The effective Young's modulus of such a structure can be determined based on Christensen and Waals model [7].

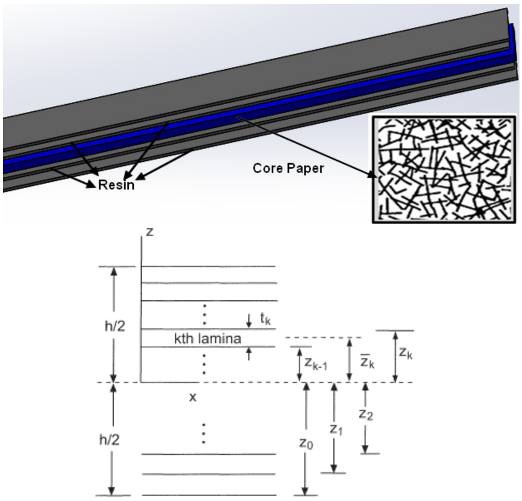

Figure 4. Core paper with added isotropic resin layers

Christensen and Waals examined the behavior of a composite system with a three-dimensional random fiber orientation. Both fiber orientation and fiber-matrix interaction effects were considered. For low fiber volume fractions, the modulus of the 3-D composite was estimated to be:

$$E_{2D} = \frac{V_f}{6} \cdot E_f + \left[1 + (1 + \nu_m) \cdot V_f\right] \cdot E_m \tag{1}$$

where V_f is the fiber volume fraction in the composite, ν_m is the matrix Poisgson's ratio E_f an Em are the Young's moduli of the fiber and matrix phase respectively. Since it can be considered that the core composite (paper) is isotropic (fibers are evenly distributed) the basic relation between Young's modulus (E),Shear modulus (G) and Poisson's ratio holds. Manera [8] proposed approximate equations to predict the elastic properties of randomly oriented short fiber composites. Using this approach, it can be shown that for this type of composites (thin plates, with randomly distributed short fibers) Poisson's ratio is ν=1/3. Using relation:

$$G_{2D} = \frac{E_{2D}}{2 \cdot (1 + 2\nu)} \tag{2}$$

In-plane shear modulus for the paper of the composite honeycomb structure can be calculated.

As described in previous section, the paper core is dipped in resin (several times until the desired density of the paper used for honeycomb structure is achieved).

This process adds rigidity to the paper and equations (1) and (2) have to be modified to account for this effect. Resin properties (Elasticity Modulus, Shear Modulus) are known and classical lamination theory (CLT) can be applied in this case. Also, resin itself is considered to be isotropic. Theretofore, it follows using CLT, stresses in each core paper layer can be expressed as follows:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_1 & Q_2 & 0 \\ Q_2 & Q_1 & 0 \\ 0 & 0 & Q_6 \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{3}$$

where,

$$Q_1 = \frac{E_i}{1 - \nu_i^2},\ Q_2 = \frac{E_i \cdot \nu_i}{1 - \nu_i^2},\ Q_6 = G_i \tag{4}$$

In equations (4), index 'i' denotes the elastic coefficient for the corresponding layer, for example if the central mid-layer is in question the value of E_i is calculated according to equation (1) and all other layers (resin) assume their intrinsic values of E. Same applies to shear modulus and Poisson ratio in equations (4).

Expressing relation (3) in terms of in-plane resultant forces one can obtain:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \cdot dz \tag{5}$$

Where t denotes thickness of the layer (Figure 4)

For the whole lay-up it follows:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \sum_{i=1}^{n} \int_{-t/2}^{t/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \cdot dz \tag{6}$$

In the previous relation n denotes the total number of layers (core paper and resin layers), therefore it can be written:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \sum_{i=1}^{n} \int_{z_{k-1}}^{z_k} [Q]_k \cdot \{\varepsilon\} dz \tag{7}$$

Or rewriting the previous relation,

$$\{N\}=[A]\cdot\{\varepsilon\} \tag{8}$$

Matrix A is stiffness matrix and all coefficients Aij (i=1,2,6 j=1,2,6) can be easily calculated once all the propreties of all the layers are known.

$$A_{ij}=\int_{-t/2}^{t/2} Q_{ij}^{(k)}\cdot dz=\sum_{k=1}^{n} Q_{ij}^{(k)}\cdot\left(z_k-z_{k-1}\right) \tag{9}$$

Since the lay-up in this case can be considered to be symmetric, it can be concluded that the effective shear modulus for this structure can be expressed as:

$$G_{xy}^{eff}=\frac{A_{66}}{h} \tag{10}$$

In equation (10) h denotes total thickness of the laminate (core paper and resin layers).

3.2 Honeycomb core with orthotropic material

During certain manufacturing process, the floc fibers can be aligned in the direction of the paper coming off the machine. This leads to orthotropic mechanical properties of the paper.

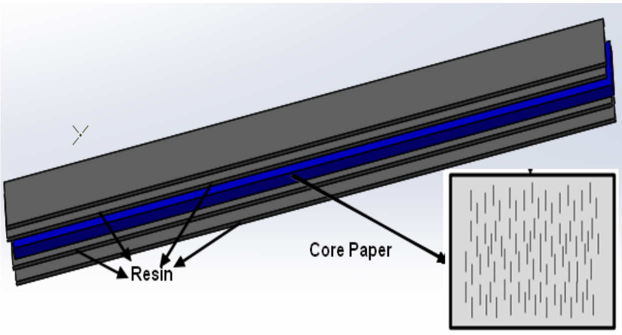

Figure 4a. Core paper (orthotropic) with added isotropic resin layers

For this type of composite equation (3) assumes the following form:

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{21} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} \tag{11}$$

Where Q_{ij} are given in the following form:

$$\begin{aligned}
&Q_{11}=\frac{E_{i1}}{1-\nu_{i12}\cdot\nu_{i21}},\\
&Q_{12}=Q_{21}=\frac{E_{i2}\cdot\nu_{i21}}{1-\nu_{i12}\cdot\nu_{i21}},\\
&Q_{22}=\frac{E_{i2}}{1-\nu_{i12}\cdot\nu_{i21}},\\
&Q_{66}=G_{ixy}
\end{aligned} \tag{12}$$

Coefficients Qij for the mid-layer (core paper) can be calculated using standard rule of mixture theory (ROM). Using rule of mixtures theory shear modulus Gxy is obtained using known relation:

$$G_{12}=\frac{G_f\cdot G_m}{V_f\cdot G_m+V_m\cdot G_f} \tag{13}$$

where Vf and Vm are fiber and matrix volume fractions, and Gf and Gm are shear mduli of the fiber and matrix phases. This relation, as experiments have shown, tends to underestimate the value of G12, and semi-experimental Halpin-Tsai theory is used instead. According to this theory the G12 modulus can be obtained using the following relation:

$$G_{12}=\frac{\left(V_f+\eta_{12}\cdot V_m\right)\cdot G_f\cdot G_m}{\left(G_m V_f+\eta_{22}\cdot V_m G_f\right)} \tag{14}$$

Correction coefficients ν_{12} and ν_{22} depend on the fiber type used in the core paper and are given for typical fibers used in honeycomb structure production.

	ν_{12}	ν_{22}
Carbon	0.500	0.400
Glass	0.516	0.316
Aramid	0.516	0.400

Once elastic coefficients (equation 14) are computed effective shear modulus for this type of composite (core paper with resin layers) is calculated using equation (10).

3.3 Effective in-plane and out-of-plane shear moduli of hexagonal honeycomb cores

Many authors have developed theoretical approaches for obtaining the equivalent material properties of honeycomb cores [9]. The nine core material properties are: two in-plane Young's moduli Ex, Ey, the out-of-plane Young's modulus Ez, in-plane shear modulus Gxy, out-of-plane shear moduli Gxz, Gyz, and three Poisson ratios nxy, nxz, nyz. Using a sensitivity analysis, Schwingshackl et al. [9] reported the major influence of the out-of-plane shear moduli Gxz, Gyz on displacement, stress and strain fields of the honeycomb core type thin plates. One of the analytical approaches mentioned in this work [9] was developed by Gibson and Ashby [10]. They described the honeycomb core material as a cellular solid made up of an interconnected network of solid structures which form the edges and faces of the cells. These formulae were later slightly modified by Zhang and Ashby [11] to include double thickness walls for the out-of-plane values, Ez, Gxz, and Gyz.

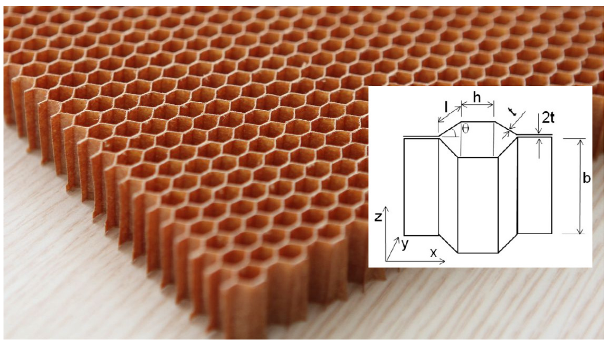

Figure 5. Hexagonal honeycomb core and cell geometry

The honeycomb cell geometry is presented in the following Figure (Figure 5) where l and h indicate the length of the hexagon face; b indicates the height; t indicates the thickness of the face; θ indicates the semi-angle between two faces.

The equations (15)-(18) represent the equivalent material out-of-plane shear moduli according to [12], and are depicted in the following Figure (Figure 6).

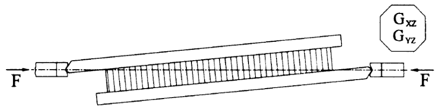

Figure 6. Hexagonal honeycomb core out-of-plane shear moduli (Gxz and Gyz)

$$G_{yz} = \frac{\cos\theta}{h/l + \sin\theta} \cdot \left(\frac{t}{l}\right) \cdot G_{xy}^{eff} \tag{15}$$

$$G_{xz}^{lower} = \frac{h/l + \sin\theta}{(1+2h/l)\cdot\cos\theta} \cdot \left(\frac{t}{l}\right) \cdot G_{xy}^{eff} \tag{16}$$

$$G_{xz}^{upper} = \frac{1}{2} \cdot \frac{h/l + 2\cdot\sin^2\theta}{(h/l + \sin\theta)\cos\theta} \cdot \left(\frac{t}{l}\right) \cdot G_{xy}^{eff} \tag{17}$$

$$G_{xz} = G_{xz}^{upper} + \frac{0.787}{b/l} \cdot \left(G_{xz}^{upper} - G_{xz}^{lower}\right) \cdot G_{xy}^{eff} \tag{18}$$

The in-plane shear modulus is presentented in the following Figure (Figure 6).

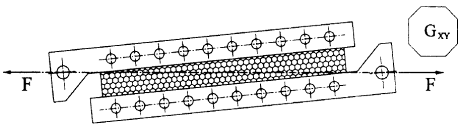

Figure 6. Hexagonal honeycomb core in-plane shear modulus (Gxy)

The equivalent in-plane shear modulus for the honeycomb composite core may be expressed by the following equation (19).

$$G_{xy} = 0.2910 \cdot \frac{E_f \cdot t}{a} \tag{19}$$

4. NUMERICAL MODEL

In order to verify the validity of the proposed model for shear material properties of composite honeycomb panel numerical approach, finite elements approach was used. Two FEA models were constructed [13-14]. First mezo-scale model, where complete structure (composite panel with honeycomb core) was modeled using plate finite elements (based on Kirchoff's thin plate theory). In this model each honeycomb cell (Figure 5.) was modeled using plate elements according to geometry which corresponds to geometry of HexWeb EC engineered core, with, HRH 10 Nomex, with cell size h=13 [mm].

Facings (stress skins, Figure 1.) where thin plates, 1 mm thick made of carbon T-300. Composite lay-up for the facings (lower and upper) was [00/450/-450/900]s.

In the second verification finite element model (model 2), the core was modeled using solid elements, where material properties (elastic coefficients in stiffness matrix) were determined using the equivalent material approach, as described in section 3.

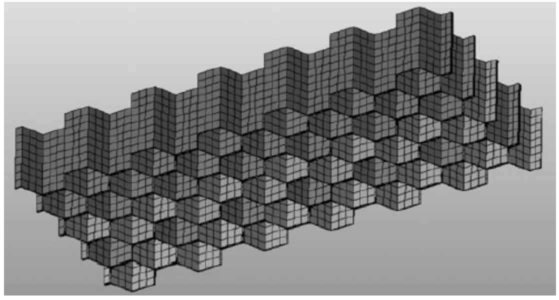

Figure 6. Hexagonal honeycomb core Finite element model

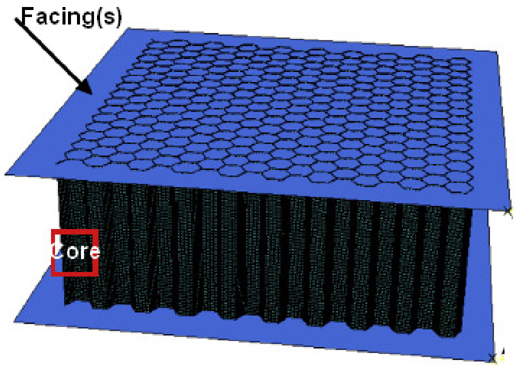

Figure 7. Hexagonal honeycomb core with facings and refined core mesh finite element model

Displacement field was calculated for both models and results were compared. Boundary conditions under which displacements of the plate tip were determined were the same in both cases. The plate was clamped at one end (all degrees of freedom of end nodes for both translation and rotation ere set to zero), whereas the loading moment was applied at the plate's free end. These boundary conditions applied, enforced honeycomb core and facings twist, where shear moduli of the complete structure have an important role and great influence on the complete structure displacement.

The results are shown in the following Figure (Figure 8.)

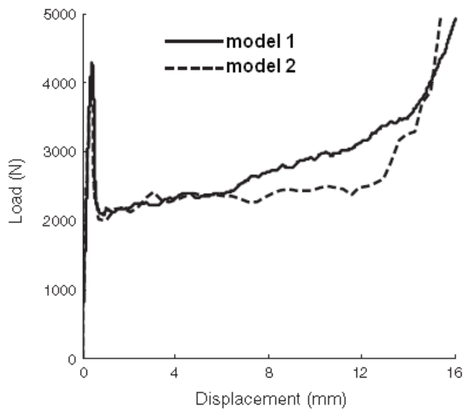

Figure 8. Displacement of plate tip as a function of applied load

5. CONCLUSION

In the present paper Shear properties of the honeycomb plate were investigated. Approach based on the material equivalent model was proposed for the honeycomb cores made of different types of composites. In general, due to complex geometry of hexagonal composite cores, prediction of the elastic coefficients for these type of structures is relatively complicated. One of the approaches is to use numerical approach starting from constituent material properties and creating honeycomb true scale model in order to determine stress-strain and displacement fields. As experiments have confirmed, this approach renders correct results, however these (true scale) models require quite a long development time (each honeycomb cell is modeled independently with a large number of elements and hence a large number of degrees of freedom). Furthermore, in order to solve and obtain desired fields large computing power and time is necessary. Many researchers have focused on finding methodology to find equivalent material models for different types of plates with honeycomb cores with different cell geometries, sizes and constituent materials. In this paper Shear moduli (in-plane and out-of-plane) for the hexagon honeycomb core made of different types of composite materials (with aligned and randomly distributed fibers) were investigated. The equivalent material model was presented. To verify the equivalent material model validity the numerical approach was deployed, by creating two different types of models. The true scale model and 'lumped' model where the core material of the honeycomb was defined using equivalent material properties. Displacement field was calculated for both models and results compared. It was found that in the linear region both models yield same results. Since the computing time for the lumped model is significantly lower it's application is recommended. In non-linear region the equivalent material model yields lower results when compared to the true-scale model which is probably due to very complex core crushing mechanism, and if the results are required in this region the true-scale model has to be developed.

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ПРОРАЧУН ЕФЕКТИВНОГ МОДУЛА СМИЦАЊА КОМПОЗИТНИХ СЕНДВИЧ ПАНЕЛА

Л. Ребхи, М. Динуловић, М. Дуњић, А. Грбовић, Б. Крстић

У раду је представљена метода прорачуна ефективног модула смицања композитних панела са сендвич испуном. Овај коефицијент еластичности представља врло важну карактеристику материјала, посебно код конструкција које су изложене чистој торзији или комбинацији савијања и торзије. Структуре које су испитиване у овом истраживању су равне плоче начињене од горњег и доњег композитног лица и испуном (језгром) од шестоугаоног композитног сендвич материјала. Полазећи од класичне теорије ламината одређен је ефективни модул смицања горње и доње плоче за сваки слој у стоку (ламинату). Ови слојеви платана се своде у један композитни слој са различитим оријентацијама влакана и дебљине. Да би се верификовао овај прилаз коришћен је Метод коначних елемената да би се одредила померања, деформације и напони у композитним плочама и сендвич испуни. Два типа нумеричких модела су упоређивани: почетни модел, где су сви спојеви плоче моделирани са својим карактеристикама и други модел код којег су спојени у једну целину са својим еквивалентним карактеристикама.