Extensive Flow Investigation of a Single-Stage Axial Flow Turbine

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Reliable flow calculation is very important part of every turbomachinery design system. A method for very fast and accurate flow calculation and performance prediction of multistage axial flow turbines at design and off-design conditions was developed by the author. The through-flow code is based on a stream function model and a finite element solution procedure for compressible steady state inviscid flow with high fidelity loss and mixing models. The code is able to calculate flow in axial turbines at subsonic and transonic conditions. The reliability of the method is verified by calculations for several gas and steam turbines. Results of flow calculation and performance prediction of single-stage turbine are presented herein. Low load operation with flow reversal in the hub region behind the last rotor blade row is also analyzed. The numerical results are compared to the results of experimental investigations. The correspondence, even for low loads, is very good.

Keywords: *Axial Turbine, flow calculation, finite elements, flow prediction.*

1. INTRODUCTION

Through-flow calculation methods are widely used in turbomachinery design because of their speed and efficiency. Nevertheless, the application of through-flow methods for flow calculations in axial flow gas and steam turbines at part and low loads is still connected with considerable limitations and uncertainty. Flow separation and reversal, which occur in the hub region behind the last rotor blade row at low load operation, cannot usually be handled, and correlations for calculation of loss coefficients at off-design conditions are not sufficiently accurate. Full three-dimensional methods for flow calculations, developed in recent years, are applicable for detailed analysis of single blade rows. The application of 3D-methods for calculating multistage turbomachines meets several serious problems. Therefore, especially in industry, there exists significant interest in further improvement of through-flow methods for economical flow calculation of multistage turbines in design phase.

This paper presents the development of computational method for quick and accurate flow analysis and performance prediction of multistage gas and steam turbines over very wide range of operating conditions. The method is based on the through-flow theory and finite element procedure. It includes considerable extensions and improvements in form of new combination of loss and deviation correlations, losses radial distribution, spanwise mixing, new procedure for density calculation, and extension to encompass local reverse meridional flow.

The results of calculations of a single-stage turbine are presented herein, as well as the comparison of numerical data with results of extensive experimental investigations.

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2. THROUGH-FLOW CODE

The code was developed by Petrovic (1995). An extensive description of the analytical background and the applied numerical procedures is given in Petrovic and Riess, 1997. In this paper, the method will be presented in short.

The steady three-dimensional flow through a turbomachine is governed by conservation equations of mass, momentum and energy and thermodynamic equation of state, which, using the usual nomenclature may be written as (Hirsh, 1988):

$$\frac{D\rho}{Dt}+\rho\nabla\vec{w}=0 \ , \tag{1}$$

$$\frac{D\vec{w}}{Dt}=-\frac{1}{\rho}\nabla p-2(\vec{\omega}\times\vec{w})+\omega^2\vec{r}+\frac{1}{\rho}\nabla\overline{\overline{\tau}} \ , \tag{2}$$

$$\rho\frac{D}{Dt}(h+\frac{w^2}{2}+\frac{u^2}{2})=\frac{\partial p}{\partial t}+\nabla(k\nabla T)+\nabla(\overline{\overline{\tau}}\,\vec{w}) \ , \tag{3}$$

$$\rho=f(h,s) \ . \tag{4}$$

The flow is assumed to be steady, adiabatic and axisymmetric, thus, two dimensional description is achieved. 2D calculation is made within meridional hub-to-shroud plane, at which momentum equation is projected. Body forces are introduced to replace turbine blades. Flow loss effects are approximated by a friction force.

In this case, the resulting momentum equation has the following form:

$$\frac{\partial^2\psi}{\partial r^2}+\frac{\partial^2\psi}{\partial z^2}=\left[\frac{2\pi}{\dot{m}}q-\frac{\partial\psi}{\partial r}\frac{\partial}{\partial r}(\frac{1}{\rho rb})-\frac{\partial\psi}{\partial z}\frac{\partial}{\partial z}(\frac{1}{\rho rb})\right](\rho rb) \tag{5}$$

where q is

$$q=-\frac{1}{w^2}\left[(w_z+w_u\,\mathrm{tg}\nu')(T\frac{\partial s}{\partial r}-\frac{\partial I}{\partial r})-(w_r+w_u\,\mathrm{tg}\varepsilon')\times\right.$$
$$\left.\times(T\frac{\partial s}{\partial z}-\frac{\partial I}{\partial z})\right]-\frac{1}{r}\left[\mathrm{tg}\nu'\frac{\partial}{\partial r}(rc_u)-\mathrm{tg}\varepsilon'\frac{\partial}{\partial z}(rc_u)\right] \quad (6)$$

The stream function defined by relations:

$$\frac{\partial\psi}{\partial r}=\frac{2\pi}{\dot m}\rho\, r\, b\, w_z\ ,\quad \frac{\partial\psi}{\partial z}=-\frac{2\pi}{\dot m}\rho\, r\, b\, w_r \quad (7)$$

satisfies the continuity equation in meridional plane. m is mass flow rate and b is tangential blockage factor (b=1-δ/t, where δ is tangential blade thickness and t is blade pitch). Outside of blade rows b is equal to 1.

The boundary conditions for turbines without extractions are defined by: ψ=0 along hub, ψ=1 along shroud and $\partial y/\partial n$=0 at the inlet and the exit, where n is direction normal to boundary. For turbines with extraction, the value of stream function at shroud downstream of extraction locations has to be reduced according to the extracted mass flow rate.

In adiabatic flow rothalpy I in rotor as well as stagnation enthalpy H in stator remains constant along a streamline. Solving eq. (5) for a bladed region the velocity components w_z and w_r are estimated first. For calculation of circumferential velocity component it is necessary to estimate flow angle β_2 at the outlet of the blade row using empirical correlations. The increase of entropy s has to be estimated by loss models (Traupel, 1988, Petrovic, 1995). For present calculation a new models for loss determination, loss distribution along a blade height, spanwise mixing and new deviation model for outlet flow angle calculation were developed (Petrovic, 1997).

The governing through-flow equation (5) is second order, quasi-linear partial differential equation. To solve this one, finite element method, with eight-node, isoparametric, quadrilateral elements and biquadratic interpolation functions, is used. The Galerkin procedure of weighted residual is applied to form finite element equations, while frontal solving method is used to solve the resulting system of linear equations.

3. RESULTS OF FLOW ANALYSIS OF A SINGLE-STAGE TURBINE

The single-stage uncooled turbine described by Foerster and Kruse, 1990, is selected for verification of the developed method. The turbine is test version of the first stage of a helicopter engine turbine. The well-documented set of experimental data (Test case E/TU-3, AGARD AR 275) has been used to check numerical results.

The turbine design data are given in the Table 1.

The turbine flow path and calculated domain with elements distribution is shown in Fig. 1. The grid refinements study was performed to make computing time shorter. It was concluded, that use of ten elements in the radial direction and four elements per blade row in the axial direction, gives very good results.

The flow in the turbine was calculated at the turbine design point and very wide range of the off design conditions to find turbine part load behavior. Fig. 2

Table 1. Design data of the single-stage turbine

The nominal rotating speed n	7800 rpm
Reduced rotating speed n_r	6.968 s/K$^{0.5}$
Reduced mass flow rate $mp/\sqrt{T}$	97 kgbar/ $\sqrt{\mathrm{K}}$ /s
Reduced total enthalpy drop $\Delta h^0/T$	135 J/kgK.
Turbine tip radius	225 mm
Turbine hub-to-tip ratio	0.756
Static tip clearance of rotor	0.25 mm.
Stotor clearance	Without clearance

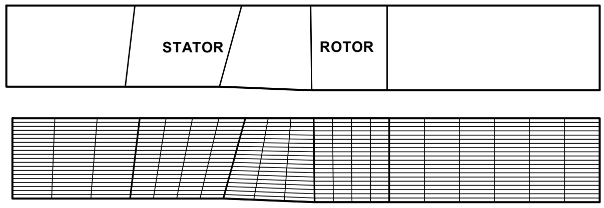

Figure 1. Single-stage turbine flow path and finite element distribution.

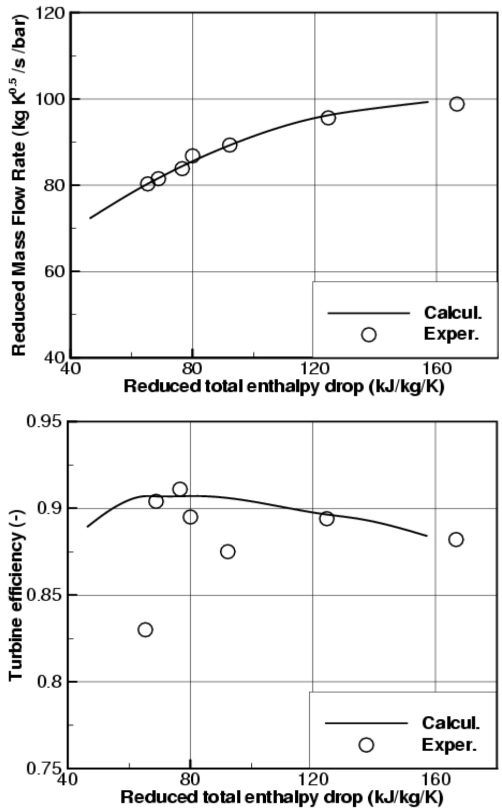

Figure 2. Overall performance of the single-stage turbine.

shows the calculated turbine overall performance: reduced mass flow rate: $mp/\sqrt{T}$ and turbine efficiency total-to-total η_{tt} as function of reduced total enthalpy drop $\Delta h^0/T$. The comparison with experimental data shows good agreement. The reason for large disagreement in η_{tt} at two operating points can be only due to experiments. At the design conditions it was calculated: η_{tt} =89.52% and measured η_{tt}=89.4%.

The distributions of total temperature and total pressure at stator (SE) and rotor (RE) exit are given in Fig. 3. The comparison with the experimental data shows good agreement. Fig. 4 shows the radial profile of the flow loss coefficient calculated for guide vanes exit (stator) and rotating blades (rotor) exit. It clearly to notice two zones with increased flow losses (the hub zone and the tip zone) due to secondary flow and clearance. According to that, the aerodynamic efficiency and the aerodynamic work are lower within these zones (Fig. 5). Fig. 6 shows the calculated variation of the stage reaction grad over blade height at design conditions.

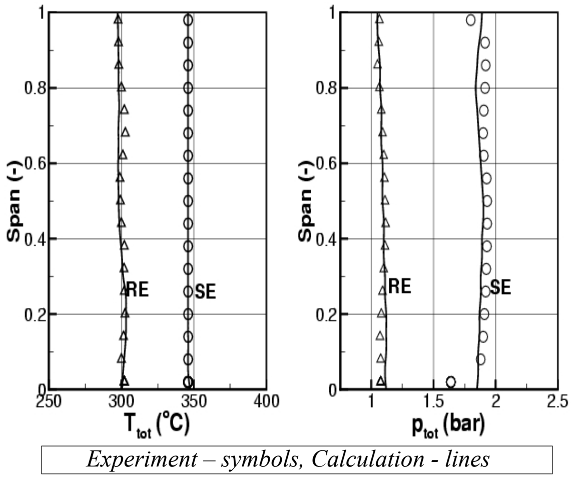

Figure 3. Radial variation of total temperature and total pressure at stator (SE) and rotor (RE) exit of the single-stage turbine.

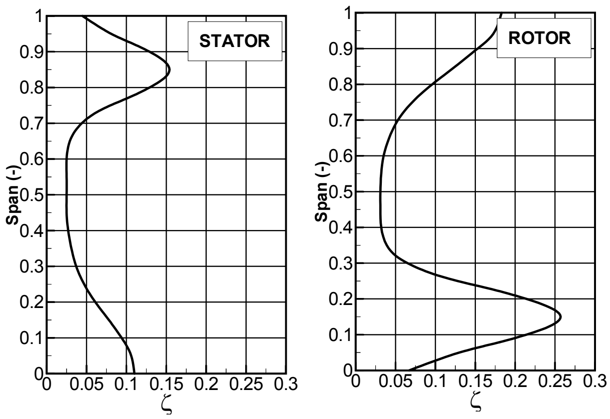

Figure 4. Radial profile of the flow loss coefficient calculated for guide vanes exit (stator) and rotating blades (rotor) exit.

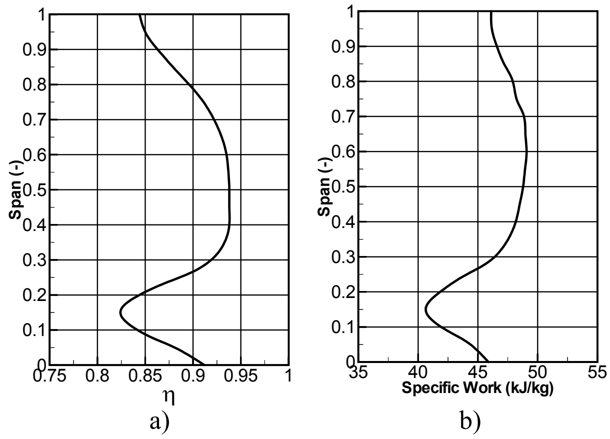

Figure 5. Radial profile of the aerodynamic efficiency (a) and the aerodynamic work (b).

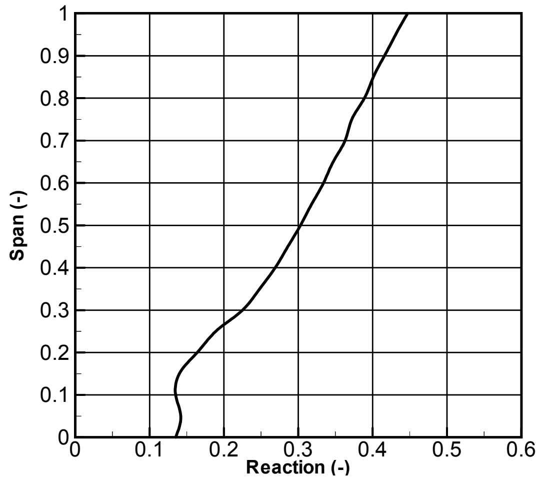

Figure 6. Variation of the stage reaction grade over the blade height.

The turbine flow field (streamlines, meridional velocities and Mach number distribution) calculated for design condition is presented in the Fig. 7. Distribution of thermodynamic parameters in the meridional plane is given in the Fig. 8. The similar results could be ploted for different off design operating points. The results presented here could be used during turbine design phase for parameter optimisation and performance prediction of a new machine.

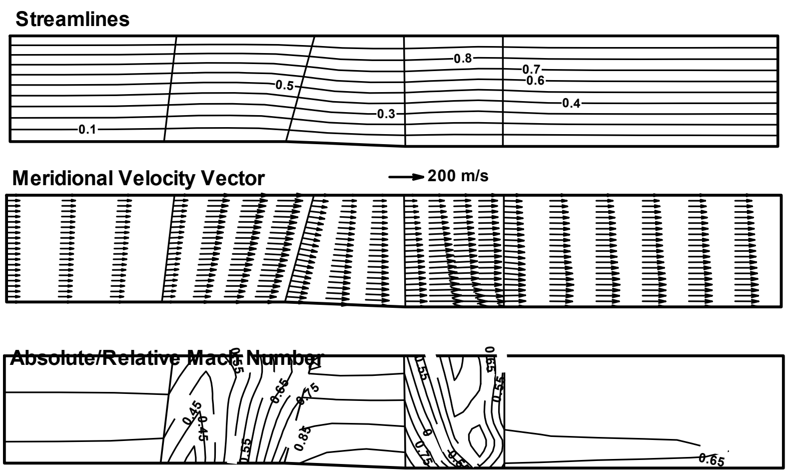

Figure 7. Turbine flow field.

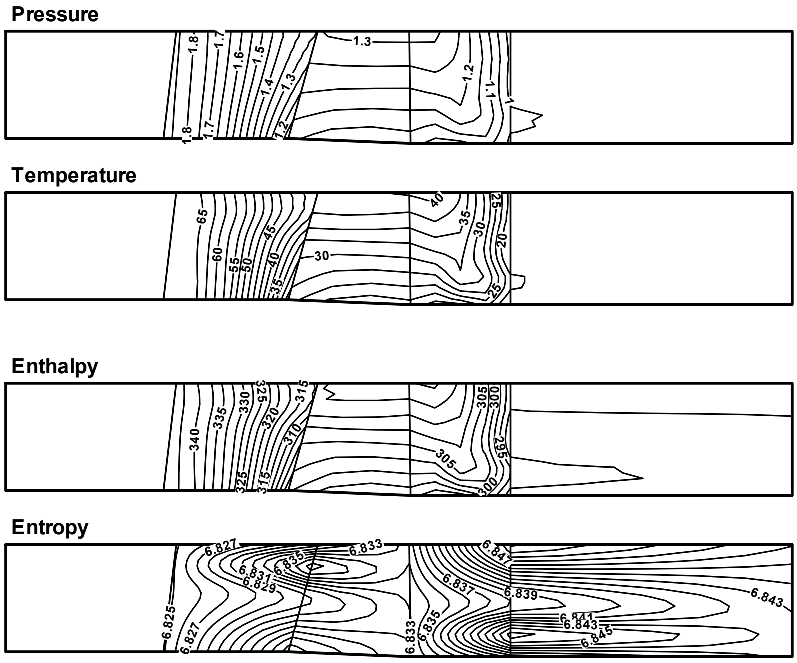

Figure 8. Distribution of the thermodynamic parameters in the turbine meridional plane.

4. CONCLUSIONS

A procedure for through-flow calculation in axial flow turbines based on finite element method is presented. The validation of the model and computer program has been done by comparison of the numerical results with experimental data for a single-stage turbine. The overall performance of the turbine is well predicted, as well at part and low load. Radial profiles of the most important stage characteristics, the calculated turbine flow field and distribution of the thermodynamic parameters are very useful information in the turbine design phase.

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СВЕОБУХВАТНО ИСТРАЖИВАЊЕ СТРУЈАЊА У ЈЕДНОСТУПНОЈ АКСИЈАЛНОЈ ТУРБИНИ

Милан Петровић

Поуздан прорачун струјања је веома важан део сваког система за пројектовање аксијалних турбина. Развијен је метод за веома брзе и тачне прорачуне струјања у вишеступним аксијалним турбинама при раду на номиналном и парцијалним оптерећењима. Метод је базиран на моделу стационарног струјања стишљивог флуида са моделима за прорачуне губитака и радијално мешање. Развијни математички програм је базиран на моделу струјне функције, а за решавање система једначина којима се описује струјање примењена је метода коначних елемената. Програм може да прорачунава подзвучна и околозвучна струјања. Метод је верификован прорачунима више гасних и парних турбина. У раду су приказани резултати прорачуна струјања и предвиђања радних карактеристика за једну једноступну гасну турбину. Прорачун је спроведен и за веома мала оптерећења на којима долази до одвајања струје у каналу иза ротора турбине. Нумерички резултати су поређени са резултатима експерименталних истраживања. Слагање, чак и за режиме рада турбине са малим оптерећењима, је врло добро.