Calculation of the Turbulent Flow in a Plane Diffuser by using the Integral Method

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An incompressible, turbulent flow in plane diffusers is analysed in this paper. For calculation we use equations of turbulent boundary layer in integral form, adjusted for internal flows. For closing the system of equations turbulent viscosity model based on the mixing length theory is used. Velocity profile in every cross-section of the diffuser is approximated by a six-order polynomial, while the coefficients of the polynomial depend on three form parameters. By this transformation system of governing equations is reduced to three ordinary differential equations for form parameters, which can be relatively easily solved numerically. Solutions are obtained for the flow in a plane diffuser for different values of the Reynolds number and the diffuser angle, starting with quasi-developed velocity profile in the inlet cross section up to the downstream cross section in which turbulent flow separates from the wall. The obtained results are in a good agreement with experiments.

Keywords: *Turbulent flow, plane diffuser, coefficient of pressure, friction factor, separation point.*

1. INTRODUCTION

A diffuser, as an element where the stream cross section changes from inlet to outlet, either plane or axisymetrical, has a great importance as an adapter of a pipe-line, or in an ejector for changing velocity and pressure, in a chimney, as the inlet or outlet of a stream engine, or in a rocket engine etc. The flow structure in the diffuser, whether laminar or turbulent, has been the theme of many investigations. Part of these investigations show that the velocity profile at the inlet, which is ordinarily developed, transforms into the velocity profile with stream separation, which is defined where the value of tangential stress on the wall is equal to zero, then after this, the cross section stream starts to separate from the channel wall. In the diffuser can exist several regime flows, which depend on the geometry and Reynolds number, and which are defined by Kline's diagram [1], [2] and [3] for turbulent flow and his appendix for laminar flow [4]. The global parameters, friction coefficient and pressure recovery coefficient, are very interesting for applying to results of investigations. Distributions of these variables are given in the experimental data [5] for pressure recovery coefficient in plane diffusers and [6] for friction coefficient in conical diffusers.

The problem which is examined in this paper is stationary turbulent flow in a plane diffuser. In other works this problem has most often been analysed by experimental methods or numerical methods of fluid mechanics. However, very rarely is this problem solved by integral boundary method, as in the reference [2] which gives very good results. Integral equations of the boundary layer are applied in reference [2] for computation of the turbulent flow, where the appropriate initial conditions are adopted for the problem of interior flow. With this calculation it is necessary to make an approximation of the velocity profile. Very different approaches can be used for approximation of velocity profiles, as for example by the profile of velocity deficit, based on the transverse coordinate and the boundary layer displacement thickness, which is described by an asymptotic seventh – order series [8] or by a sinus function [9]. In this paper we use an approximation of the velocity profile by the six-ordrer polynomial based in the eddy turbulent viscosity. This gives the system of nonlinear simple differential equations, which are solved by classical numerical method of the Runge-Kutte IV. Numerical solution for the turbulent flow in plane diffusers is obtained in this paper between the inlet cross section with a developed velocity profile and the downstream cross section with the flow separation from the wall.

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2. PRESSURE COEFFICIENT AND FRICTION FACTOR

In order to have a detailed knowledge of the flow structure in a diffuser it is necessary to know the velocity field and the pressure field, which requires that the appropriate physical-mathematical model be solved. The global parameters – pressure recovery coefficient (C_p), friction factor (C_f) and local loss of energy are interesting in engineering practice, and in determining which we use concrete results of calculation. In this paper we will talk about pressure recovery coefficient (C_p), which represents dimensionless pressure field in a diffuser and it is defined by the equation:

$$C_p(x) = \frac{p(x) - p(0)}{\rho u_m^2(0)/2} = 1 - \left(\frac{u_m(x)}{u_m(0)}\right)^2 , \quad (1)$$

where subscript m represents the mean value in the cross section, and friction factor:

$$C_f(x) = 2\,\tau_w(x) / \rho\, u_m^2(x) , \quad (2)$$

where τ_w is shear stress on the wall, and the other values are given in the Fig. 1. The mean value of velocity in the cross section at the distance x is determined classically:

$$v_m(x) = \delta^{-1}(x) \int_0^\delta \bar{u}(x,y)\,\mathrm{d}y \ .$$

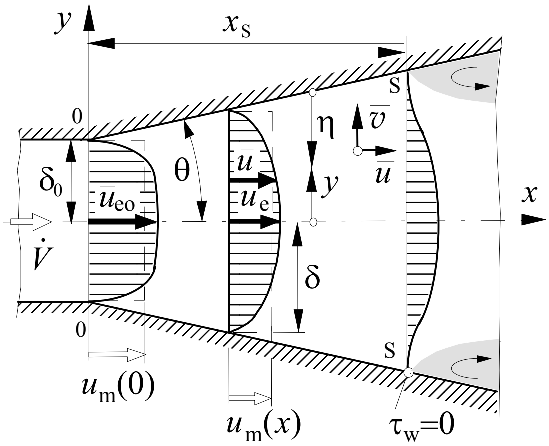

Figure 1. Flow through a diffuser.

3. PHYSICAL-MATHEMATICAL MODEL

Two dimensional stationary turbulent flow in the diffuser, shown in Fig. 1, is described by Reynolds equations and equation of continuity, which are written in common notation:

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left[(\nu + \nu_t)\frac{\partial \bar{u}}{\partial y}\right] , \quad (3)$$

$$0 = \frac{\partial p}{\partial y} \ , \quad (4)$$

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \ . \quad (5)$$

According to equation (3), through quantity ν_t, (introduced by Boussinesq) in the present paper turbulent stresses are determined by turbulent viscosity. Equations (3)-(5) are applied from the diffuser inlet 0-0, to the cross section S-S where the boundary layer started to separate from the channel wall ($\tau_w = -\rho\nu(\partial\bar{u}/\partial y)_{y=0} = 0$). These equations are solved by satisfying the boundary conditions:

$$y = 0 ,\ \bar{u}(x,0) = \bar{u}_e(x) \ ,\ \partial\bar{u}/\partial y = 0 \ ; \quad (6.1)$$

$$y = \delta \ ,\ \bar{u}(x,\delta) = 0 \ ,\ \bar{v}(x,y) = 0 \ , \quad (6.2)$$

$$\dot{V} = 2\int_0^\delta \bar{u}\, dy = \text{const.}; \quad (6.3)$$

$$0 = \bar{u}_e\bar{u}_e' + \frac{\partial}{\partial y}\left[(\nu + \nu_t)\frac{\partial \bar{u}}{\partial y}\right]_\delta - \nu\left(\frac{\partial^2 \bar{u}}{\partial y^2}\right)_e \ . \quad (6.4)$$

The boundary conditions (6.4) was obtained by satisfying the momentum equation (3) for the axis (subscript e) and for channel wall (subscript δ), whereby the condition for turbulent stress on the axis $(\rho\nu_t\partial\bar{u}/\partial y)_e = 0$ has been employed. If we apply classical procedure for transition on integral equations then partial differential equations (3)-(5) will take the form:

$$\frac{\mathrm{d}\delta_2}{\mathrm{d}x} + (2\delta_2 + \delta_1)\frac{\bar{u}'_e}{\bar{u}_e} = \frac{\tau_w}{\rho\bar{u}_e^2} + \frac{\nu\,\delta}{\bar{u}_e^2}\left(\frac{\partial^2\bar{u}}{\partial y^2}\right)_e \ , \quad (7)$$

$$\frac{\mathrm{d}\delta_3}{\mathrm{d}x} + 3\delta_3\frac{\bar{u}'_e}{\bar{u}_e} = 2\int_0^\delta \frac{\bar{u}}{\bar{u}_e}\left[\frac{\nu\,\delta}{\bar{u}_e^2}\left(\frac{\partial^2\bar{u}}{\partial y^2}\right)_e - \frac{\partial}{\partial y}\left(\frac{\tau}{\rho\bar{u}_e^2}\right)\right]\mathrm{d}y \quad (8)$$

where (7) represents momentum equation and (8) represents mechanical energy equation, where total shear stress τ is identified as:

$$\tau = \rho(\nu + \nu_t)\frac{\partial\bar{u}}{\partial y} \ , \quad (9)$$

variable $\bar{u}'_e$ represents the derivative $\bar{u}'_e = \mathrm{d}\bar{u}_e/\mathrm{d}x$, on the end the variables: δ_1, δ_2 and δ_3 displacement thickness, momentum thickness and energy thickness respectively, are defined in the classical way:

$$\delta_1 = \int_0^\delta \left(1 - \frac{\bar{u}}{\bar{u}_e}\right)\mathrm{d}y \ ,\ \delta_2 = \int_0^\delta \frac{\bar{u}}{\bar{u}_e}\left(1 - \frac{\bar{u}}{\bar{u}_e}\right)\mathrm{d}y \ ,$$
$$\delta_3 = \int_0^\delta \frac{\bar{u}}{\bar{u}_e}\left[1 - \left(\frac{\bar{u}}{\bar{u}_e}\right)^2\right]\mathrm{d}y \quad (10)$$

With the aim of solving equations (7)-(8) we predicted that velocity profile is the six-order polynomial:

$$u^+(x,y) = a(x) + b(x)y^{+2} + c(x)y^{+4} + d(x)y^{+6} \ . \quad (11)$$

The velocity profile has to be symmetrical in relation to the axis of the channel and for this reason we only applied even numbers in the power ratios, and $a(x)$, $b(x)$, $c(x)$ and $d(x)$ denote the coefficient of the polynomial. Analysis of turbulent flow shows that it is useful if we introduce: a coordinate measured positive from the wall: $\eta = \delta - y$, friction velocity $u^*(x) = \sqrt{\tau_w(x)/\rho}$ and dimensionless variables:

$$u^+ = \frac{\bar{u}}{u^*} \ ,\ y^+ = \frac{y\,u^*}{\nu} = \delta^+ - \eta^+ \ ,$$
$$\eta^+ = \frac{\eta\,u^*}{\nu} \ ,\ \delta^+ = \frac{\delta\,u^*}{\nu} \ . \quad (12)$$

If we use velocity profile (11) and satisfy boundary conditions (6) we will determine polynomial coefficients:

$$a(x) = u_e^+ \ , \quad (13.1)$$

$$b(x) = \frac{-8q + 5.25\,\mathrm{Re} - 0.0333\,\lambda\delta^{+3}}{\delta^{+3}} \quad (13.2)$$

$$c(x) = \frac{70q - 52.5\,\mathrm{Re} + 0.667\,\lambda\delta^{+3}}{6\delta^{+5}} \ , \quad (13.3)$$

$$d(x) = \frac{-4.67\,q + 3.5\,\mathrm{Re} - 0.0778\,\lambda\delta^{+3}}{\delta^{+7}} \ ; \qquad (13.4)$$

in which: $\mathrm{Re} = 2\delta\overline{u}_e / \nu$ is the Reynolds number, and parametrical forms:

$$\lambda(x) = \frac{\overline{u}_e\overline{u}_e{}'\nu}{u^{*3}} = \frac{\overline{u}_e{}'\delta^2}{\nu}\frac{u_e^+}{\delta^{+2}} \ , \ q(x) = \delta^+\overline{u}_e^+ . \quad (14)$$

If we use formula (14) we find a relationship between the parametrical forms:

$$q' = \frac{1}{\delta q}(\lambda\delta^{+3} + q^2\delta') \ . \qquad (15)$$

For closing the physical-mathematical model which describes turbulent flow for eddy viscosity we used mixing length model where $\nu_t = l^2 \mathrm{d}\overline{u} / \mathrm{d}\eta$, where mixing length $l(\eta)$ is defined by Michel's model [7]:

$$\frac{l(\eta)}{\delta} = 0{,}085 \tanh\left(\frac{\kappa}{0{,}085}\frac{\eta}{\delta}\right) \ , \qquad (16)$$

where $\kappa = 0{,}4$. For simplicity, the expression (16) is represented in the form of the fourth-order polynomial:

$$\frac{l}{\delta} = 0{,}472\frac{\eta}{\delta} - 0.98\left(\frac{\eta}{\delta}\right)^2 + 0.894\left(\frac{\eta}{\delta}\right)^3 - 0.301\left(\frac{\eta}{\delta}\right)^4 . \ (17)$$

By linearization the expression (17) is not reduced on the well know Prandtl's expression for the mixing length: $l = \kappa y$, but in spite of that it approximates the function (16) very well in the whole cross section. Velocity profile (11) is defined by formula (13) as a function of parametrical forms, as $\overline{u} = \overline{u}(\lambda, q, \delta^+)$. After a huge mathematical process, which is dictated by equations (7) and (8), and by formula (10), a system of three simple differential equations are obtained:

$$\begin{aligned} \frac{\mathrm{d}\lambda}{\mathrm{d}x} &= f(x,\lambda,q,\delta^+) \ ; \\ \frac{\mathrm{d}q}{\mathrm{d}x} &= g(x,\lambda,q,\delta^+) \ ; \\ \frac{\mathrm{d}\delta^+}{\mathrm{d}x} &= h(x,\lambda,q,\delta^+) \ . \end{aligned} \qquad (18)$$

The functions which depend on the longitudinal coordinate and parametrical forms: $f(x,\lambda,q,\delta^+)$, $g(x,\lambda,q,\delta^+)$ and $h(x,\lambda,q,\delta^+)$, and which are in the system of differential equations (18), are given in the Appendix.

Initial conditions of the parametrical forms are firstly defined by the prediction that in the diffuser inlet exists fully developed turbulent velocity profile, where $\overline{u}_e'(0) = 0$ from where it follows that: $\lambda(0) = 0$. However, experimental results, as in reference [3] show that the fluid stream adapts before entering the diffuser inlet and follows the geometry of the diffuser, the outcome of which is:

$$\lambda(0) \neq 0 \ , \qquad (19)$$

but which is very near zero. Secondly, from the definition of the parametrical form q follows the equation: $q = (\overline{u}_e / u_m)\cdot \mathrm{Re}/2$, in which $\mathrm{Re} = 2\delta u_m / \nu$ Reynolds number and u_m mean velocity in the cross section of the channel, and to use a well known relation between velocities: $u^*/u_m = \sqrt{C_f / 2}$ where C_f is friction coefficient, we obtain the initial condition:

$$q(0) = \frac{\mathrm{Re}}{2}(1 + 3{,}75\sqrt{C_f(0)/2}) \ . \qquad (20)$$

The third initial condition follows from the definition (12) where the variable δ^+ is defined as:

$$\delta^+(0) = \frac{\mathrm{Re}}{2}\sqrt{C_f(0)/2} \ . \qquad (21)$$

Finally, if we want to find solutions it is necessary to solve a system of simple differential equations (18) by satisfying initial conditions (19)-(21). This system of differential equations is solved by the application of the Runge-Kutte IV method. Numerical calculations are stopped in the downstream cross section in which the flow separates from the wall of the diffuser, that is when on a distance x shear stress becomes $\tau_w(x_s) = 0$. In this regard, an appropriate computer program was organised.

4. NUMERICAL RESULTS AND DISCUSSION

In order to have concrete numerical results we have to define the geometry of the diffuser and the value of the Reynolds number at the diffuser inlet. In this paper the geometry of the diffuser (see Fig. 1) is defined as a straight walled slope of half-angle θ, and the cross section change is defined by the linear function: $\delta(x) = \delta_0 + tg\theta \cdot x$.

Fig. 2 and Fig. 3 show the results of the development of velocity profile, defined in relation to the maximal velocity of the inlet cross section, for the value of Reynolds number Re=50000 and half-angles of the diffuser $\theta = 15°$ (Fig. 2) and $\theta = 30°$ (Fig. 3). From these diagrams can clearly be seen, the development of the inlet velocity profile to the velocity profile at the separation point of the boundary layer. It is seen in these Figures that velocity profile in the inlet cross section deviates from the fully developed one for appr. 5% - the discrepancy being the consequence of the inlet boundary condition (19): $\lambda(0) \neq 0$, that is due to the adjustment of the flow in the channel with parallel walls in front of the diffuser variable geometry.

After the separation point in the diffuser, a different type of turbulent flow begins, which cannot be described using the applied model. If we compare velocity profiles for half-angles $\theta = 15°$ with $x_s / \delta_o = 1{,}445$ and $\theta = 30°$ with $x_s / \delta_o = 0{,}66$, using a constant Reynolds number, then we can see that the position of the separation point moves nearer the diffuser inlet as the half-angle increases.

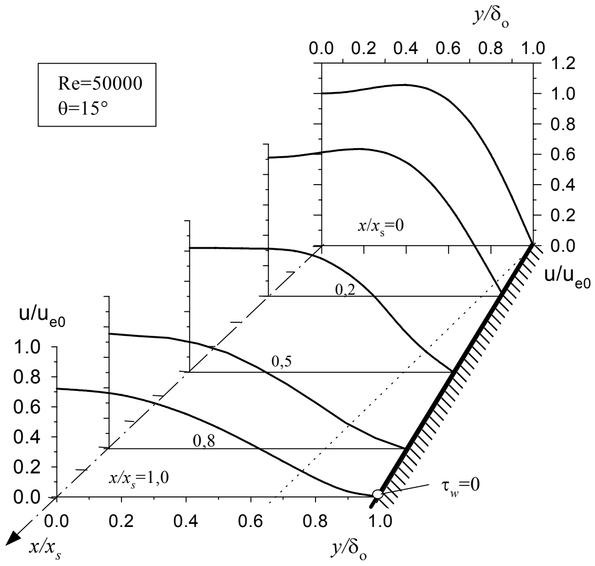

Figure 2. Velocity profiles in a diffuser at $\mathrm{Re} = 50000$ **and for** $\theta = 15°$, $x_s / \delta_o = 1{,}445$.

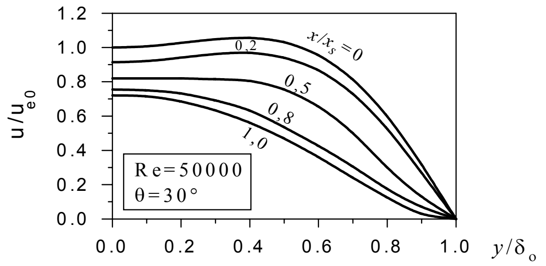

Figure 3. Velocity profiles in a diffuser at $\mathrm{Re} = 50000$ **and for** $\theta = 30°$, $x_s / \delta_o = 0{,}66$.

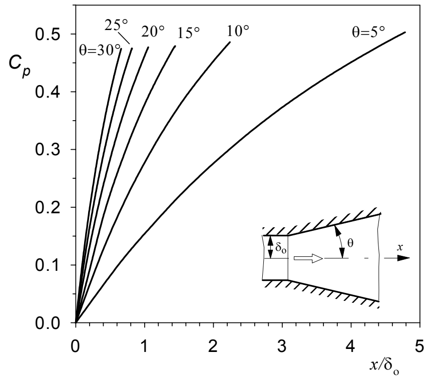

Figure 4. Pressure recovery coefficient in a diffuser at $\mathrm{Re} = 50000$ **and for different values of** θ.

Fig. 4 shows the details of pressure recovery coefficient distribution along the length of the diffuser determined by expression (1). From Fig. 4 we can see that the value of the pressure increases along the length of the diffuser and it's change is identified by the pressure recovery coefficient, with more intensity for a diffuser with a greater angle.

Fig. 5 illustrates a comparison between the results obtained for pressure recovery coefficient, for a half-angle of the diffuser $\theta = 4°$, with the results from reference [5], in which the problem of turbulent flow in a plane diffuser is solved by the application of $k-\varepsilon$ model of turbulence and Prandtl's mixing length model, with uniform distribution in some parts of the cross section. From this figure we can see that both models defined by Prandtl's mixing length gave similar results, but, that after all, a difference does exist which is probably a consequence of the uniform distribution of mixing length accepted in reference [5]. And here, as in reference [5], exists a difference in values of pressure recovery coefficient which are obtained in $k-\varepsilon$ model of turbulence and Prandtl's mixing length model.

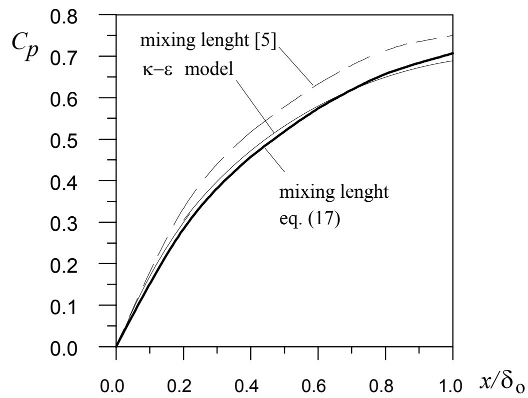

Figure 5. Comparison of results with reference [5] for $\theta = 4°$.

By using the results for the calculation of velocity field, and expressions (2) and (9), the friction factor can be determined. In Fig. 6 the dependence of the friction factor on the angle θ is shown, for $\mathrm{Re} = 10000$. Typical drop of the friction factor from the initial value $C_f(0)$ to the value: $C_f(x_s) = 0$ in the separation cross section is clearly noticed, whereby it is obvious that the drop is more intense in the diffusers with larger angles.

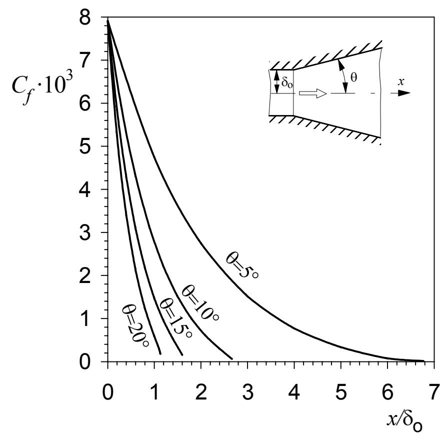

Figure 6. Friction factor in a diffuser at $\mathrm{Re} = 10000$ **and for different values of** θ.

Comparison of our results obtained for a plane diffuser at $\mathrm{Re} = 6000$ and $\theta = 5°$ with the corresponding experimental results for a conical diffuser, stated in [2], are shown in Fig. 7, in nondimensional form. At that the value of the Reynolds number from [2] of 4335, defined by means of the maximum velocity in the inlet cross section, was redefined to the value of 6000 by using the average velocity. The Figure shows very good agreement between numerical and experimented results.

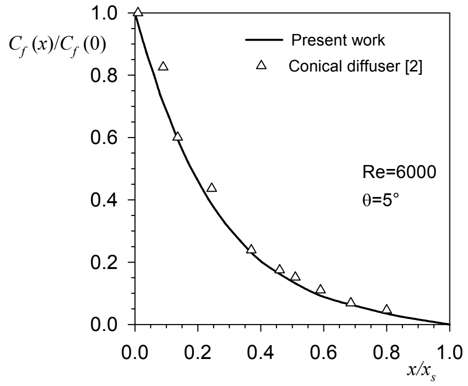

Figure 7. Friction factor in a diffuser at $\mathrm{Re} = 6000$ and for $\theta = 5°$.

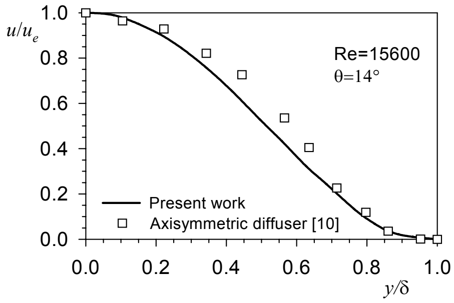

Figure 8. Velocity profiles in the cross section with separation flow for $\mathrm{Re} = 15600$ and $\theta = 14°$.

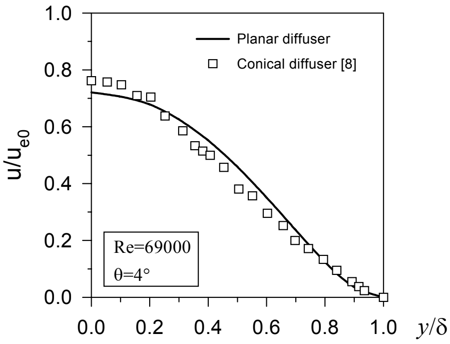

Figure 9. Velocity profiles in the cross section with separation flow for $\mathrm{Re} = 69000$ and $\theta = 4°$.

The comparison of velocity profiles in the separation cross section between plate and conical diffusers is shown in Fig. 8 and 9 – for $\mathrm{Re} = 15600$ and $\theta = 14°$ ([10]) in Fig. 8, and for $\mathrm{Re} = 69000$ $\theta = 4°$ ([8]) in Fig. 9. These diagrams show that separation velocity profiles in plane and conical diffusers are very similar and that they differ up to 8%, whereby the difference decreases with the Reynolds number.

5. CONCLUSION

It is shown in the paper that the method of integral equations of the boundary layer theory, suitably adjusted for the calculation of turbulent flow in plane diffusers. The results obtained by our calculations contain the development of velocity profiles and changes of pressure coefficient and friction factor from the inlet cross section of the diffuser up to the separation cross section. These quantities are functions of the Reynolds number and the diffuser angle, and their changes are more intense in the diffusers with greater angles. For the fixed value of the Reynolds number the position of the separation cross section is postponed for smaller diffuser angles. Comparison of our calculations for plane diffusers with the experimental results for conical diffusers shows that between both of them very good agreement exists in all relevant flow quantities, and that the deviations between these two types of diffusers are relatively small.

In this paper the method of integral equations of boundary layer is used for calculation of flow in plane diffuser with different angle, and it gives very good results for relatively small angles of diffuser. For relatively great angles of diffuser the basic hypothesis of boundary layer theory may become invalid. This is the reason why the proposed method for calculation of turbulent flow in plane diffuser is not recommended for use for relatively high angles of diffuser.

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ПРОРАЧУН ТУРБУЛЕНТНОГ СТРУЈАЊА У РАВАНСКОМ ДИФУЗОРУ ПРИМЕНОМ ИНТЕГРАЛНЕ МЕТОДЕ

М. Вујичић, Ц. Црнојевић

У раду се анализира нестишљиво турбулентно струјање флуида у раванским дифузорима. За прорачун се користе интегралне једначине турбулентног граничног слоја прилагођене за унутрашња струјања. За затварање система једначина користи се модел турбулентне вискозности базиран на путањи мешања. Профил брзина у попречном пресеку дифузора апроксимиран је полиномом шестог степена, а коефицијенти полинома су функције од три параметра облика. Овом апроксимацијом полазни систем диференцијалних једначина трансформише се на три обичне диференцијалне једначине по параметрима облика које је релативно лако решити нумеричким поступком. У раду су добијена решења за струјање у раванском дифузору при различитим вредностима Рејнолдсовог броја и угла ширења и то од улазног квази-развијеног турбулентног профила брзина до низструјног пресека дифузора у коме настаје одвајање флуидне струје са зида. Резултати прорачуна су у доброј сагласности са експерименталним подацима из литературе.

APPENDIX

$$f(x,\lambda,q,\delta^+) = \frac{GD-BQ}{AD-CB} \quad , \quad g(x,\lambda,q,\delta^+) = \frac{\lambda\,\delta^{+3} + q^2\,\mathrm{tg}\,\theta}{\delta^+ q} \quad , \quad h(x,\lambda,q,\delta^+) = \frac{AQ-GD}{AD-CB} \quad ,$$

$$A = \delta_o\,(1.5\,\mathrm{Re}\,\delta^{+3} + 9.3\,q\,\delta^{+3} - 0.119\,\lambda\delta^{+6})\cdot 10^{-4}\,q^{-2} \;\; ; \; B = \delta_o\,(4.6\,\mathrm{Re}\,\lambda\delta^{+2} + 27.78\,q\lambda\delta^{+2} - 0.36\,\lambda^2\delta^{+5})\cdot 10^{-4}\,q^{-2}$$

$$\begin{aligned} C = {} & \delta_o\,(2.78\cdot 10^{-4}\,\mathrm{Re}^2\,\delta^{+3} + 0.00108\,q\,\mathrm{Re}\,\delta^{+3} - 8.8\cdot 10^{-5}\,q^2\delta^{+3} - 1.97\cdot 10^{-5}\,\mathrm{Re}\,\lambda\delta^{+6} + \\ & + 9.44\cdot 10^{-6}\,q\,\lambda\delta^{+6} - 2.87\cdot 10^{-8}\,\lambda^2\delta^{+9})\,/\,q^3 \end{aligned}$$

$$\begin{aligned} D = {} & \delta_o\,(8.37\cdot 10^{-4}\,\mathrm{Re}^2\,\lambda\delta^{+2} + 0.0032\,\mathrm{Re}\,q\lambda\delta^{+2} - 2.65\cdot 10^{-4}\,q^2\lambda\delta^{+2} - 5.9\cdot 10^{-5}\,\mathrm{Re}\,\lambda^2\delta^{+5} + \\ & + 2.83\cdot 10^{-5}\,q\,\lambda^2\delta^{+5} - 8.6\cdot 10^{-8}\,\lambda^3\delta^{+8})\,/\,q^3 \end{aligned}$$

$$Q = Q_1 - Q_2 - Q_3 - Q_4 \;\; ; \; n = \lambda\,\delta^{+3} \; ;$$

$$\begin{aligned} Q_1(x,\lambda,q,\delta^+) = {} & \{-0.21q^3 + 0.0165\,\mathrm{Re}^3 + q^2(13.36 + 0.3393\,\mathrm{Re} - 4.96\cdot 10^{-4}\,n) + \mathrm{Re}^2(19.52 - 4.09\cdot 10^{-5}\,n) + \\ & + \mathrm{Re}\,n\,(-0.045 - 3.9\cdot 10^{-6}\,n) + n^2(8.89\cdot 10^{-4} + 3.39\cdot 10^{-8}\,n) + q\,[-0.166\,\mathrm{Re}^2 + n(-0.0287 - 3.14\cdot 10^{-6}\,n) + \\ & + \mathrm{Re}(-36.04 + 7.14\cdot 10^{-4}\,n)]\}/q^3 \end{aligned}$$

$$\begin{aligned} Q_2(x,\lambda,q,\delta^+) = {} & 3\,n\,[-0.0926q^3 - 0.2727\,\mathrm{Re}^3 + 2.79\cdot 10^{-4}\,\mathrm{Re}^2\,n - 9.85\cdot 10^{-6}\,\mathrm{Re}\,n^2 - 9.56\cdot 10^{-9}\,n^3 + \\ & + (0.1229\,\mathrm{Re} - 8.83\cdot 10^{-5}\,n)\,q^2 + (0.44\,\mathrm{Re}^2 + 0.00108\,\mathrm{Re}\,n)\,q + 4.72\cdot 10^{-6}\,n^2]/q^5 \end{aligned}$$

$$\begin{aligned} Q_3(x,\lambda,q,\delta^+) = {} & [-0.0926q^3 - 0.273\,\mathrm{Re}^3 + 2.79\cdot 10^{-4}\,\mathrm{Re}^2\,n - 9.85\cdot 10^{-6}\,\mathrm{Re}\,n^2 - 9.56\cdot 10^{-9}\,n^3 + \\ & + (0.123\,\mathrm{Re} - 8.8\cdot 10^{-5}\,n)q^2 + (0.441\,\mathrm{Re}^2 + 0.00108\,\mathrm{Re}\,n + 4.72\cdot 10^{-6}\,n^2)\,q]\,\mathrm{tg}\,\theta\,/\,q^3 \end{aligned}$$

$$\begin{aligned} Q_4(x,\lambda,q,\delta^+) = {} & \frac{\mathrm{Re}\;\;\mathrm{tg}\,\theta}{q^3}\left(0.818\,\mathrm{Re}^2 - 0.883\;\;q\,\mathrm{Re} - 0.1229q^2 - 8.4\cdot 10^{-4}\,\mathrm{Re}\;\;n - 2.16\cdot 10^{-3}\;\;n\;\;q\right) - \frac{0.1229\,\mathrm{Re}\;\;n}{q^3} + \\ & + \frac{8.8\cdot 10^{-5}\,n\,\mathrm{tg}\,\theta}{q} + (8.8 + \frac{2.95\,\mathrm{Re}}{q^2}\;\;)\cdot 10^{-5}\,\frac{n^2}{q^3} - \frac{9.44\cdot 10^{-6}\,n^2\,\mathrm{tg}\,\theta}{q^2} + \frac{2.87\cdot 10^{-8}\,n^3\,\mathrm{tg}\,\theta}{q^3} + \frac{n}{q^4}(-0.883\,\mathrm{Re}^2 - \\ & - 0.0022\,\mathrm{Re}\,n - 9.4\cdot 10^{-6}\,n^2) + \frac{n}{q^5}(0.818\,\mathrm{Re}^3 - 8.3\cdot 10^{-4}\,\mathrm{Re}^2\,n + 2.95\cdot 10^{-5}\,\mathrm{Re}\,n^2 + 2.87\cdot 10^{-8}\,n^3) \end{aligned}$$

$$G(x,\lambda,q,\delta^+) = G_1 - G_2 - G_3 - G_4$$

$$G_1(x,\lambda,q,\delta^+) = (-16\,q + 10.5\,\mathrm{Re}\,\delta^{+2} - 0.0667\,n)\,/\,q^2 \;\; ,$$

$$G_2(x,\lambda,q,\delta^+) = (0.341q^2 + 1.13q\,\mathrm{Re} - 0.704\,\mathrm{Re}^2 + 0.00185\,q\,n + 3.04\cdot 10^{-4}\,\mathrm{Re}\,n - 1.12\cdot 10^{-5}\,n^2)\,n/q^4$$

$$G_3(x,\lambda,q,\delta^+) = \mathrm{tg}\,\theta\,(-0.329q^2 + 0.817\,\mathrm{Re}\,q - 0.352\,\mathrm{Re}^2 + 9.3\cdot 10^{-4}\,q\,n + 1.5\cdot 10^{-4}\,\mathrm{Re}\,n - 5.96\cdot 10^{-6}\,n^2)/q^2$$

$$\begin{aligned} G_4(x,\lambda,q,\delta^+) = {} & (q^2\,\mathrm{tg}\,\theta + n)\,[0.8162\,\mathrm{Re}\,/\,q^3 - 0.6586\,/\,q^2 + 9.26\cdot 10^{-4}\,n\,/\,q^3 - 2\,(-0.352\,\mathrm{Re}^2 + 0.8162\,\mathrm{Re}\,q - \\ & - 0.329q^2 + 1.52\cdot 10^{-4}\,\mathrm{Re}\,n + 9.23\cdot 10^{-4}\,q\,n - 5.96\cdot 10^{-6}\,n^2)\,/\,q^4] \end{aligned}$$