Effects of Hard Stopping of the Ski Lift on Dynamical Strain of the Pulling Rope

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The ski lift has been modeled as per of the Theory of Small Oscillations of a material system with finite number of degrees of freedom of movement, as shown at the Fig. 1. The material system of the ski lift is allowed to six degrees of freedom of movement, that take into account the elastic properties of the pulling ropes and the lower foothold, as well as the mobility of rotating and translatory masses at the upper foothold. Upon determination of the main forms of oscillation and corresponding modal columns, on the basis of initial parameters, the laws of ski lift movement were set. Dynamical forces at lower parts of the up-going and down-going rope tracks were determined by means of the said laws of movement. The ratios of dynamical rope strains amount to 3.96 and 2.26 respectively, implying that considerable dimensions of the pulling ropes must be set if their breakage at hard stops is to be avoided.

Keywords: *Ski lift, pulling rope, static force, dynamic force, laws of movement.*

1. INTRODUCTION

Ski lift is a machine where elastic elements (ropes) play an important role, as their task is to transfer movement to relatively large masses originating from the skiers' presence and from the weight of the ropes themselves. Consequently, the ski lift is extremely prone to oscillations that are translated along the full length of the rope line and affect the elements of the ski lift structure in a very complex way. Any change in the uniformity of ski lift movement is, therefore, a strong impulse for appearance of oscillations. This paper is a research into the effects of hard stopping which can occur, for example, as a result of power supply failure. The research was made on the basis of the Theory of Small Oscillations and accompanied by a numerical example.

2. ELASTIC–INERTIAL MODEL OF THE SKI LIFT

The Fig. 1 shows an elastic inertial material system of the ski lift, that is the system comprising elastic and inertial elements. Moment of inertia of the lower pulley is neglected, because his movement around vertical axe is dumped by AC drive and gear unit. Longitudinal oscillations of the up-going and down-going ropes are neglected because users of the ski lift are leaned on the ground and represent longitudinal rigidity during the movement.

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Where :

Lower foothold	m= 450 kg
Lower foothold gear mass	m_1= 500kg
lower foothold height	l= 2.5 m
Lower foothold moment of inertia	J_0= 1000 kg m^2
Rope rigidity	C= 30 kN/m
Reduced mass of the down-going rope	m_3= 5300 kg
Reduces mass of the up-going rope	m_4= 11300 kg
Moment of inertia of the upper foothold pulley	J_2= 234 kg m^2
Lower foothold pulley diameter	r= 2.5 m
Upper foothold gear mass	m_2= 400kg
Tensile rope rigidity	C_1= 45 kN/m
Counter weight mass	m_5= 10000 kg

m_3 represents the sum of masses of the pulling rope of the down-going track and the empty towing devices concentrated at the mid-span.

m_5 represents the masses of the pulling rope of the up-going track, towing devices and the skiers concentrated at the mid-span.

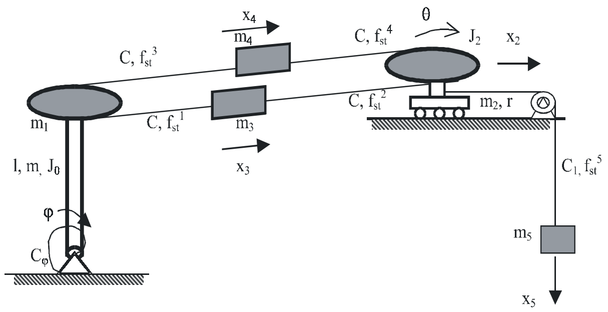

Fig. 1 Dynamic model of the ski lift

3. DETERMINATION OF COEFFICIENTS OF RIGIDITY AND INERTIA OF THE MATERIAL SYSTEM SHOWN AT THE FIGURE 1

Potential energy of the fictive spiral spring of the lower foothold

$$E_{p1} = \frac{1}{2} C_{\varphi} (\varphi_{st} + \varphi)^2 - \frac{1}{2} C_{\varphi} \varphi_{st}^2 \ .$$

Potential spring energy of the down-going track

$$E_{p2} = \frac{1}{2} C(f_{st}^1 + x_3 - l\,\varphi)^2 - \frac{1}{2} C(f_{st}^1)^2 + + \frac{1}{2} C(f_{st}^2 + x_2 - x_3 - r\,\theta)^2 - \frac{1}{2} C(f_{st}^2)^2$$

Potential spring energy of the up-going track

$$E_{p3} = \frac{1}{2} C(f_{st}^3 + x_4 - l\varphi)^2 - \frac{1}{2} C(f_{st}^3)^2 + + \frac{1}{2} C(f_{st}^4 + x_2 - x_4 + r\theta)^2 - \frac{1}{2} C(f_{st}^4)^2$$

Potential spring energy of the tension rope

$$E_{p4} = \frac{1}{2} C_1 (\varphi_{st}^5 + x_5 - x_2)^2 - \frac{1}{2} C(f_{st}^5)^2 \ .$$

Potential weight energy of the lower foothold

$$E_{p5} = m_1 gl \cos(\varphi + \varphi_{st}) - m_1 gl \cos\varphi_{st} + \frac{1}{2} mgl \cos(\varphi + \varphi_{st}) - \frac{1}{2} mgl \cos\varphi_{st}$$

Potential weight energy of the down-going track

$$E_{p6} = m_3 g(h_3 + x_3 \sin\beta) - m_3 g h_3 \ .$$

Potential weight energy of the up-going track

$$E_{p7} = m_4 g(h_4 + x_4 \sin\beta) - m_4 g h_4 \ .$$

Potential weight energy of the tension weight

$$E_{p8} = m_5 g(h_t - x_t) - m_5 g h_t \ .$$

Total potential energy

$$E_p = \sum_{i=1}^{8} E_{pi} \ .$$

As all generalized forces are equal to zero in the point of stable balance the following six conditions are obtained:

$$\left(\frac{\partial E_p}{\partial \varphi}\right)_0 = 0 \Rightarrow C_{\varphi}\varphi_{st} - C f_{st}^1 l - C f_{st}^3 l - - m_1 gl \sin\varphi_{st} - \frac{1}{2} mgl \sin\varphi_{st} = 0$$

$$\left(\frac{\partial E_p}{\partial \theta}\right)_0 = 0 \Rightarrow - C f_{st}^2 r - C f_{st}^4 r = 0$$

$$\left(\frac{\partial E_p}{\partial x_2}\right)_0 = 0 \Rightarrow C f_{st}^2 + C f_{st}^4 - C_1 f_{st}^5 = 0 \ ,$$

$$\left(\frac{\partial E_p}{\partial x_3}\right)_0 = 0 \Rightarrow C f_{st}^1 - C f_{st}^2 + m_3 g \sin\beta = 0 \, ,$$

$$\left(\frac{\partial E_p}{\partial x_4}\right)_0 = 0 \Rightarrow C f_{st}^3 - C f_{st}^4 + m_4 g \sin\beta = 0 \, ,$$

$$\left(\frac{\partial E_p}{\partial x_5}\right)_0 = 0 \Rightarrow C_1 f_{st}^5 - m_5 g = 0 \ .$$

On the basis of said six conditions the expression for the total potential energy is deduced to homogenous square form:

$$E_p = \frac{1}{2}\Big\{ \Big[C_{\varphi} + 2Cl^2 - \Big(\frac{m}{2} + m_1\Big) gl \Big] \varphi^2 + 2Cr^2\theta^2 + + (2C + C_1) x_2^2 + 2C x_3^2 + 2C x_4^2 + C_1 x_5^2 - - 2Cl\varphi x_3 - 2Cl\varphi x_4 + 2Cr\theta x_3 - 2Cr\theta x_4 - - 2C x_2 x_3 - 2C x_2 x_4 - 2C_1 x_2 x_5 \Big\}$$

If the generalized coordinates are numerically marked as follows:

$$q_1 = x_5 \ ; \ q_2 = x_4 \ ; \ q_3 = x_3$$
$$q_4 = x_2 \ ; \ q_5 = \theta \ ; \ q_6 = \varphi$$

On the basis of the expression for potential energy the rigidity matrix $[c]$ is obtained, whose minors meet the Sylvester criterion.

Kinetic energy of the material system of the ski lift

The kinetic energy system reads

$$E_k = \frac{1}{2}\Big[\big(J_0 + m_1 l^2\big)\dot{\varphi}^2 + J_2 \dot{\theta}^2 + + m_2 \dot{x}_2^2 + m_3 \dot{x}_3^2 + m_4 \dot{x}_4^2 + m_5 \dot{x}_5^2 \Big]$$

and from this expression the matrix of inertia (a) coefficients is obtained according to already introduced numerical expression of generalized coordinates.

4. DETERMINATION OF THE FREQUENCY OF MAIN FORMS OF OSCILLATION AND CORRESPONDING MODAL COLUMNS

We start from the matrix

$$[A] = [c]^{-1}[a]$$

After elimination of m-1 form of oscillation m-th form of oscillation is obtained by means of iteration formula:

$$[A]_m \{\mu_{jm+1}\} = \Lambda_{m+1} \{\mu_{jm+1}\} ,$$

where $\Lambda_r = 1/\omega_r^2$ is the reciprocal value of the square of angular speed of frequency of the r-th main oscillation form, to which corresponds the modal column $\{\mu_{jr}\}$ - r-th main form of system oscillation.

The matrix

$$[A]_m = [A]_{m-1}[S]_m .$$

It can be further shown that

$$(v_m) = const.(\mu_m)[a] .$$

For m defined modal columns the Gauss elimination procedure gives:

$$v_{mm}^{(m-1)} q_m + ... + v_{ms}^{(m-1)} q_s = 0 .$$

The matrix $[S]_m$ is diagonal except in the m-th series which looks as follows:

$$0 \;\; 0 \;\; 0 \;\; ... \; -\frac{v_{mm+1}^{(m-1)}}{v_{mm}^{(m-1)}} \; ... -\frac{v_{ms}^{(m-1)}}{v_{mm}^{(m-1)}} .$$

In such way modal matrix $[\mu]$ and angular speeds are determined.

$\omega_1 = 1{,}206\ \mathrm{s}^{-1}$; $\omega_2 = 1{,}932\ \mathrm{s}^{-1}$; $\omega_3 = 2{,}825\ \mathrm{s}^{-1}$;
$\omega_4 = 16{,}63\ \mathrm{s}^{-1}$; $\omega_5 = 41{,}423\ \mathrm{s}^{-1}$; $\omega_6 = 197{,}192\ \mathrm{s}^{-1}$.

5. DETERMINATION OF LAWS OF MOVEMENT OF THE SKI LIFT MATERIAL SYSTEM

When all model columns and frequencies of main forms of oscillations are defined the laws of movement of the observed material system can be determined. For this determination initial parameters are used, which, for the hard stop case (j=3 m/s^2) read:

$$x_4(0) = \frac{m_4 j}{C} = \frac{11300 \cdot 3}{30000} = 1{,}13 \text{ m} ,$$

$$x_3(0) = -x_4(0) = -1{,}13 \text{ m} ,$$

$$\theta(0) = \frac{x_3(0)}{r} = \frac{1{,}13}{2{,}5} = 0{,}452 \text{ rad} .$$

In tabular presentation, the initial parameters read:

Table 1.

i	1	2	3	4	5	6
$q_i(0)$	0	1,13	-1,13	0	0,452	0
$\dot{q}_i(0)$	0	0	0	0	0	0

Laws of movement, presented as matrix, look as follows:

$$\{q\} = [\mu]\{A\cos\omega t\} + [\mu]\{B\sin\omega t\}$$

The matrices of the column of constants $\{A\}$ and $\{B\}$ are determined as follows:

$$\{A\} = [\mu]^{-1}\{q(0)\} ,$$

$$\{B\omega\} = [\mu]^{-1}\{\dot{q}(0)\} = \{0\} \Rightarrow \{\mathbf{B}\} = 0 .$$

The matrix of coefficients with cosines is :

$$[K] = [\mu]\lfloor A_{dij} \rfloor =$$

$$\begin{pmatrix} 0.349 & -0.506697 & 0.157885 & -0.631\cdot10^{-4} & -0.128\cdot10^{-8} & 0.208\cdot10^{-13} \\ 0.188 & 1.045217 & -0.10307 & -0.376\cdot10^{-4} & 0.485\cdot10^{-5} & -0.697\cdot10^{-11} \\ 0.115 & -0.720186 & -0.52517 & -0.809\cdot10^{-4} & -0.104\cdot10^{-4} & -0.148\cdot10^{-10} \\ 0.237 & -0.125912 & -0.11525 & 0.368\cdot10^{-2} & 0.308\cdot10^{-6} & 0.631\cdot10^{-13} \\ 0.0145 & 0.353827 & 0.0848 & 0.104\cdot10^{-4} & -0.117\cdot10^{-2} & -0.662\cdot10^{-13} \\ 0.14\cdot10^{-3} & 0.15\cdot10^{-3} & -0.29\cdot10^{-3} & -0.56\cdot10^{-7} & -0.270\cdot10^{-8} & 0.409\cdot10^{-7} \end{pmatrix}$$

Where in the matrix $\lfloor A_{dij} \rfloor$ $A_{jj} = A_j$ $\;j = 1,....,6$ and $A_{kj} = 0$ for $k \neq j$.

The matrix of coefficients with sines is a zero matrix. Taking into account the law of movement, the material system of the ski lift model in the form of matrix reads for this set of initial parameters

$$\{q\} = [K]\{\cos\omega t\} .$$

6. CONCLUSION

The static force at the lower end of the up-going track is:

$$F_{odl}^{stat} = \frac{m_5 g}{2} - m_4 g \sin\beta = 13441 \text{ N} .$$

The static force at the lower end of the down-going track is:

$$F_{dol}^{stat} = \frac{m_5 g}{2} - m_3 g \sin\beta = 32360 \text{ N} .$$

Dynamic force at the lower end of the up-going track is:

$$\Delta F_{odl}^{din} = C(x_4 - l\varphi) \approx Cx_4 = Cq_2 .$$

Dynamic force at the lower end of the down-going track is:

$$\Delta F_{dol}^{din} = C(x_3 - l\varphi) \approx Cx_3 = Cq_3 .$$

From the matrix $[K]$ it can be deduced that:

$$q_2 \approx 0{,}187\cos 1{,}206t + 1{,}04\cos 1{,}932t - 0{,}103\cos 2{,}825t$$

$$\Rightarrow q_{2\max} \approx 1{,}33 \text{ m}$$

$$q_3 \approx 0{,}115\cos 1{,}206t - 0{,}72\cos 1{,}932t 0{,}525\cos 2{,}825t$$

$$\Rightarrow q_{3\max} \approx 1{,}36 \text{ m}$$

It follows that:

$$\Delta F_{odl\max}^{din} = Cq_{2\max} = 39900 \text{ N}$$

$$\Delta F_{dol\max}^{din} = Cq_{3\max} = 40800 \text{ N}$$

so that the dynamic coefficients at the lower ends of up-going and down-going tracks amount to:

$$K_{odl}^{stat} = \frac{F_{odl}^{stat} + \Delta F_{odl\,\max}^{stat}}{F_{odl}^{stat}} = 3{,}96\ ,$$

$$K_{odl}^{din} = \frac{F_{dol}^{stat} + \Delta F_{dol\,\max}^{stat}}{F_{dol}^{stat}} = 2{,}26\ .$$

Accordingly, the pulling ropes must have large dimensions in order to sustain the accidental stoppages.

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УТИЦАЈ НАГЛОГ ЗАУСТАВЉАЊА СКИ ЛИФТА НА ДИНАМИЧКО ОПТЕРЕЋЕЊЕ НОСЕЋЕ-ВУЧНОГ УЖЕТА

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Ски лифт је моделиран теоријом малих осцилација материјалног система са коначним бројем степени слободе кретања како је то проказано на слици 1. Материјалном систему ски лифта је дозвољено шест степени слободе кретања који узимају у обзир еластична својства носеће-вучних ужади и доњег ослонца, као и покретљивости обртних и транслаторних маса на горњем ослонцу. После одређивања главних облика осциловања и одговарајућих модалних колона, на основу почетних услова постављени су закони кретања ски лифта. Применом закона кретања одређене су динамичке силе у ужадима на доњим деловима грана успона и спуста. Коефицијенти динамичких оптерећања респективно износе 3,96 и 2,26 што показује да ако се морају одредити велике димензије носеће-вучних ужади ски лифта ако се жели избећи њухово кидање при наглом заустављању.