Formation Control of Robotized Aerial Vehicles Based on Consensus-Based Algorithms

In this paper, consensus based formation control of multi robotized quadrotor vehicles is presented. The method is suitable in the case when information space is not huge, but includes as many states by how many degrees of freedom are achieved by consensus, or by the number of states in which the formation is achieved. Several extensions of the basic consensus algorithm are shown that include second-order consensus, as well as the case where the formation required to follow a predetermined external reference. Simulation results are fully consistent with the theory, but the implementation in real systems would have more details to take into consideration such as technical details about mutual communication.

Keywords: Formation Control, Multi Agent Robotic Systems, Consensus, Quadrotor, Dynamic Model.

1. INTRODUCTION

Autonomous vehicles, especially Unmanned Aerial Vehicles (UAV) have attracted great attention from researchers in the field of robotics [1]. Flying opens new opportunities to robotically perform services and tasks like search and rescue, observation, mapping or even inspection and maintenance. Potential applications consider remote surveillance, monitoring and ground imaging from above, border patrolling, disaster management during floods, earthquakes, fire, commercial missions like aerial photography, television and cinema shootings. Key areas in research to be addressed include innovative UAV design, autonomous missions, guidance, navigation and control, and multi-vehicle coordination.

In this research, a kind of rotary-wing UAV called Quadrotor is considered. Quadrotor is a system consisting of four independent propellers attached at each corner of a cross frame (Fig. 1). This platform has vertical take-off and landing (VTOL) capabilities, which gives higher maneuverability and hovering capabilities. This kind of platform shows the characteristics of a unique ability for vertical, stationary and low speed flight. The Quadrotor architecture has been chosen for this research for its low dimension, good maneuverability, simple mechanics and payload capability. As main drawback, the high energy consumption can be mentioned. However, the trade-off results are very positive.

Quadrotor is a particularly interesting configuration because of its simplicity and low cost of production. One UAV could be used in a variety of applications. For example, this flying platform represents a remote mobile sensor that could collect a variety of information: monitoring a specific area to locate lost or people injured in an accident, to collect samples from inaccessible or potentially dangerous locations (sample smoke from the volcano). Another important application is transport of a very limited cargo weight. However, the execution of any of these tasks depends entirely on this single autonomous robotic system, and if the robot fails to perform at least one task, the entire mission will fail. However, there are tasks that an aircraft independently may not perform as in the case when flying platform try to transfer the weight greater than it can raise itself. On the other hand, several robotic systems with equally small transport capacity, in certain circumstances, have ability to raise a little more cargo weight. The multi robots would allow a greater chance for success in the execution of the single task. In case of aircraft fault, other aircrafts could finish the complete task. For example, if an aircraft for any reason (sensor fault) fails to establish the existence of a forest fire, one or more other aircraft will be in possibility to notice the same fire and report for appropriate further action.

![Fig. 1 Quadrotor configuration](image-url)
autonomously perform any task, it is better to have simpler systems to perform the same task in joint action and cooperation. Cooperation and joint efforts of individual autonomous agents (capable of independent decisions on actions to be taken) are research topic in the field of Multi Agent Systems. Multi Agent Systems are a very fertile area of research in computer science past few decades.

One of the possible ways in which the agents in the group can achieve cooperation is to agree on some parameters of action. For example, to agree on what is their common goal. This agreement or negotiation is achieved by Consensus, when any agent is able to make its own decisions, suggest a value of the parameter (or multiple parameters). Consensus algorithms have the aim that on the basis of the parameter values of other agents, change their proposed value in order to converge to identical values. In this way, the algorithm should provide a similar dynamic information space for each agent [2].

Considering the problem of multiple ground or aerial vehicles, information consensus guarantees that vehicles sharing information over a network topology, have a consistent view of information that is critical for the coordination task. To achieve the consensus, there must be a shared variable of interest, called the information state, as well as appropriate algorithmic methods for negotiating to reach consensus on the value of this variable. The information state represents an instantiation of the coordination variable for the team. Examples include a local representation of the centre and shape of a formation, the rendezvous time, the length of a perimeter being monitored, the direction of motion for a multi vehicle swarm, and the probability that a military target has been destroyed. By necessity, consensus algorithms are designed to be distributed, assuming only neighbour-to-neighbour interaction between vehicles. Vehicles update the value of their information states based on the information states of their neighbours. The goal is to design an update law so that the information states of all of the vehicles in the network converge to a common value.

Consensus algorithms have recently been studied extensively in the context of cooperative control of multiple autonomous vehicles [3-5], while our attention is oriented toward consensus based formation control [6-7]. Notice that in the literature, most consensus algorithms consider the case where the communicating/cooperating vehicles come to consensus about the value of the consensus variable. Although the consensus variable may be a vector, such algorithms are effectively first-order, as the typical consensus algorithm adapts the first derivative of the consensus variable on each node based on the value of the consensus variable of its neighbours. There is idea of a second-order consensus algorithm under directed information exchange, where each vehicle adapts the second-order derivative of its local consensus variable based on both zero-order and first-order derivatives of the consensus variables of its nearest neighbours.

Formation control is an important field in multi-robot coordinated control, which has recently triggered great interest of the research community. A Team of robots can efficiently solve tasks such as space exploration, transportation of large objects, security tasks, group hunt, etc. Approaches in the formation control can be divided into several categories: behavior based approaches, virtual structures methods, leader – follower approaches, potential fields and generalized coordinates methods [8].

Our research problem is oriented to cooperative control of multiple Quadrotors, where consensus algorithms were also applied [9-10]. The remainder of the paper is organized by presentation of Quadrotor dynamic model and basic first and second order consensus control algorithm with tracking of determined external reference. The effectiveness of the proposed algorithms is illustrated throughout the paper by simulations studies.

2. DYNAMIC MODEL OF QUADROTOR

Quadrotor can be described as four independent rotated propellers that are connected to a solid (inflexible) structure in the shape of a cross as shown in Fig. 1. Propellers are directly connected to the engine and unlike the example of helicopter propellers, they are fixed and can not change the angle of pitch . The only quantity that can be changed is the speed of rotation of the propeller. It is shown that the thrust force generated by the propeller is directly proportional the square of the angular velocity of rotation. Four forces and their moments have an effect on the center of mass with a total force $F_T$, which is responsible for the translational movement, while three moments which affect the rotational movement of the solid body are $\tau_x$, $\tau_y$, and $\tau_z$. As a solid body in three-dimensional has six degrees of freedom, the three coordinates of the position $x$, $y$, and $z$ expressed relation to a global coordinate system, while three degrees of freedom in the orientation (rotation) represented Euler angles $\phi$, $\theta$, and $\psi$. Although there are some different representations as quaternions and rotation matrices, this representation is adopted, beside drawbacks (singularities). The model expressed by (1) is partially decoupled and simplified to include only the most important effects. Denominator $i$ indicates which of $n$ aircraft state belongs to:

$$
\begin{align*}
\ddot{x}_i &= -F_{T,i} \sin(\theta) \\
\dot{y}_i &= F_{T,i} \cos(\theta) \sin(\phi) \\
\dot{z}_i &= F_{T,i} \cos(\theta) \cos(\phi) - 1 \\
\dot{\phi}_i &= \tau_{\phi,i} \\
\dot{\theta}_i &= \tau_{\theta,i} \\
\dot{\psi}_i &= \tau_{\psi,i}
\end{align*}
$$

This model is adopted from [10] and clearly shows that quadrotor is not actuated in all directions, while translation depends on the angles of yaw and pitch, so take that position in the plane ($x$ and $y$ coordinates) operated indirectly through the angles $\phi$ and $\theta$, which are controlled directly.

3. ACHIEVING CONSENSUS FORMATIONS

In order to know the mutual agent information space for consensus algorithms, it is necessary that an appropriate
communication network exists between them. The assumption is that the bandwidth of communication channel is large enough and capable enough to be quickly refreshed, so it is possible to model the information space of each agent’s by differential equation. The communication network between agents is usually modeled by graphs that can be unidirectional or bidirectional. Unidirectional graph represents communication in only one direction, while in the case of bidirectional link between two nodes, this graph means two-way communication. The links in the graph can have thier specific weight, and if it is not important, any weight can equal 1. Connectivity of a graph can be represented by square matrix, where 1 means that between the i-th and j-th agent there is connection, while 0 denotes that connection does not exist. In accordance with the appearance of the graph, there is a variety of possible topologies. These topologies can be fixed or variable. They are variable, when quadrotor goes so far from another and communication is lost, or they are sufficiently close, imposes that communication is established. In our research, a highly-networked fixed topology is chosen, when at every moment there is a two-way communication between each quadrotor. Strong networking implies that the final value of consensus depends of the initial proposed values, which is not the case in some other topologies. Strong networking also accelerates the convergence of the consensus algorithm.

A typical equation for first order consensus [2] (consensus by position of every quadrotor) in scalar form is represented by:

$$\dot{x}_i = - \sum_{j=1}^{n} a_{ij} (x_i - x_j) \quad (2)$$

where $a_{ij}$ is the coefficient of the matrix that describes the mutual communication, while $n$ is number of agents within the system. In this case, coefficient $a_{ij}$ can be omitted, because the coefficient is always equal to 1, except for the case when $i = j$ coefficient is 0.

Similarly, second order consensus [3] (consensus by position and velocity of every quadrotor) is defined by the following equation:

$$\dot{x}_i = - \sum_{j=1}^{n} a_{ij} \left[ (x_i - x_j) + \gamma (\dot{x}_i - \dot{x}_j) \right] \quad (3)$$

Usually, if we want that there is some spatial desired separation between quadrotors defined by value $\delta$, our equations (2) and (3) are now in the following forms:

$$\dot{x}_i = - \sum_{j=1}^{n} a_{ij} \left[ (x_i - \delta) - (x_j - \delta) \right] \quad (4)$$

$$\dot{x}_i = - \sum_{j=1}^{n} a_{ij} \left[ (x_i - \delta) - (x_j - \delta) \right] + \gamma (\dot{x}_i - \dot{x}_j) \quad (5)$$

Finally, it is also possible to modify the consensus control law, so as to enable the flying quadrotor formation with external reference trajectory. These references may be forwarded to all quadrotors in the formation, and then the consensus ensures that the entire formation closely follows the given trajectory $x_i^d$. However, we will here give only one reference aircraft which can then be regarded as the leader, and the control algorithm will ensure that the quadrotor precisely follows the given trajectory. Consensus will then ensure that other quadrotors follow the trajectory (though not so accurate, but with a slight delay).

$$\dot{x}_i = - \sum_{j=1}^{n} a_{ij} \left[ (x_i - \delta) - (x_j - \delta) \right] + x_i^d \quad (6)$$

$$\dot{x}_i = - \sum_{j=1}^{n} a_{ij} \left[ (x_i - \delta) - (x_j - \delta) \right] + \gamma (\dot{x}_i - \dot{x}_j) + x_i^d \quad (7)$$

Considering formations of the multiple quadrotors, a consensus is achieved on mutual schedule, exactly on the position/velocity coordinates. Quadrotors make an agreement on the position/velocity of the formation virtual center, in accordance with the selected formation. Regarding quadrotor, which has four degrees of actuation freedom (three coordinates of the position, and an orientation), the information space of quadrotor will essentially have four states. The separation in space is achieved by adding a constant member $\delta$ to determine the position of each particular quadrotor with respect to a virtual center about which the consensus is realized.

4. CONSENSUS-BASED FORMATION CONTROL ALGORITHMS

We assume that the dynamics of each aerial robotized vehicle is defined by:

$$\ddot{x}_i = v_i, \dot{v}_i = u_i \quad (8)$$

where $x_i$ and $v_i$ represent the position and velocity of vehicle $i$, and $u_i$ is the control input.

Based on previous equations, in the case of first order consensus, control input for quadrotors which ensures the stabilization of the system around the origin [10] is synthesized in the form:

$$F_x = \frac{-k_1 \dot{z} - k_2 z + 1}{\cos(\phi)\cos(\theta)}$$

$$\tau_\phi = -\sigma_4 (\dot{\phi} + \sigma_3 (\dot{\phi} + \phi +$$

$$\sigma_3 (\dot{\phi} + 2\phi - \dot{y} + \sigma_3 (\dot{\phi} + 3\phi - 3\dot{y} - y)))$$

$$\tau_\theta = -\sigma_4 (\dot{\theta} + \sigma_3 (\dot{\theta} + \theta +$$

$$\sigma_3 (\dot{\theta} + 2\theta - \dot{x} + \sigma_3 (\dot{\theta} + 3\theta - 3\dot{x} - x)))$$

$$\tau_\psi = -k_4 \psi$$

where $k_{1,4}$ are control gains, $\sigma_{1,4}$ are control saturations, which are chosen to limit the amplitude of the yaw and pitch angles $\phi$ and $\theta$ on a certain range around 0, where linear approximation of translational dynamics is valid. This range provides stability of quadrotor, during large deviations of the position in relation to the reference position. The selected control law is called a nested saturation control and has the properties of global stability that have great importance for the problem of formation control [10].
The next case is to design the control law that reaches a consensus of first-order together with achieving spatial separation by constants $\delta$ that are determined by the mutual positions of quadrotors in the formation. The formation is defined by the positions of quadrotor in relation to the virtual center of the structure, and the consensus is achieved by the location of the virtual center. To achieve first order consensus formation, equation (9) will expand to multiple quadrotors by adding a subscript $i$, and by incorporating the member that provides a first-order consensus defined by equation (4):

$$ F_{\tau i} = \frac{-k_{\tau i}\dot{z}_i - k_{\tau i}(\sum(z_i - \delta_j) - (z_j - \delta_j))}{\cos(\phi)\cos(\theta)} + 1 $$

$$ \tau_{\phi i} = -\sigma_1(\dot{\phi} + \sigma_1(\dot{\phi} + \phi + \sigma_1(\dot{\phi} + 2\dot{\phi} - \dot{y}_i) + (\sum(y_j - \delta_j) - (y_j - \delta_j)))}) $$

$$ \tau_{\theta j} = -\sigma_1(\dot{\theta} + \sigma_1(\dot{\theta} + \theta + \sigma_1(\dot{\theta} + 2\theta - \dot{x}_j) + (\sum(x_j - \delta_j) - (x_j - \delta_j)))}) $$

$$ \tau_{\psi i} = -k_{\psi i}\dot{\psi}_i - k_{\psi i}(\sum((\psi_j - \delta_{\psi}) - (\psi_j - \delta_{\psi})) $$

Control algorithm, which includes second-order consensus (3), achieves convergence to the formation position and to a common speed in the plane (applied to guarantee that $x_i - x_{i+1}$ and $v_i$ is based on the following equations:

$$ F_{\tau i} = \frac{-k_{\tau i}\dot{z}_i - k_{\tau i}(\sum(z_i - \delta_j) - (z_j - \delta_j))}{\cos(\phi)\cos(\theta)} + 1 $$

$$ \tau_{\phi i} = -\sigma_1(\dot{\phi} + \sigma_1(\dot{\phi} + \phi + \sigma_1(\dot{\phi} + 3\dot{\phi} - 3\dot{y}_i) - (\sum(y_j - \delta_j) - (y_j - \delta_j)))}) \sum(y_j - \delta_j) - (y_j - \delta_j)))) $$

$$ \tau_{\theta j} = -\sigma_1(\dot{\theta} + \sigma_1(\dot{\theta} + \theta + \sigma_1(\dot{\theta} + 3\theta - 3\dot{x}_j) - (\sum(x_j - \delta_j) - (x_j - \delta_j)))}) \sum(x_j - \delta_j) - (x_j - \delta_j))) $$

$$ \tau_{\psi i} = -k_{\psi i}\dot{\psi}_i - k_{\psi i}(\sum((\psi_j - \delta_{\psi}) - (\psi_j - \delta_{\psi})) $$

It is also possible to modify the control law to enable the quadrotors in the formation to track an external reference trajectory. These reference trajectories may be forwarded to all quadrotors in the formation, and then the consensus ensures that the entire formation closely follows the given trajectory. However, in our research, only one quadrotor has reference trajectory, which can be regarded as the leader, while control algorithm will ensure that the quadrotor precisely tracks the given trajectory. This control law is defined by the following equations:

$$ F_{\tau j} = \frac{-k_{\tau j}\dot{z}_j - k_{\tau j}(\sum(z_j - \delta_j) - (z_j - \delta_j)) + z_{d,j}}{\cos(\phi)\cos(\theta)} + 1 $$

$$ \tau_{\phi j} = -\sigma_1(\dot{\phi} + \sigma_1(\dot{\phi} + \phi + \sigma_1(\dot{\phi} + 2\dot{\phi} - \dot{y}_j) + (\sum(y_j - \delta_j) - (y_j - \delta_j)))}) \sum(y_j - \delta_j) - (y_j - \delta_j)))) $$

$$ \tau_{\theta j} = -\sigma_1(\dot{\theta} + \sigma_1(\dot{\theta} + \theta + \sigma_1(\dot{\theta} + 3\theta - 3\dot{x}_j) - (\sum(x_j - \delta_j) - (x_j - \delta_j)))}) \sum(x_j - \delta_j) - (x_j - \delta_j))) $$

$$ \tau_{\psi j} = -k_{\psi j}\dot{\psi}_j - k_{\psi j}(\sum((\psi_j - \delta_{\psi}) - (\psi_j - \delta_{\psi})) + \psi_{d,j}) $$

It differs from the control law (11) only for the added members $x_{d,i}$, which constitute the reference trajectory, and which are only different from zero for only one $i$ (it is considered as first or the quadrotor $i = 1$).

### 5. Simulation Results

As simulation example, the case of the six quadrotors is presented, where in the initial moment, each of quadrotors proposes its position, but later in accordance with the algorithm, this position is changed until reaching the given spatial configuration (formation). The initial selection of positions was realized by random function, i.e. by selecting a value from a certain interval with uniform distribution. Formation is defined by the choice of $\delta$ coefficients for each state in which consensus is achieved. For coordinates $x_0 = [1 0 0 0 0 0]$, for coordinates $y_0 = [0 1 0 0 0 0]$, for coordinates $z_0 = [0 0 0 0 0 0]$, while for coordinates $\psi, \delta_0 = [0 0 0 0 0 0]$.

In Fig. 2, the convergence of relevant state variables of six quadrotors to common values in the case of first order consensus, is shown. This is best illustrated by using the orientation angle $\psi$. For the remaining three coordinates of the position $x, y$ and $z$, it is shown how the four quadrotors converge to common values, while in the case of other two, their coordinates are shifted by a constant value to ensure spatial separation (desired shape formation). For the remaining two graphs, the velocity in the plane is displayed (i.e., they can be seen converging to zero, when movement stops to achieve the desired formation).

In Fig. 3, the second order consensus example is shown, presenting how the velocities of the six quadrotor are converging towards the common identical value (which is generally different from zero, as opposed to the previous examples). At the same time, position of the quadrotor is changed in accordance with the speeds obtained by a consensus, while still maintaining mutual relationship that requires a desired formation.

In Fig. 4, there are state variables of 6 quadrotors in the case with external reference input. The blue line denotes a first quadrotor (leader). It is obvious that leader better follows the reference trajectory. Other quadrotors with a slight delay follow the leader, while they trying to better keep up a formation. With regard for other quadrotors, the reference trajectory is not available (this information is extracted from the consensus algorithm), it can be said that other quadrotors very well track the reference trajectory.

Although, the reference trajectory is provided only for $x$ and $y$, it is possible in the same way to realize for the remaining two degrees of freedom $z$ and $\psi$. The
Chosen shape of formation is *diamond formation*, which is shown in Fig. 5. All control algorithms are valid for the arbitrary form of formation, where some examples are shown in Fig. 6 and 7.
6. CONCLUSION

Consensus algorithm is a powerful tool for a variety of applications in multi-agent systems and sensor networks. In this paper, movement of the aerial vehicles in the formation is successfully achieved by consensus algorithms. This method is suitable, because the information space is not huge, but includes as many states as how many degrees of freedom are achieved by consensus or by the number of states in which the formation is achieved. The problem of dynamic conditions under which consensus is achieved is avoided by suitable choice of the control law that serves to stabilize the dynamics. Extensions of the basic consensus algorithm by a second-order consensus, together with the case, where the formation required to follow a predetermined external reference are realized. Simulation results are fully consistent with the theory of consensus algorithms, but for the real-time implementation it is necessary to take into consideration communication channel, the results and performance of the method would be highly dependent on the delay in the communication channel, reliability of the network and loss of packet information.

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