Romeo Meštrović
Professor
University of Montenegro
Maritime Faculty Kotor
Montenegro

Branislav Dragović
Professor
University of Montenegro
Maritime Faculty Kotor
Montenegro

Nenad Zrnić
Professor
University of Belgrade
Faculty of Mechanical Engineering

Dražen Dragojević
PhD candidate
University of Montenegro
Maritime Faculty Kotor
Montenegro

A Relationship Between Different Costs of Container Yard Modelling in Port Using Queuing Approach

In this paper we consider batch arrivals of containers at a port container yard which is modeled as a multi-server queue $M^X/M/c$ with c yard cranes for the service. It is assumed that the related group size (the number of containers in an arriving group), X is distributed by a Poisson-like distribution. Using a more general formula for such queue models, here it is deduced the expression for the specific cost ratio involving the state probabilities, the utilization factor, the mean and the variance of the group size. Applying this expression, related numerical and graphical results are presented when the number of yard cranes at container yard in port is 1, 2 *or* 3. *We also point out that this expression allows us to discuss and compare the values of specific cost ratio concerning the considered $M^X/M/c$ queues with different port performance parameters, as well as with specific cost ratio of the queues investigated in earlier authors' papers on this topic.*

Keywords: *Container yard modelling, $M^X/M/c$ queue, Specific cost ratio, Poisson-like distribution, Utilization factor.*

1. INTRODUCTION

In this paper, we focus our attentiom on solving or at least simplifying the problems concerning container yard in the port by applying some analytical models of queuing theory. The objective is to describe this model for defining the strategies at yard and to calculate the total cost of the system. It is obvious that the arrival and service processes of containers at container yard must give the input data in the shape of some statistical distribution.

A container yard in the port can be considered as a queuing system defined by basic parameters: the container arrival rate and the container service rate in an observed period of time. It is evident that the optimal number and capacity of servers must be of greater importance in real system. The total cost of the system can be also determined by the specific number of servers. In ship-berth link, quay cranes are servers and, on the other hand, servers at container yard are specific types of yard cranes. Therefore, the costs of yard cranes make important point for obtaining total costs of containers at the terminal.

It is well known that containers arrive at yard in batches and its behavior must fit some statistical distribution. Considering that, it is noted that for valid analysis some studies used batch arrival queuing model and batch arrival multi-server queuing system. Several authors have investigated this system using different techniques. Mennis et al. in [1] used Markov theory and reliability models for the estimation of the associated risks and costs that can result in delays due to machinery breakdowns in container terminals. Gaver in [2] used Markov chain method for defining the arrival in batches to a single channel with arbitrary service time distribution. Also, using renewal theorem of batch arrival, Burke in [3] solved a single-server queuing system. All these authors have carried out their own theory about queuing systems, specifying the best approach for modelling container terminals in the port. Speaking of batch arrival of customers, Kozan in [4] made a comparison of analytical and simulation planning models of container terminals and shows the advantages of simulation because it is able to capture all details and the complexity of a real system. He also pointed out the importance of better approach to modeling the container port system by batch arrivals of containers as opposed to ships. He considered the ships as a batch of containers.

The analysis of a queue with batch arrivals and batch-dedicated servers is explained in [5]. In this paper, it is considered the $M/G/\infty$ queuing system with batch arrivals whose jobs belonging to a batch have to be processed by the same server. In [6] it was developed an analytical methodology of bulk queuing system. This methodology determines the optimum number and capacity of berths within seaports and river ports. In [7], the authors deal with the port storage locations as queuing systems with bulk arrivals and a single service. The optimal numbers of servers in queuing system with bulk arrivals by minimizing the total costs of system are determined in [8]. In the papers [9] and [10] authors discussed about the anchorage-ship-berth link at the port utilizing queuing theory with bulk arrivals. The developed process is described by the non-stationary multi-channel queuing system. In the same manner, Laxmi and Gupta in [11] analyzed a multi-server queue

Received: January 2018, Accepted: March 2018
Correspondence to: Dr. Romeo Meštrović, Professor
Maritime Faculty of the University of Montenegro,
Dobrota 36, 85330 Kotor, Montenegro
E-mail: romeo@ac.me
doi:10.5937/fmet1803367M

with bulk arrivals and finite-buffer space. Lagoudis and Platis in [12] used birth-and-death modelling in order to examine the improvement of container terminal opera–tions in two stages of the container transportation pro–cess. Dynamic system performance evaluation in the port utilizing queuing models with batch arrivals was studied by Dragović et al. in [13] (for similar topics, see [14-17]). Furthermore, the total cost for different port systems was extensively studied in literature in the last fifteen years ([18-28]) (for earlier related study, see [29] and [30]).

The remainder of the paper is organized as follows. The mathematical model of considered batch arrival multi-server queue $M^X/M/c$ is described in Section 2. Moreover, in this section it is derived the analytical expression for the specific cost ratio related to this port queue model. Related numerical and graphical results are presented in Section 3. Concluding remarks, inclu–ding the potential subject for the further investigations in related topic, are presented in the last section.

2. MATHEMATICAL MODEL

Here we examine a batch arrival multi-server queue $M^X/M/c$ described as follows. Containers arrive at a port container yard in batches according to a time-homogeneous (stationary) Poisson process with mean arrival rate λ. A container yard is a single or multi-channel system with c yard cranes for the service. The c yard cranes have independent, exponentially distributed service times with common average service time $1/\mu$ (μ is the service rate). The queue discipline is first come first served by tows batch and random within the tow batch. The number of containers that arrive for service at the same time is a discrete random variable X with distribution given by $a_k = P(X = k)$, $k \geq 1$ (where k = number of containers in group), whose mean is $E(X)=\overline{a}$ and the variance $\sigma^2(X) = \sigma^2$. Furthermore, the interarrival times, the batch sizes and service times are mutually independent. The service times of service batches (containers) are independent of the arrival process. The utilization factor or server occupancy of considered queue model is $\rho=(\lambda\overline{a})/(c\mu)$, and its traffic intensity is usually defined as $\theta = \lambda/\mu$.

Let P_n (n = 0,1,2,…) be the (steady-state) probability that there are n containers in a port container yard. Let $L = L(c)$ be the average number of containers at the yard, and let $L_q = L_q(c)$ denote the average number of containers in queue. Then by (Chaudhry and Tampleton [31], Chapter 5, Section 5.2, p. 244), we have

$$L-L_q = c+\sum_{n=0}^{c}(n-c)P_n \ . \tag{1}$$

Further, by ([31], Chapter 5, Section 5.2, p. 244, Eq. (5.2.14)), we find that

$$L-L_q=\theta\overline{a}=\frac{\lambda}{\mu}\overline{a}=c\rho \ . \tag{2}$$

Comparing (1) and (2), we immediately obtain

$$P_0 = 1-\rho-\frac{\sum_{n=1}^{c}(c-n)P_n}{c} \tag{3}$$

It is known that for the probabilities P_n (n = 1,2,…) the following Kabak's recurrence relations ([10] and [32]) are satisfied:

$$P_n = y(n)\sum_{k=0}^{n-1}P_k A_{n-k} \ , \ n=1,2,..., \tag{4}$$

where

$$y(n)=\frac{\lambda}{\mu(n)} \text{ with } \mu(n) = \mu \cdot \min\{n,c\},\ n = 1,2,\ldots \tag{5}$$

and the coefficients A_i are recursively defined as

$$A_j = 1-\sum_{i=0}^{j-1}a_i \text{ with } a_i = P(X=i) \text{ for each } j=2,3,..., \text{ and}$$
$$A_1 = 1 \ . \tag{6}$$

For our purposes, it is suitable the following expression for the average number of containers present in considered queuing system with c yard cranes $L=L(c)$ concerning any arbitrary queue model $M^X/M/c$ described above ([31], Chapter 5, Section 5.2, p. 244):

$$L(c)=\frac{\theta(\overline{a}+(\sigma^2+(\overline{a})^2-\overline{a})/2)+\sum_{n=0}^{c-1}n(c-n)P_n}{c-\theta\cdot\overline{a}}, \tag{7}$$

where $\overline{a}$ and σ^2 are the mean and the variance of random variable X, respectively.

Taking $\theta = c\rho/\overline{a}$ into (7), we obtain

$$L(c)=\frac{\rho(\sigma^2+\overline{a}^2+\overline{a})}{2\overline{a}(1-\rho)}+\frac{\sum_{n=0}^{c-1}n(c-n)P_n}{c(1-\rho)} \tag{8}$$

2.1 Total annual cost related to a queuing system with *c* yard cranes

Let us note that the total cost for numerous port systems was extensively investigated by many authors. In particular, this concerns the problem of determination and minimization of the so-called the specific cost ratio which is rarely investigated in the literature. Namely, under some given constraints, the determination of specific cost ratio and the optimal numbers of berths in a port in the sense of minimization of related specific cost ratio for certain single arrival and bulk arrival queueing systems were studied by several authors in [18-20], [22-30]. By considering the total annual cost for queuing systems with c yard cranes, in [27] (also see [28]) it was established the following equality:

$$C_{qs} = OT_{yc}\cdot T_e\cdot c + OT_c\cdot T_e\cdot L(c) \tag{9}$$

where

C_{qs} - total annual cost for queuing system with c

yard cranes;

OT_{yc}	- the daily operating cost of yard crane;
T_e	- defined period of year (365 days);
c -	- number of yard cranes;
OT_c	- the daily cost of containers;
T_e	- defined period/considered period of year (365 days);
OT_c	- daily cost of containers and
$L(c)$	- the average number of containers that are present in a queuing system.

Then dividing the equation (9) by $OT_c \cdot T_e$, we find that (cf. [28])

$$R_c = \frac{C_{qs}}{OT_c \cdot T_e} = \frac{OT_{yc}}{OT_c} \cdot c + L(c) = r_{cy} \cdot c + L(c) \tag{10}$$

where R_c is the *specific cost ratio* (i.e.., the quotient of total annual cost for queuing system with c yard cranes and total annual cost of containers) and $r_{cy} = \frac{OT_{yc}}{OT_c}$ is the daily yard crane–container cost ratio (the quotient of daily operating cost of yard crane and the daily cost of containers).

The equality (10) shows that the value of specific cost ratio depends on the average number of containers at the yard, $L = L(c)$.

Now we consider the case when containers arrive and service at a port container yard in batches according to the batch arrival multi-server queue $M^X/M/c$ described above. Recall that in [28] it was studied the specific cost ratio concerning the queue model $M^X/M/c$, where X is a constant or a geometric distribution. As applications, some numerical examples are presented in [28] for the port of Bar, Montenegro. Moreover, in [20] it was considered the case when a group of containers that arrive at yard has the shifted-Poisson distribution X with the parameter a, that is,

$$P(X = i) = a_i = e^{-a} \cdot \frac{a^{i-1}}{(i-1)!}, \ i = 1,2,... ; \ a > 0$$

Motivated by the Poisson distribution ([32, 33]) and the shifted-Poisson distribution (for more information on the genearilzed random variables involving Poisson distribution, see [34]), here we consider the case when a group of containers that arrive at yard has the Poisson-like distribution X with the parameter $a > 0$, whose probability law is defined as (cf. [17])

$$P(X = i) = a_i = \frac{1}{e^a - 1} \cdot \frac{a^i}{i!}, \ i = 1,2,... \tag{11}$$

Note that the random variable X is well defined in view of the fact that

$$\sum_{i=1}^{\infty} \frac{1}{e^a - 1} \cdot \frac{a^i}{i!} = \frac{1}{e^a - 1}\left(-1 + \sum_{i=0}^{\infty} \frac{a^i}{i!} \right) = 1.$$

For our computational purposess we will need the following result (cf. [17]).

Proposition. *The mean and the variance of* X *are respectively given by*

$$\overline{a} = E(X) = \frac{ae^a}{(e^a - 1)} \tag{12}$$

and

$$\sigma^2 = \sigma^2(X) = \frac{ae^a(e^a - a - 1)}{(e^a - 1)^2}. \tag{13}$$

Proof. Taylor expansion of the function $x \to e^x$, (x □ $(-\infty,+\infty)$)is given as a sum

$$e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!} \tag{14}$$

Then from (11) and (14) we immediately obtain

$$E(X) = \frac{1}{e^a - 1} \sum_{i=1}^{\infty} i \cdot \frac{a^i}{i!} = \frac{a}{e^a - 1} \sum_{i=1}^{\infty} \frac{a^{i-1}}{(i-1)!} =$$
$$= \frac{a}{e^a - 1} \sum_{k=0}^{\infty} \frac{a^k}{k!} = \frac{ae^a}{e^a - 1},$$

which yields the equality (12).

Differentiating the equation (14) multiplied by x, we find that for all $x \neq 0$

$$e^x + xe^x = \frac{d}{dx}(xe^x) = \frac{d}{dx} \sum_{i=0}^{\infty} \frac{x^{i+1}}{i!} = \sum_{i=0}^{\infty} \frac{(i+1)x^i}{i!}$$
$$= \frac{1}{x} \sum_{i=0}^{\infty} \frac{(i+1)x^{i+1}}{i!} = \frac{1}{x} \sum_{i=1}^{\infty} \frac{ix^i}{(i-1)!},$$

whence it follows that

$$\sum_{i=1}^{\infty} \frac{ix^i}{(i-1)!} = xe^x(1+x) \tag{15}$$

From the equalities (12) and (15) we find that

$$\sigma^2(X) = E(X^2) - (E(X))^2$$
$$= \frac{e^a}{e^a - 1} \sum_{i=1}^{\infty} i^2 e^{-a} \frac{a^i}{i!} - \frac{a^2 e^{2a}}{(e^a - 1)^2}$$
$$\frac{1}{e^a - 1} \sum_{i=1}^{\infty} \frac{ia^i}{(i-1)!} - \frac{a^2 e^{2a}}{(e^a - 1)^2}$$
$$= \frac{ae^a(1+a)}{e^a - 1} - \frac{a^2 e^{2a}}{(e^a - 1)^2} = \frac{ae^a(e^a - a - 1)}{(e^a - 1)^2},$$

which implies the equality (13). This completes the proof of proposition.

Taking (12) and (13) into (8), and then into (10) after a routine calculation, we obtain the following expression for R_c:

$$R_c = r_{cy} \cdot c + \frac{(a+2)\rho}{2(1-\rho)} + \frac{\sum_{n=0}^{c-1} n(c-n) P_n}{c(1-\rho)}, \tag{16}$$

where

$$L(c)=\frac{(a+2)\rho}{2(1-\rho)}+\frac{\sum_{n=0}^{c-1} n(c-n)P_n}{c(1-\rho)} \quad (17)$$

3. NUMERICAL RESULTS

Here, we consider container's arrivals at a port container yard as a particular batch arrival multi-server queue $M^X/M/c$ described in Section 2, where the batch size (X) has a Poisson-like distribution (cf. [28], where it was assumed that X has a constant or a geometric distri–bution, and in [20], where it was supposed that X has a shifted-Poisson distribution). A container yard is a sin–gle or multi-channel system with c yard cranes for the service (c = 1,2,3). The number of containers that arrive for service at the same time is a Poisson-like distri–bution X whose probability law is defined by (11) with parameter a = 2 or a = 5 (cf. [17]). For a related discu–ssion it is supposed that the value of utilization factor ρ varies between 0.2 and 0.8. It is assumed that the daily yard crane–container cost ratio is equal to r_{cy} = 0.4. For these purposes, by applying the formulae (3) - (6), (12) and (13), here we deduce the formulae for the specific cost ratio R_c with c = 1,2,3 related to the $M^X/M/c$ queue in which a group of containers that arrive at yard has the Poisson-like distribution X defined by (11). From (12) we have

$$\rho=\frac{\lambda\bar{a}}{c\mu}=\frac{\theta a e^a}{c(e^a-1)}. \quad (18)$$

From the equality (18) we find that

$$\rho=\frac{\lambda\bar{a}}{c\mu}=\frac{\theta a e^a}{c(e^a-1)}. \quad (19)$$

Now we consider the cases when c = 1,2,3.

Case 1: c = 1. Then from the equality (16) with c = 1 we immediately obtain

$$R_1=r_{cy}+\frac{(a+2)\rho}{2(1-\rho)}. \quad (20)$$

Case 2: c = 2. Then from (3) we find that $P_0 = 1 - \rho - P_1/2$, while from (4) - (6) and (19) we get $P_1 = \theta P_0 A_1 = 2\rho P_0(e^a\text{-}1/(ae^a)$. The previous two equa–lities implies that

$$P_0=\frac{(1-\rho)ae^a}{ae^a+\rho e^a-\rho} \text{ and } P_1=\frac{2\rho(1-\rho)(e^a-1)}{ae^a+\rho e^a-\rho}.$$

Taking the above two values for P_0 and P_1 into (16) with c =2 immediately yields

$$R_2=2r_{cy}+\frac{(a+2)\rho}{2(1-\rho)}+\frac{\rho(e^a-1)}{(ae^a+\rho e^a-\rho)}. \quad (21)$$

Case 3: c = *3*. Then from (3) we obtain $P_0 = 1 - \rho - (2P_1+P_2)/3$, while from (4) - (6) and (19) we find that

$P_1=\theta P_0 A_1=3\rho(e^a-1)P_0/(ae^a)$ and

$$P_2=(\theta/2)(P_0A_2+P_1A_1)$$
$$=3\rho(e^a-1)(P_0(1-e^{-a}-ae^{-a})+P_1)/(2ae^a).$$

The expressions for the solutions P_0, P_1 and P_2 of the previous system of three linear equations are very large (complicated), and hence, here they are omitted. If e^a takes large values, then $(e^a\text{-}1) \approx e^a$. In this case, taking e^a-1 instead of e^a in the above system, leads to its following approximate solutions:

$$P_0\approx\frac{2a^2(1-\rho)e^a}{2a^2e^a+5a\rho e^a+3\rho^2e^a-a\rho-a^2\rho},$$
$$P_1\approx\frac{6a\rho(1-\rho)e^a}{2a^2e^a+5a\rho e^a+3\rho^2e^a-a\rho-a^2\rho}$$
$$P_2\approx\frac{3(1-\rho)\rho(ae^a+3\rho e^a-a-a^2)}{2a^2e^a+5a\rho e^a+3\rho^2e^a-a\rho-a^2\rho}.$$

Taking the above values for P_0, P_1 and P_2 into (16) with c = 3 yields

$$R_3\approx 3r_{cy}+\frac{(a+2)\rho}{2(1-\rho)}+\frac{2\rho(3ae^a+3\rho e^a-a-a^2)}{2a^2e^a+5a\rho e^a+3\rho^2e^a-a\rho-a^2\rho}. \quad (22)$$

Applying the expressions (20), (21) and (22), we obtain the numerical results given in Tables 1 and 2. Also, by using the expressions (20), (21) and (22), we obtain the numerical results given in Tables 3 and 4.

Tables 1 - 4 and Figures 1 - 3 show that the values of R_c (i.e., the quotient of total annual cost for queuing system with c yard cranes and total annual cost of conta–iners) increase with respect to both variables (parameters) c (for a fixed ρ) and ρ (for a fixed c). Moreover, the same fact is true with respect to the parameter a when a > 1. Accordingly, in order to decrease the total annual cost for queuing system with c yard cranes with respect to the total annual cost of containers, it is necessary to decrease at least one of these three parameters.

Table 1. Specific cost ratio (R_c) related to the queuing system $M^X/M/c$ with $P(X = i) = a_i = a^i/((e^a\text{-}1)i!$, i = 1,2,...,

	R_c for a = 2 and r_{cy} = 0.4		
c / ρ	1	2	3
0.20	0.90000	1.37959	1.93251
0.30	1.25714	1.77195	2.38842
0.40	1.73333	2.28077	2.95376
0.50	2.40000	2.97774	3.70120
0.60	3.40000	4.00597	4.77464
0.70	5.06667	5.69899	6.50831
0.80	8.40000	9.05698	9.90298

Table 2. Specific cost ratio (R_c) related to the queuing system $M^X/M/c$ with $P(X = i) = a_i = a^i/((e^a\text{-}1)i!$, i = 1,2,...

	R_c for a = 5 and r_{cy} = 0.4		
c / ρ	1	2	3
0.20	1.27500	1.71321	2.18682
0.30	1.90000	2.35624	2.86321
0.40	2.73333	3.20695	3.74523
0.50	3.90000	4.39035	4.95809
0.60	5.65000	6.15650	6.75197
0.70	8.56667	9.08875	9.71038
0.80	14.40000	14.93710	15.58350

Table 3. Specific cost ratio (R_c) related to the queuing system $M^X/M/c$ with $P(X = i) = a_i = a^i/((e^a-1)i!$, i = 1,2,…

	R_c for $\rho = 0.4$ and $r_{cy} = 0.4$		
c / a	1	2	3
0.5	1.23333	1.66385	2.648
1.0	1.40000	1.84814	2.791
1.5	1.56667	2.02518	2.934
2.0	1.73333	2.19803	3.078
2.5	1.90000	2.36841	3.225
3.0	2.06667	2.53731	3.337
3.5	2.23333	2.70533	3.526
4.0	2.40000	2.87282	3.680
4.5	2.56667	3.03998	3.836
5.0	2.73333	3.20695	3.993

Table 4. Specific ratio (R_c) related to the queuing system $M^X/M/c$ with $P(X = i) = a_i = a^i/((e^a-1)i!$, i = 1,2,…

	R_c for $\rho = 0.6$ and $r_{cy} = 0.4$		
c / a	1	2	3
0.5	2.27500	3.12009	4.003
1.0	2.65000	3.52051	4.404
1.5	3.02500	3.91027	4.781
2.0	3.40000	4.29401	5.148
2.5	3.77500	4.67422	5.512
3.0	4.15000	5.05235	5.876
3.5	4.52500	5.80539	6.239
4.0	4.90000	6.18108	6.604
4.5	5.27500	6.18108	6.970
5.0	5.65000	6.55650	7.335

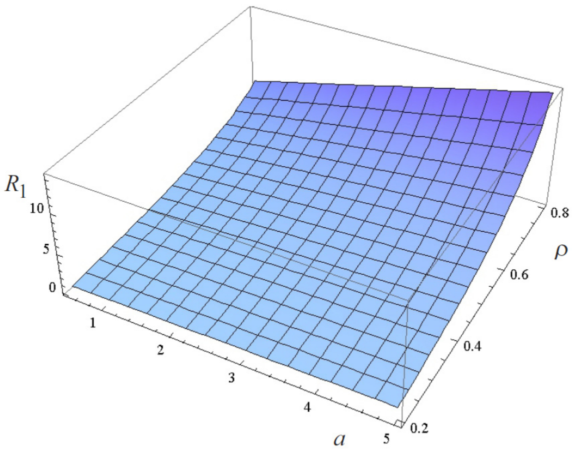

Figure 1. The graphic of the function ($a,\rho \rightarrow R_1(a,\rho) = R_1$ with $r_{cy} = 0.4$, $a \in (0.5,5)$ and $\rho \in (0.2,0.8)$

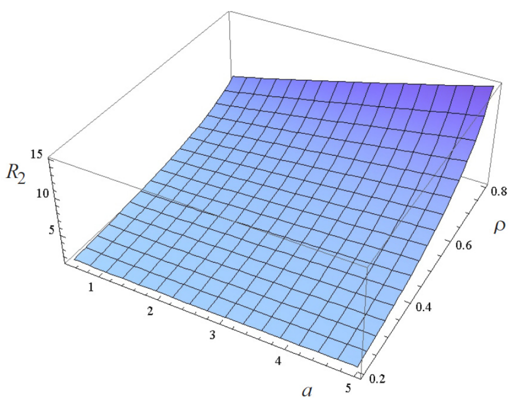

Figure 2. The graphic of the function ($a,\rho \rightarrow R_2(a,\rho) = R_2$ with $r_{cy} = 0.4$, $a \in (0.5,5)$ and $\rho \in (0.2,0.8)$

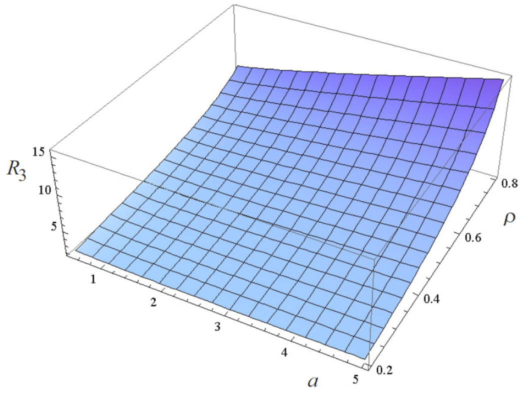

Figure 3. The graphic of the function ($a,\rho \rightarrow R_3(a,\rho) = R_3$ with $r_{cy} = 0.4$, $a \in (0.5,5)$ and $\rho \in (0.2,0.8)$

Let us notice that by the equality (10) it follows that $R_c > L(c)$, and thus, $R_c > 1$ whenever $L(c) > 1$ However, it can be seen from Table 1 that it is possible the case when $R_c < 1$ (especially, $R_1 = 0.9$ for $a = \overline{a} = 2$, $r_{cy} = 0.4$ and $\rho = 0.2$. It is clear that this situation can occur in port systems with low utilization. Note also that to the case of the port of Bar there correspond (approximately) only the values of R_c from Tables 1 and 2 for ρ with $0.2 \le \rho \le 0.5$.

Taking into account the previously mentioned facts, it can be believed that further study of the concept of specific cost ratio involved in different types of costs in various port systems (considered as some queuing systems), would be very appropriate and useful for improving related port efficiency.

4. CONCLUSION

Since containers arrive at a port container yard in batches, their arrivals can be often described as a particular batch arrival multi-server queue. In this paper we consider batch arrivals of containers at a port con–tainer yard which is a multi-channel system with c yard cranes for the service. More precisely, it is modelled as a batch arrival multi-server queue $M^X/M/c$ in which the size X of arriving group is distributed by the Poisson-like distribution. For such a multi-server queue, here it is deduced the explicit expression for the specific cost ratio (R_c) defined as a ratio of total annual cost to total annual cost of containers. Using this expression, some numerical and graphical results are presented.

Notice that the queuing modeling approach to the analysis of specific cost ratio can be important for under–standing various kinds of port operations and determining the optimal values of their performances. In particular, further study of the concept of specific cost ratio con–cerning the investigations of different types of costs in several port systems (considered as certain queuing sys–tems) would be important for improving the correspon–ding port efficiency. Some results in this direction were established in the literature during last ten years. Accor–dingly, we believe that the obtained results of this paper should be useful for further research in this direc–tion.

ACKNOWLEDGMENT

The study was carried out within the Project MNE-HERIC-81180, financed within the scope of "Higher

Education and Research for Innovation and Competitiveness in Montenegro" – ("HERIC") project, from the International Bank for Reconstruction and Development loan, in accordance with the Decision of the Ministry of Science of Montenegro on awarding the grant: Number: 01-1062 from 29th May 2014.

Also, the study was carried out within the Bilateral project (Montengro and Serbia) entitled "Modeling and sustainable development of Montenegro and Serbia intermodal connections (road and railway)", financed by the Ministry of Science of Montenegro and the Ministry of Education, Science and Technological Development of Serbia.

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МЕЂУЗАВИСНОСТ РАЗЛИЧИТИХ ТРОШКОВА МОДЕЛИРАЊА КОНТЕНЕРСКИХ СКЛАДИШТА У ЛУЦИ ПРИМЕНОМ ТЕОРИЈЕ РЕДОВА ЧЕКАЊА

Р. Мештровић, Б. Драговић, Н. Зрнић, Д. Драгојевић

У овом раду разматрани су групни доласци контенера на контенерском складишту који су моделирани вишеканалним системом теорије редова чекања $M^X/M/c$ са c складишних дизалица као каналима опслуживања. Претпостављено је да је величина групе контенера, X (број контенера у групи која долази на контенерско складиште) дистрибуирана у складу са Пуасон-овским типом расподеле.

Користећи општи израз за разматрани модел редова чекања, изведен је израз за однос специфичних трошкова који садржи вероватноће стања система, фактор искоришћења система, средњу вредност и дисперзију величине групе. Примењујући тај израз, представљени су одговарајући нумерички и графички резултати за 1, 2 и 3 складишне дизалице на контенерском складишту у луци. Такође истичемо да добијени аналитички израз омогућава дискусију и поређење вредности односа специфичних трошкова разматраних модела редова чекања $M^X/M/c$ са различитим параметрима перформансе луке, као и поређења истих са односом специфичних трошкова у односу на редове чекања истраживаних у претходним радовима аутора који се односе на овај проблем.