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Calculation Method to Determine the Throughput and the Energy Consumption of S/R Shuttle Systems

Shuttle systems are used in high performance automated storage/retrieval unit load systems. Each storage level is serviced by one shuttle. On the front side of the rack there is a lift for storage and retrieval. A different number of buffer slots can be located between the lifts and the respective levels. These systems are already treated in various scientific papers. In this paper, the calculation of the throughput of a shuttle system and, more particularly, the calculation of the energy consumption for the handling of a tote in such a system is presented. The results of these calculation methods are shown.

Keywords: shuttle-system, storage/retrieval unit load systems, performance, analytical calculation, energy consumption

1. INTRODUCTION AND PROBLEM DEFINITION

Due to the development of the economy to demand-oriented delivery and storage of consumables and pro–ducts, the requirements for the storage technology are becoming higher and higher. In order to meet these requirements, shuttle systems are increasingly installed. They are flexible and achieve highest performances. The focus of this paper is not only on the calculation of the throughput, but on the calculation of the energy consumption for storage and retrieval.

A number of research papers have been published in the past to investigate these systems [3, 4, 5].

In detail, the following research questions will be treated:

 ➢ Which throughput can be achieved by shuttle systems?
 ➢ What is the energy consumption for one double cycle?

2. DESCRIPTION OF THE SHUTTLE SYSTEM

The shuttle system treated here is a system with container lifts. This means that the shuttles can not leave the individual levels and the lifts transport only the totes. So there must be a shuttle in each level of the system. Between lifts and shuttles there is a buffer installed. See Figure 1.

3. ANALYTICAL CALCULATION OF THE THROUGHPUT

In order to calculate the throughput of an entire shuttle system the utilization rate of a single level must be determined first. This is done with the following equation:

$$\rho = \frac{\lambda}{\mu} = \frac{t_o}{t_a} \qquad (1)$$

It includes the relationship between the inter–arrivaltime of the lifts t_a and the operationg time of the shuttles t_o.
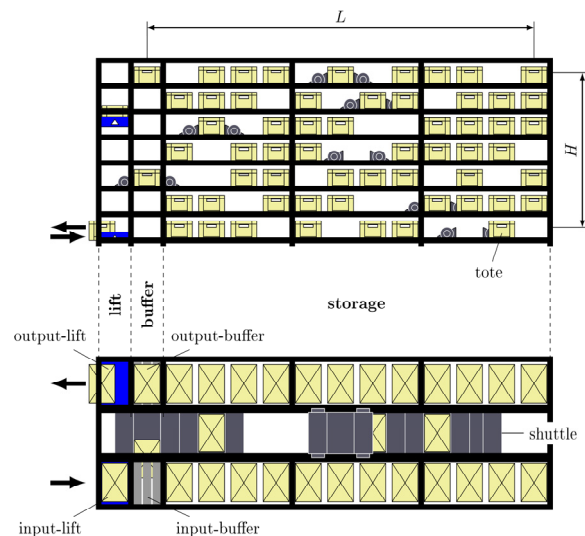

Figure 1. Shuttle-System

The interarrival time is the mean cycle time of the lifts converted to the individual levels. This conversion is done by the multiplication of the cycle time by the number of levels in the rack:

$$t_a = t_{lift} \cdot n_{level} \qquad (2)$$

The mean cycle time of the lifts is defined as follows [1]:

$$t_{lift} = \frac{H}{v_{lift}} + 2\frac{v_{lift}}{a_{lift}} + t_{load/unload\_lift} \qquad (3)$$

The second part in determining the utilization rate of one level is the mean operating time of the shuttles. This is calculated as follows [1].

$$t_o = t_{shuttle} \qquad (4)$$

$$t_{shuttle} = \frac{4}{3}\frac{L}{v_{shuttle}} + 3\frac{v_{shuttle}}{a_{shuttle}} + t_{load/unload\_shuttle} \qquad (5)$$

In order to calculate the throughput a queuing system M|G|1|K is used. The throughput of one level is given by the following equation [2]:

$$\vartheta_{level} = \frac{1}{t_a}\left(1-p_K\right) = \frac{1}{t_b}\left(1-p_0\right) \qquad (6)$$

The result of this queuing must be converted with the following relationship, to get the result for the whole system.

$$\vartheta_{system} = \vartheta_{level} \cdot n_{level} \qquad (7)$$

Equation (6) shows two methods for determining the throughput. On the one hand by the interarrivaltime and the blocking probability, on the other hand, via the operating time and the vacancy probability.

The basic equation to calculate the blocking probability is [2]:

$$p_K = \frac{\rho^{\left(\frac{\sqrt{\rho}\cdot s_s^2 - \sqrt{\rho} + 2K}{2+\sqrt{\rho}\cdot s_s^2 - \sqrt{\rho}}\right)}\cdot\left(\rho-1\right)}{\rho^{\left(2\frac{\sqrt{\rho}\cdot s_s^2 - \sqrt{\rho} + K + 1}{2+\sqrt{\rho}\cdot s_s^2 - \sqrt{\rho}}\right)}-1} \qquad (8)$$

The vacancy probability is quite similar [3]:

$$p_0 = \frac{\left(\rho-1\right)}{\rho^{\left(2\frac{\sqrt{\rho}\cdot s_s^2 - \sqrt{\rho} + K + 1}{2+\sqrt{\rho}\cdot s_s^2 - \sqrt{\rho}}\right)}-1} \qquad (9)$$

These two equations contain the utilization rate of the individual levels, the variance of the operating times of the shuttles and the capacity of the considered system. The capacity K of the queuing system can be defined as follows:

$$K = n_{buffer} + n_{capShuttle} \qquad (10)$$

This means that the capacity of the considered waiting system is always the number of input buffer slots in one level (n_{buffer}=1) plus the capacity of the shuttle ($n_{capshuttle}$=1). In this case, the capacity of the shuttle itself is always provided with one tote.

The variance of the operating times by the shuttle results from the probability distribution of the operating times. These operating times are subjected to a triangular distribution [2]. The variance can therefore be calculated using the following equation.

Table 1. System parameters

	lift	shuttle
velocity	5 m/s	2 m/s
acceleration	7 m/s²	2 m/s²
load-/unload time	2.8 s	4.0 s
vertical pitch	p_v=0.4 m	
horizontal pitch	p_h=0.5 m	

Based on the data in Table 1 the throughput results are shown in Figure 2 for single deep racks with a length of 100 m. The result could be confirmed by simulation with SIMIO.
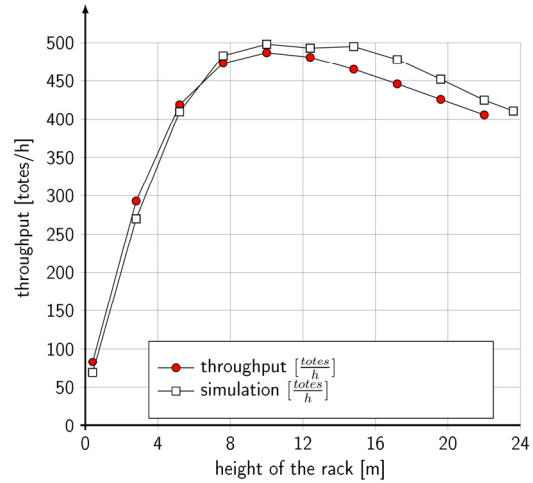

Figure 2. Throughput of a shuttle-system

4. ANALYTICAL CALCULATION OF THE ENERGY CONSUMPTION

The calculation of the energy consumption of a shuttle system consists of the energy consumption for the acceleration or decceleration, the energy for a ride with constant speed, the energy for lifting movement and the consumptions for the storage and retrieval. These individual consumptions are listed below. Also the limits of validity for the given equations are shown. This calculation method has been introduced to compare different systems and configurations.

4.1 Energy consumption of the whole shuttle-system

The energy consumption of the whole shuttle system att–ributed to one double cycle of a tote is as follows. In add–ition to the energy consumption of the lifts and the shuttle, the standby consumptions of all lifts and shuttles including the energy consumption of the control of the whole shuttle system, are divided by the handling performance.

4.2 Energy consumption for the vehicles

In this part, the equations are shown for the two main parts of a shuttle system, the lift and the shuttle. These two equations relate to one storage and retrieval process including the energy consumption of the vehicles when riding without any load.

$$s_s = \frac{\sqrt{\frac{\left(2\frac{L}{v_{shuttle}}\right)^2}{18}}}{t_{shuttle}} \qquad (11)$$

$$E_{system} = E_{lift} + E_{shuttle}$$
$$+ \frac{E_{lifts\,tan\,dby}n_{lift} + E_{shuttles\,tan\,dby}n_{shuttle} + E_{control}}{\vartheta_{system}} \qquad (12)$$

Lift single cycle

$$E_{lift} =$$
$$+E_{Ahvu} + E_{Rhvu} + E_{liftup} + E_{Dvu}$$
$$+(E_{Avd} + E_{Rvd} + E_{down} + E_{Dhvd}) \cdot \eta_{reklift}$$
$$+E_{Ahvu} + E_{Rhvu} + E_{liftup} + E_{Dvu} \qquad (13)$$
$$+(E_{Avd} + E_{Rvd} + E_{down} + E_{Dhvd}) \cdot \eta_{reklift}$$
$$+2 \cdot E_{load/unload\_lift}$$

Shuttle double cycle

$$E_{shuttle} =$$
$$+E_{Ahvu} + E_{Rhvu} + E_{Dhvd} \cdot \eta_{reklift}$$
$$+E_{Ahvu} + E_{Rhvu} + E_{Dhvd} \cdot \eta_{reklift} \qquad (14)$$
$$+E_{Ahvu} + E_{Rhvu} + E_{Dhvd} \cdot \eta_{reklift}$$
$$+2 \cdot E_{load/unload\_shuttle}$$

4.3 Energy consumption for acceleration an deceleration

The special thing about energy consumption for accelerating and decceleration is that there is also negative consumption. This happens for example when the lift accelerates downwards and the acceleration is less than the gravity. There are four different equations to calculate the energy consumption for acceleration depending on the riding direction.

First the acceleration in horizontal directon or vertical direction upwards is described.

$$E_{Ahvu} = +(m_{tote} + m_{veh} + m_{redveh}) \frac{v_{veh}^2}{2} \frac{1}{\eta_{veh}} \qquad (15)$$

Next the acceleration in the vertical direction downwards is shown.

$$E_{Avd} = +(m_{tote} + m_{veh} + m_{redveh}) \frac{v_{veh}^2}{2} \eta_{veh} \qquad (16)$$

The following equation relates to the decceleration process downwards or in horizontal direction.

$$E_{Dhvd} = -(m_{tote} + m_{veh} + m_{redveh}) \frac{v_{veh}^2}{2} \eta_{veh} \qquad (17)$$

The fourth and last equation includes the deccele–ration process in vertical direction upwards.

$$E_{Dvu} = -(m_{tote} + m_{veh} + m_{redveh}) \frac{v_{veh}^2}{2} \frac{1}{\eta_{veh}} \qquad (18)$$

For vertical transportation the following limit is relevant:

$$(m_{tote} + m_{veh} + m_{redveh}) \cdot a_{veh} \leq$$
$$(m_{tote} + m_{veh}) \cdot g \cdot \mu_{veh} + (m_{tote} + m_{veh}) \cdot g \qquad (19)$$

This means that the mass of the vehicle, including the tote and the reduced rotating mass, multiplied by the acceleration has to be less than the mass of the vehicle

including the tote multiplied by the acceleration due to gravity and increased by the coefficient of friction.

4.4 Energy consumption for constant speed ride

The energy consumption of a ride with constant speed is the energy consumption required to overcome friction work.

Equation 20 defines this energy consumption for a ride in horizontal direction or a ride vertically upwards. In this case, energy always must be put into the system, so the efficiency of the vehicle is in the denominator.

$$E_{Rhvu} = (m_{tote} + m_{veh}) \cdot g \cdot \mu_{veh} \cdot L \cdot \frac{1}{\eta_{veh}} \qquad (20)$$

Equation 21 defines this energy consumption for a ride vertically downwards. Here the efficiency is in the numerator, because this energy consumption reduces the output from the system.

$$E_{Rvd} = -(m_{tote} + m_{veh}) \cdot g \cdot \mu_{veh} \cdot L \cdot \eta_{veh} \qquad (21)$$

The limit for the validity of the second part of the energy consumption equation is that the friction force is less than the weight force of the vehicle including the tote.

$$(m_{tote} + m_{veh}) \cdot \mu_{veh} \cdot g \leq (m_{tote} + m_{veh}) \cdot g \qquad (22)$$

4.5 Energy consumption for lifting and ride down–wards

For the lifting movement the potential energy must be added to the system and therefore the efficiency of the vehicle is in the denominator.

$$E_{liftup} = (m_{tote} + m_{veh}) \cdot g \cdot H \cdot \frac{1}{\eta_{veh}} \qquad (23)$$

When riding downwards, the potential energy is released from the system, so the efficiency is here in the numerator.

$$E_{down} = -(m_{tote} + m_{veh}) \cdot g \cdot H \cdot \eta_{veh} \qquad (24)$$

5. RESULTS FROM THE CALCULATION OF THE ENERGY CONSUMPTION

Based on the data in Table 1 and Table 2 the energy consumption and the throughput results are shown in Figure 3 for single deep racks with a length of 100 m.

Table 2. System parameters

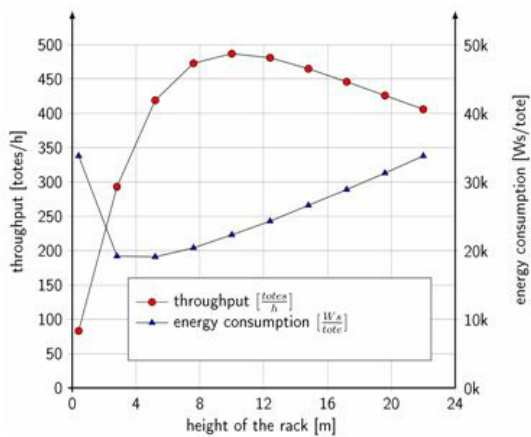|	lift	shuttle
mass	5 m/s	2 m/s
red. rot. mass	7 m/s²	2 m/s²
coefficient of friction	0.1	0.1
efficiency	0.7	0.75
recuperation efficiency	0.5	0,5
load/unload energy	300 J	800 J
standby energy	0.010 kWh/h	0.010 kWh/h
control energy	0.5 kWh/h	
mass of the tote	25 kg	

Figure 3. Energy consumption and throughput of a shuttle-system

On the right side of the figure the influence of the lift is dominating and so the energy comsumption increases with increasing height of the rack. On the left side the constant parts of the energy comsumption are dominating and so the curve is decreasing with increasing throughput. The two curves are nearly mirrored. This is because, in one calculation, the lengths of the rack is in the denominator and in the other it is in the numerator. There is a difference in the position of the extremes (the minimum energy consumption or the maximum handling capacity). This difference results from the waiting time in the calculation of the troughput.

6. CONCLUSION

With the present calculation methods the throughput and the energy consumption of shuttle systems can be calculated. These two calculation methods are shown here for single deep racks. They are also available for multi deep racks. In future these methods shall be used to optimize the geomety of the racks.

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NOMENCLATURE

a_{lift}	acceleration of the lift
$a_{shuttle}$	acceleration of the shuttle
a_{veh}	acceleration of the vehicle
E_{Ahvu}	Energy consumption for acceleration in horizontal and vertical direction upwards
E_{Avd}	Energy consumption for acceleration in vertical direction downwards
$E_{control}$	Energy consumption of the control station of the shuttle system
E_{Dhvd}	Energy consumption for deceleration in horizontal and vertical direction downwards
E_{down}	Energy consumption for lifting a tote downwards
E_{Dvu}	Energy consumption for deceleration in vertical direction upwards
E_{lift}	Energy consumption for one cycle of the lift
$E_{lifts\,tan\,dby}$	Energy consumption of the lift while waiting
E_{liftup}	Energy consumption for lifting a tote upwards
$E_{load/unload\_lift}$	Energy consumption for loading and unloading the lift
$E_{load/unload\_shuttle}$	Energy consumption for loading and unloading the shuttle
E_{Rhvu}	Energy consumption for constant ride in horizontal and vertical direction upwards
E_{Rvd}	Energy consumption for constant ride in vertical direction upwards
$E_{shuttle}$	Energy consumption for one cycle of the shuttle
$E_{shuttles\,tan\,dby}$	Energy consumption of the shuttle while waiting
E_{system}	Energy consumption for one handling cycle of the whole shuttle system
g	gravitational acceleration
H	Height of the rack
K	capacity of the queuing system
L	Length of the rack
m_{lift}	Mass of the lift
$m_{redlift}$	reduced rotating masses of the lift
$m_{redshuttle}$	reduced rotating masses of the shuttle
m_{redveh}	reduced rotating masses of the vehicle
m_{tote}	Mass of the tote
$m_{shuttle}$	Mass of the shuttle
m_{veh}	mass of the vehicle

n_{buffer}	number of buffer slots between lifts and shuttle in one storage level
$n_{capshuttle}$	transporting capacity of a shuttle
n_{level}	number of storage levels
n_{lift}	number of lifts in the system
$n_{shuttle}$	number of shuttles in the system
p_0	Unconditional probability that there is no tote in the service channel
p_k	blocking probability
$t_{load/unload\_lift}$	loading/unloading time of the lifts
$t_{load/unload\_shuttle}$	loading/unloading time of the shuttles
t_a	interarrival time
t_o	operating time
t_{lift}	cycle time of the lifts
$t_{shuttle}$	cycle time of the shuttles
s_s	Variance of the operation time
v_{lift}	velocity of the lift
$v_{shuttle}$	velocity of the shuttle
v_{veh}	velocity of the vehicle

Greek symbols

ϑ_{level}	throughput of a storage level
ϑ_{system}	throughput of the whole shuttle system
ρ	utilization rate

λ	arrival rate
μ	service rate
μ_{lift}	friction coefficient of the lift
$\mu_{shuttle}$	friction coefficient of the shuttle
μ_{veh}	friction coefficient of the vehicle
η_{lift}	efficiency of the lift
$\eta_{shuttle}$	efficiency of the shuttle
$\eta_{reklift}$	efficiency of recuperation of the lift
$\eta_{rekshuttle}$	efficiency of recuperation of the shuttle
η_{veh}	efficiency of the vehicle

МЕТОД РАЧУНСКОГ ОДРЕЂИВАЊА ПРОТОКА И ПОТРОШЊЕ ЕНЕРГИЈЕ КОД ШАТЛ СИСТЕМА СКЛАДИШТЕЊА

М. Едер, Г. Картниг

Шатл системи су аутоматски системи великих перформанси за ускладиштење складишних једи–ница. Сваки ниво складишта опслужује један шатл. С предње стране регала налази се лифт за ускла–диштење и искладиштење. Различит број бафера се може налазити између лифтова и њима одгова–рајућих нивоа. О овим системима се већ расправља у различитим научним радовима.

У овом раду је приказано израчунавање протока код шатл система и, посебно, израчунавање потрошње енергије у функционисању целокупног шатл система. Приказани су резултати примене рачунске методе.