Determination of the Acceleration of a Characteristic Group by Method of Coupled Centers

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This paper presents the procedure of determination of acceleration of the characteristic points of dyad by Method of coupled centers. The dyad, by which a mechanism is spread applying the structural synthesis procedure, forms a four-member closed contour with the members of the mechanism. The contour is a slider-crank mechanism, in which all members are movable. A line that passes through the momentary centers of rotation, formed by the opposing contour members (i.e. members which are not in direct contact), has the key role in defining of dyad acceleration.

Keywords: *Mechanism, acceleration, coupled centres.*

1. INTRODUCTION

It is well known that every mechanism can be spread by statically defined kinematics group by the method of static synthesis, so that it still remains a mechanism. In this paper, we will discuss the determination of acceleration of a such two-member group, by Method of coupled centers.

2. THESIS

Suppose that a six-member planar mechanism OABDCH is given (Fig. 1). It basically consists of: the foundation (1), the drive member (2), connecting rod (3) and rocker (4), which form closed kinematics chain OABD. Determination of characteristic accelerations of such a mechanism by Method of coupled centers has already been presented in [1] and [2], and here it will only be interpreted.

Let the IP (Fig. 2) be known, prescribed, or unit acceleration of point A of the drive member 2, which rotates at a constant angular velocity.

$$\mathrm{IP} = \left|\vec{a}_{\mathrm{A}\,n}\right| = \mathrm{AO}\,\omega_2^2 \ . \tag{1}$$

By Method of coupled centers we will define all characteristic accelerations of mechanism OABD: line through D (Fig. 1), parallel with AO, crosses the member AB at point G. A line through I, parallel with LW and line through P, parallel with GL, by their intersection define point T.

$$\mathrm{DG}//\mathrm{AO} \ , \tag{2}$$

$$\mathrm{IT}//\mathrm{LW} \ , \tag{3}$$

$$\mathrm{PT}//\mathrm{GL} \ , \tag{4}$$

Line through P, parallel with BD and line through T, parallel with AB, define point V, by which accelerations of mechanism OABD are defined.

$$\mathrm{PV}//\mathrm{BD} \ , \tag{5}$$

$$\mathrm{TV}//\mathrm{AB} \ , \tag{6}$$

$$\mathrm{PV} = \left|\vec{a}_{\mathrm{B}\,n}\right| \ , \tag{7}$$

$$\mathrm{TV} = \left|\vec{a}_{\mathrm{B}\,n}^{\mathrm{A}}\right| \ . \tag{8}$$

For determination of characteristic accelerations of members 5 and 6, the contour group HBCH is formed, and its members 3, 4, 5 and 6 form a closed kinematics hain.

A line which passes through momentary centers of rotation M(46) and N(35) (formed by members of the contour group which are not in direct contact), have the key role in determination of the acceleration of members 5 and 6. All components of the normal acceleration vectors (including the Coriolis acceleration as well) within the four-member closed contour, form a polygon which is closed by a vector PARALLEL with mentioned line MN.

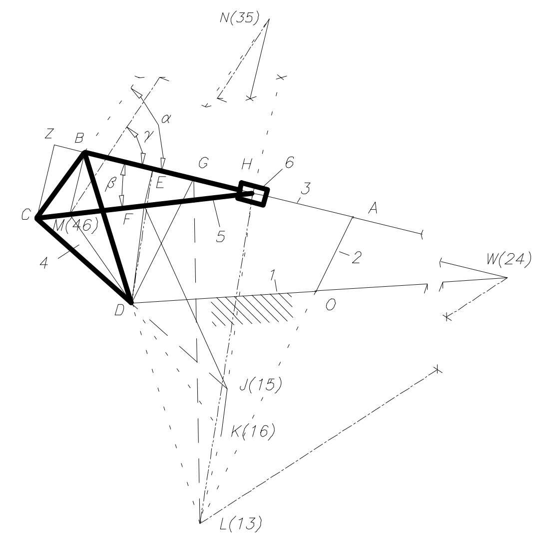

Figure 1. Six-member planar mechanism OABDCH.

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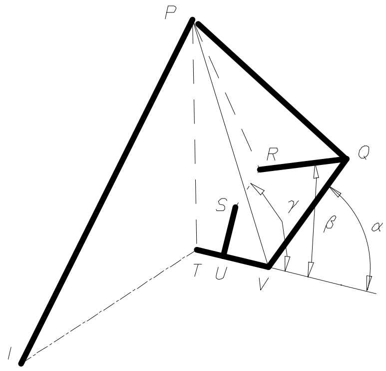

Figure 2. Plan of Accelerations

Procedure is the following: components of normal acceleration of the point C, $\vec{a}_{Cn}$ and $\vec{a}_{Cn}^{B}$ are determined from the similarity conditions of triangles $\Delta DCB \cong \Delta PQV$, where:

$$PQ = |\vec{a}_{Cn}| , \quad (9)$$

$$QV = |\vec{a}_{Cn}^{B}| . \quad (10)$$

The normal component of relative acceleration $\vec{a}_{H3n}^{B}$ of point H of member AB can be determined from the similarity conditions:

$$\frac{UV}{TV} = \frac{BH}{AB} \Rightarrow UV = TV\frac{AH}{AB} , \quad (11)$$

where

$$UV = |\vec{a}_{Hn}^{B}| . \quad (12)$$

A line through D, parallel with JH, and member CH define point F (Fig. 1). Line through P (Fig. 2), parallel with FJ, and line through Q, parallel with CH, define point R, i.e. acceleration $\vec{a}_{Hn}^{C}$.

$$DF//JH , \quad (13)$$

$$PR//FJ , \quad (14)$$

$$QR//CH , \quad (15)$$

$$QR = |\vec{a}_{Hn}^{C}| . \quad (16)$$

Coriolis acceleration in sliding couple H between members 6 and 3 is normal to AB, and its magnitude is determined by point S, as crossing point of line through R, parallel with coupled centers M(46) N(35) and the normal on TV through point U.

$$SR//MN , \quad (17)$$

$$SU \perp TV , \quad (18)$$

$$SU = |\vec{a}_{H6H3cor}| . \quad (a)$$

3. PROOF

In order to prove assumption (*a*), we will project the closed contours UVQRSU on direction normal to TV (normal to member 3) and on TV (direction of member 3). Coriolis acceleration, which is represented by SU, is determined by equation:

$$SU = QV\sin(\alpha) - QR\sin(\beta) - RS\sin(\gamma) . \quad (19)$$

At the same time, projections of the mentioned quantities on direction of member 3 give equation (20), which will enable determination of the value of RS,

$$UV + QV\cos(\alpha) - QR\cos(\beta) - RS\cos(\gamma) = 0 . \quad (20)$$

The following relations are known:

$$\sin(\alpha) = \frac{HN}{BN};\ \cos(\alpha) = \frac{BH}{BN};\ \sin(\beta) = \frac{BM}{HM};$$

$$\cos(\beta) = \frac{BH}{HM};\ \sin(\gamma) = \frac{HN + BM}{MN};\ \cos(\gamma) = \frac{BH}{MN};$$

The normal component of relative acceleration of point C with respect to B is determined from the similarity conditions:

$$\frac{QV}{PV} = \frac{BC}{BD} \Rightarrow QV = PV\frac{BC}{BD} . \quad (21)$$

Normal relative acceleration component of point H of member 5, with respect to point C is determined by Method of coupled centers from the similarity:

$$\frac{QR}{PQ} = \frac{CF}{CJ} \Rightarrow QR = PQ\frac{CF}{CJ} , \quad (22)$$

where $QR = |\vec{a}_{Hn}^{C}|$, and further

$$\frac{PQ}{PV} = \frac{CD}{BD} \Rightarrow PQ = PV\frac{CD}{BD} . \quad (23)$$

From the similarity of triangles $\Delta PVT \cong \Delta LBG$ the following is obtained:

$$\frac{TV}{PV} = \frac{BG}{BL} \Rightarrow TV = PV\frac{BG}{BL} . \quad (24)$$

By substituting (24), (11), (21), (22) and (23) in (20) and solving for RS, we obtain:

$$RS = PV\left(\frac{MN\cdot BG}{AB\cdot BL} + \frac{MN\cdot BC}{BN\cdot BD}\right) - PV\left(\frac{MN\cdot CF\cdot CD}{HM\cdot CJ\cdot BD}\right) \quad (25)$$

By substituting (21), (22), (23) and (25) in (19), the following is obtained:

$$SU = PV\left(\frac{HN\cdot BC}{BN\cdot BD} - \frac{BM\cdot CF\cdot CD}{HM\cdot CJ\cdot BD}\right) - $$
$$- PV\frac{HN + BM}{MN}\left(\frac{MN\cdot BG}{AB\cdot BL} + \frac{MN\cdot BC}{BN\cdot BD}\right) +$$
$$+ PV\frac{HN + BM}{MN}\frac{MN\cdot CF\cdot CD}{HM\cdot CJ\cdot BD} . \quad (26)$$

From the similarity of triangles $\Delta CMB \cong \Delta CHN$ follows the relation:

$$\frac{BC}{BN} = \frac{BM}{HN - BM} , \quad (27)$$

i.e. $\Delta CDF \cong \Delta CJH$

$$\frac{CD}{CJ} = \frac{CF}{CH} . \quad (28)$$

Substituting (27) and (28) in (26), and after some mathematics, the equation gets the form:

$$SU = \frac{PV}{BD}\left(\frac{HN \cdot CF^2}{HM \cdot CH} - \frac{BM^2}{(HN - BM)}\right) - \frac{PV}{BD}\frac{BG^2}{AB^2}(HN + BM) . \quad (29)$$

The following expressions are obtained from the similarity of triangles $\Delta\,CZH \cong \Delta\,MBH \cong \Delta\,FEH$, or $\Delta\,NBH \cong \Delta\,CBZ$:

$$\frac{BZ}{BH} = \frac{BC}{BN}, \quad (30)$$

$$\frac{CZ}{HZ} = \frac{BM}{BH} \Rightarrow CZ = HZ\frac{BM}{BH}, \quad (31)$$

$$\frac{CF}{HM} = \frac{EZ}{BH}, \quad (32)$$

$$\frac{CF}{CH} = \frac{EZ}{HZ}, \quad (33)$$

$$\frac{HN}{BH} = \frac{CZ}{BZ} . \quad (34)$$

By substituting (32), (33) and (34) in (29), with the condition that EZ=BZ+BE, we have:

$$SU = \frac{PV}{BD}\left(\frac{BZ \cdot CZ}{HZ} + \frac{2\,BE \cdot CZ}{HZ} + \frac{BE^2 CZ}{HZ \cdot BZ}\right) - \frac{PV}{BD}\left[\frac{BM^2}{HN - BM} + \frac{BG^2}{AB^2}(HN + BM)\right], \quad (35)$$

and further, using (30) and (27), after proper canceling out:

$$SU = \frac{PV}{BD}\left(\frac{2\,BE \cdot BM}{BH} + \frac{BE^2 BM}{BZ \cdot BH}\right) - \frac{PV}{BD}\left(\frac{BG^2}{AB^2}HN + \frac{BG^2}{AB^2}BM\right). \quad (36)$$

The similarity of triangles $\Delta\,BDG \cong \Delta\,BLA$ and $\Delta\,BDE \cong \Delta\,BLH$ gives the equation:

$$\frac{BG}{AB} = \frac{BE}{BH}, \quad (37)$$

while from (34) and (31) we get:

$$HN = \frac{HZ \cdot BM}{BZ} . \quad (38)$$

Substituting (37) and (38) in (36)and after systematization of members:

$$SU = \frac{PV}{BD}BM\frac{BE}{BH}\left[2 + \frac{BE(BH - HZ - BZ)}{BH \cdot BZ}\right], \quad (39)$$

and further, substituting $BZ = HZ - BZ$ and $EH = BH - BE$ in (39) :

$$SU = 2\frac{PV}{BD}\frac{BE \cdot BM \cdot EH}{BH^2} . \quad (40)$$

From the similarity of triangles $\Delta\,DBM \cong \Delta\,DLK$ and $\Delta\,BDE \cong \Delta\,BLH$ we get the expressions:

$$\frac{BD}{BL} = \frac{BE}{BH}, \quad (41)$$

$$\frac{KL}{BM} = \frac{DL}{BD} = \frac{EH}{BE} \Rightarrow EH = BE\frac{KL}{BM}, \quad (42)$$

and by substituting (42) in (40) the following is obtained:

$$SU = 2\frac{PV}{BD}\frac{BE^2}{BH^2}KL, \quad (43)$$

Equation (24) may be written in form:

$$\frac{BG}{BL} = \frac{TV}{PV} = \frac{\left|\vec{a}^{\,B}_{A\,n}\right|}{\left|\vec{a}_{B\,n}\right|} = \frac{\omega_3^2\,AB}{\omega_4^2\,BD}, \quad (44)$$

from which comes the expression:

$$\omega_4^2 = \omega_3^2\frac{AB \cdot BL}{BD \cdot BG} . \quad (45)$$

If we write (43) as:

$$SU = 2\,\omega_4^2\frac{BE \cdot BE}{BH \cdot BH}KL . \quad (46)$$

by placing (45), (41) and (37) in (46) we get:

$$SU = 2\,\omega_3^2\,KL, \quad (47)$$

which, by the fact that the magnitude of relative velocity in sliding couple is equal to the product of the common angular velocity and the distance of the absolute momentary centers of rotation

$$\left|\vec{v}_{H\,6\,H\,3}\right| = \omega_3\,KL, \quad (48)$$

proves the assumption (*a*)

$$\left|\vec{a}_{H\,6\,H\,3cor}\right| = SU = 2\,\omega_3\left|\vec{v}_{H\,6\,H\,3}\right| .$$

4. CONCLUSION

This paper treats, among other things, the problem of determination of the Coriolis` acceleration of a point. It is demonstrated that Coriolis` acceleration can be reduced to relative acceleration; by the method of teamed centres Coriolis` acceleration is treated as "normal acceleration of a point" and the magnitude and direction in space are determined, in spite of the fast that basically Coriolis` acceleration, as we know from kinematics, is not normal acceleration of a point.

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ОДРЕЂИВАЊЕ УБРЗАЊА КОД КАРАКТЕРИСТИЧНЕ КИНЕМАТСКЕ ГРУПЕ МЕТОДОМ СПРЕГНУТИХ ЦЕНТАРА

С. Ђорђевић

У раду се разматра поступак одређивања убрзања карактеристичних тачака дијаде Методом спрегнутих центара. Дијада, којом је поступком структурне синтезе проширен неки механизму, образује са члановима механизма четворочлану затворену контуру. Контура представлја клипни механизам код кога су сви чланови покретни. Права која пролази кроз тренутне центре ротацие, које образују наспрмни чланови контуре (чланови који нису у директном додиру), има кључну улогу у дефиниању убрзања дијаде.