Effect of Response Time on Stopping Distance

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The purpose of this paper is to determine the time demand for pneumatic braking processes, especially signal transmission in the form of pneumatic brake pipe pressure drop, and its effect on stopping distance. Processes that happen from the moment of achieving maximum pressure force in brake cylinders will be considered as established and familiar and described with known equations adequately determined by the brake design and research results. To the present time determination of stopping distance was putting emphasis on friction brake process (at maximum brake force), in which case braking process before achieving maximum brake force, were considered less important, since they had insignificant influence on cumulative stopping distance, and were therefore determined approximately. On this part of braking process essential influence have human factor, brake signal transmission speed, and braking from beginning of development brake force till achieving its maximum value. With more accurate determination of a part of the distance covered by the train from the moment of giving brake demand until beginning of braking with maximum brake force, will be determined how large deviation is, to the present time, approximate determination of this value on the basis of empirical data and how this time affects stopping distance.

Keywords: *railway, brake, pneumatic, stopping distance*

1. INTRODUCTION

The paper presents, in the scope of dealing with analyses of parameters which by pneumatic brake have impact on cumulative stopping distance (braking distance), usual method for determination of stopping distance by braking of railway vehicles and introduce applied hypothesis [1]. By analysis of parameters, which at pneumatic brake affect on cumulative stopping distance, this paper deals comprehensively with braking process before reaching the maximum brake force [2].

In the paper we use terms such as cumulative stopping distance (s_{uk}) and cumulative stopping time (t_{uk}).

$$t_{uk} = t + t_{ps} \ , \quad s_{uk} = s + s_{ps} \ .$$

Braking process can be divided in few segments as follows:

A) Making brake decision, i.e. it is time in which driver observes need for braking demand and put the handle of driver's brake valve in braking position. Cumulative stopping time comprises this time for making of brake decision, so called psychic second (t_{ps}). The terms such as brake application time or stopping time (t), and stopping distance (s) are related, to time and distance from the moment of putting the driver's brake valve in braking position till fully stopping of train and involve few segments such as:

$$t = t_{ok} + t_{rk} + t_{mk} = t_{pk} + t_{mk} \ ,$$

$$s = s_{ok} + s_{rk} + s_{mk} = s_{pk} + s_{mk} \ .$$

B) Transmission of the braking signal demand to the distributor valve, i. e. brake system response, from the moment of putting the handle of driver's brake valve in braking position until the beginning of braking force increment in brake cylinders. By this part of stopping distance s_{ok} and brake system response time t_{ok} , train has not been braked, thus it is easy to calculate the distance part for the known speed by which braking begins.

C) Rise of pressure force in brake cylinder, from the beginning of rise of braking force up to 95% of maximum value. Train covers distance s_{rk} in time t_{rk} for pressure force rise in brake cylinders.

D) Braking from the moment of achieving the maximum brake force until stopping. Train covers distance s_{mk} during braking time with maximum brake force t_{mk} .

Accordingly, it is possible to identify portion of the distance s_0 that vehicle reaches before starts braking with maximum pressure force, as preparation distance s_{pk} that vehicle covers during preparation time t_{pk} ($s_0 = s_{pk} = s_{ok} + s_{rk}$).

The distance portion, which vehicle covers for response time, is determined empirically and added to actual braking distance in order to provide safe stopping taking into account warning (presignalling) and signalling distances, whereby human factor is also considered to some extent.

By determination of distance s_{rk} which vehicle travels

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at development of pressure force, the most common assumption was that vehicle brakes at mean value brake force that develops from zero to maximum value.

2. THEORETICAL ANALYSIS

According to [1], equation (1) for determination of stopping distance parameters, related to stopping distance before braking with maximum brake force, are expressed by preparation distance s_{pk} .

$$s = \int_0^{V_0} \frac{\rho m V}{F_r + F_k} \mathrm{d}V + s_{pk} , \qquad (1)$$

where:

ρ - is rotational inertia coefficient,
m - train mass,
V - instantaneous train speed,
F_r - rolling resistance force,
F_k - brake force of train and
V_0 - train speed from which braking starts.

Known equations [1] that determine cumulative stopping distance are stated with expression (2) - general equation, whereby resistance force is not given as speed function but through mean value of specific resistance w_{sr}, and expression (3) - equation of vehicle equipped with disc brakes.

$$s_1 = \frac{1}{2} \frac{\rho m V_0^2}{K\mu + w_{sr}\, m g} + s_{pk} , \qquad (2)$$

$$s_2 = \frac{\rho m}{2}\left[\frac{1}{c} \ln\left(\frac{V_0{}^2 + pV_0 + q}{q} \right) - \frac{p}{c} \frac{1}{\sqrt{(4q - p^2)/4}} \times \right.$$
$$\left. \times (\arctan \frac{V_0 + p/2}{\sqrt{\frac{4q + p^2}{4}}} - \arctan \frac{p/2}{\sqrt{\frac{4q + p^2}{4}}}) \right] + s_{pk} . \qquad (3)$$

where:

K - is braking efficiency,
μ - coefficient of friction between brake block and wheel or brake lining and disc,
g - acceleration of gravity ($g = 9{,}81\ \mathrm{m/s^2}$) and
p, q, c – empirical coefficients (constants).

In equations (2, 3), a preparation distance s_{pk} is determined by multiplying speed at which braking starts (V_0) and preparation time t_{pk} (time elapsed from the moment of putting the handle of driver's brake valve in braking position until reaching 95% of maximum pressure force in brake cylinders).

In table 1 are given values for this distance portion for estimated response and vehicle circuit preparation time ($t'_{ok} = 0{,}784$ s, $t_{ok} = 0{,}49$ s, $t'_{pk} = 2{,}09$ s) [1] and cumulative stopping distance s determined by equation (2). For the vehicle equipped with disc brake (data are known and determined, analytically as well as empirically) stopping distance s_1 is determined by equation (2), and stopping distance s_2 by equation (3). By comparing ratios of stopping distances (table 1), it is easy to notice that the preparation distance - s_{pk} becomes longer as brake speed goes up, but, expressed in percentages, preparation time t_{pk} has smaller impact on stopping distance. Values for stopping distance calculated by equations (2) and (3) do not deviate significantly one from another.

Table 1. Comparation ratios of stopping distances, according to [2] and [3]

Speed [km/h]	s_1 [m]	s_{pk} [m]	s'_{ok} [m]	s_{ok} [m]	s_{pk}/s_1 [%]	s'_{ok}/s_1 [%]	s_{ok}/s_1 [%]
120	574	69,7	26,1	16,3	12,1	4,54	2,84
130	665,8	75,5	28,3	17,7	11,3	4,25	2,66
140	763,7	81,3	30,5	19,1	10,6	3,99	2,5
150	864,5	87,1	32,7	20,4	10,1	3,78	2,36
160	972,8	92,9	34,8	21,8	9,5	3,58	2,24

Stopping and preparation distance values are determined for the following input data: $m = 40\,\mathrm{t}$, $\rho = 1{,}05$, $\mu = 0{,}35$, $K = 32{,}59\,\%$, brake regime R, maximum work pressure in brake cylinders $p \approx 3{,}75$ bar etc [3].

Based on previous consideration, it is possible to select three methods for determining of stopping distance and preparation part of stopping distance s_{pk}, respectively, depending upon models of pneumatic processes in brake installation and based on accuracy level for its determination.

1) Approximate determination of preparation distance s_{pk} which train covers from giving braking signal till applying maximum pressure force (as it has been done so far), is the simplest model. For this case, it is anticipated that up to a fictive moment, which divides process of train movement in totally release/totally braked, train moves fully unbraked and none of the resistance forces F_r makes impact on its movement. By determination of preparation distance s_{pk}, approximate values are used depending on empirical data. Hence, obtained values of response time are for vehicle in the middle of a train according to equation (4).

$$s_{pk} = s_{ok} + s_{rk} = V_0 t_{ok} + V_0 t'_{rk} = V_0 t'_{pk} ,$$
$$t_{ok} = \frac{l N/2 + l_l}{u_{pr\,\min}} = \frac{(N/2+1) l}{u_{pr\,\min}} , \qquad (4)$$

where:

N - is number of vehicle of a train,
l - length of vehicles (all vehicles in the train should be of a same length),
l_1 - length of the locomotive and
$u_{pr\,\min}$ - minimum transmission speed defined by leaflets UIC 540 [4] ($u_{pr\,\min} = 250$ m/s).

Value t_{ok} is appx. 0,8 s for a train that consists of 15 passenger stocks.

Time required for pressure force development t'_{rk} in brake cylinders up to some fictive moment, which divides transient process in fully release/fully braked vehicle, amounts $t'_{rk} = k_1 t_{rk} = 1{,}6\,\mathrm{s}$, where k_1 is coefficient of distribution of marked area in diagram of air pressure rise in brake cylinder.

Preparation time t'_{pk} includes brake system response time and time for pressure force development:

$$t'_{pk} = t_{ok} + t'_{rk} = 12{,}384\,\text{s}\,.$$

In practice, preparation time is taken in the range of $t'_{pk} = 2 \div 2{,}8$ s.

2) More accurate approximate determination of preparation distance s_{pk} can be obtained when it is considered that until braking with maximum brake has taken place, on train impacts resistance force F_r. The rolling resistance force is known and calculated with mean value of specific resistance w [5]. For determining of stopping distance s_{ok} it is taken into account response part of resistance force (equation 5 according to [5]), and for determining the part of a stopping distance at pressure force rise s_{rk} is considered, in addition to resistance force, also braking effort through specific brake force b (equation 6 for braking in G regime and equation (7) for braking in P regime).

$$s_{ok} = V_0 t_{ok} - \zeta w t_{ok}^2 / 2\ , \tag{5}$$

$$s_{pk} = V_0 t_{pk} - \zeta\left[\frac{1}{2} w t_{pk}^2 + (t_{pk} - t_{sr})^2 \left(\frac{1}{3} b_{sk} + \frac{1}{6} b_{pk}\right)\right]\ , \tag{6}$$

$$s_{pk} = V_0 t_{pk} - \zeta\left[\frac{1}{2} w t_{pk}^2 + \frac{4}{15} b_{pk} (t_{pk} - t_{ok})^2\right], \tag{7}$$

where:

ζ - is coefficient of specific brake force,

b_{sk} - specific brake force at overriding and

b_{pk} - specific brake force at filling brake cylinders.

For more accurate calculation of stopping distance s, in equations (2) and (3) it is necessary to replace speed at braking start V_0 with speed which vehicle has in the moment when starts to brake with maximum brake force V_{pk}.

3) For determination of preparation distance s_{pk}, it is assumed that vehicle, until achieved maximum brake force, was braked with resistance force F_r at motion which is observed as known speed function, whereby values of coefficients are given in equation (8). According to [1], with equation (9) is also possible to calculate response part of stopping distance.

$$F_r = c(V^2 + pV + q)\ , \tag{8}$$

$$c = 0{,}000238\ ;\ p = 39{,}916\ \ ;\ q = 6050{,}42$$

$$s_{ok} = \rho m \int_{V_{ok}}^{V_0} \frac{V}{F_r(V)} \mathrm{d}V\ . \tag{9}$$

Integer (9) is determined by equation (10) analogically to the solution of stopping distance s (3).

$$s_{ok} = \rho\left[\frac{1}{c}\ln\left(\frac{V_0{}^2 + pV_0 + q}{V_{ok}{}^2 + pV_{ok} + q}\right) - \frac{p}{c}\frac{2}{\sqrt{4q - p^2}} \times \right. \\ \left. \times\left(\arctan\frac{V_0 + p/2}{\sqrt{\frac{4q + p^2}{4}}} - \arctan\frac{V_{ok} + p/2}{\sqrt{\frac{4q + p^2}{4}}}\right)\right] \tag{10}$$

Determination of part of stopping distance s_{rk} over which vehicle performs braking with inconstant brake force F_k, a problem is encountered because it is necessary to know function of pressure force rise $F(t)$ from zero to 95% of a maximum time value, as well as function of friction coefficient μ in dependance upon speed (equation 11).

$$s_{rk} = \rho m \int_{V_{po}}^{V_{ok}} \frac{V}{F_k(V,t) + F_r(V)} \mathrm{d}V\ . \tag{11}$$

In order to define this part of stopping distance it is necessary to know function of friction coefficient μ in dependance upon speed $\mu(V)$. But still remains a problem how to determine function for pressure force rise in time $F(t)$, as well as determination of speed V_{pk} at which maximum brake force would be achieved. Time t_{rk} needed for pressure force to ascend is defined by time required for filling out brake cylinders, which is set out in leaflet UIC 540 [3] and ranges from $3 \div 5$ s in P regime (continuous brake – freight train) or $18 \div 30$ s in G regime (continuous brake – passenger train).

$$\begin{aligned} F_k(t) &= F(t)\mu(V) = F_1(t) n \mu(V) = \\ &= A\,p(t)\mu(V) + B\mu(V) \end{aligned} \tag{12}$$

$$A = i\eta n D^2 \pi/2\ ,\ \ B = -(F_o + F_R) i \eta n\ ,$$

$$s_{rk} = \rho m \int_{V_{ok}}^{V_{pk}} \int_0^{t_{rk}} \frac{V}{A\,p(t)\mu(V) + B\mu(V) + F_r(V)} \mathrm{d}t\,\mathrm{d}V\ , \tag{13}$$

where:

$F(t)$ - pressure force of brake blocks acting on wheel,

$F_1(t)$ - pressure force of one brake block,

n - number of brake blocks,

$p(t)$ - air pressure function in brake cylinder,

D - brake cylinders inside diameter,

F_o - resistance force of spring,

F_R - resistance force of regulator,

i - amplification ratio of brake rigging,

η - brake-rigging efficiency and accordingly

A, B - known constants.

For more accurate calculation of stopping distance s, in equations (2) and (3) start speed V_0 has to be replaced with speed which vehicle has in the moment when starts to brake with maximum brake force V_{pk}.

For determination of equations (11) and (13), it will not go without knowing speed V_{ok} whereat starting pressure force rise in brake cylinders as prior to that occurrs only resistance force, and but also will not be possible without knowing speed V_{pk} at which pressure force achieves 95% of maximum value. Speed V_{ok} should be determined considering that this braking segment includes psychic second and time necessary to induce braking, namely transmit signal along brake pipe vehicle circuit/train circuit, through distributor valve and transmitter valve to brake cylinder. According to leaflet UIC 540 [4], that deals with review of conditions for modern compressed-air brake (after 1989), brake must react at brake pipe pressure drop in 1,2 s max. Since determination of psychic second is not thoroughly studied and may have very wide range we will apply value 1,2 s for train and time t_{ok} at which brake force in braking process does not operate.

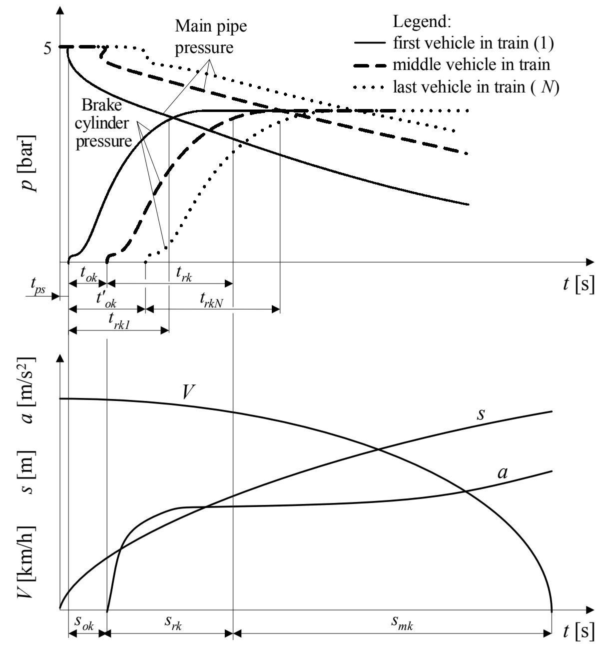

Figure 1. Dividing of braking process into few segments, according to [1] and [2]

Table 2 presents preview of equations for determination of stopping distance for different methods of braking and times required [2].

3. ANALYSIS BASED ON EMPIRICAL DATA

3.1. Brake testing in place and at motion

For vehicles with automatic compressed-air brake for passenger stock (brake regime P), according to leaflet UIC 544-1 [6], braked weight is determined by testing and depends on stopping distance measured after quick braking which is to be executed on straight and flat track in clear weather without wind. Tests can be conducted with:

- train with 60 axles consists of a same type vehicle, which are tracted by unbraked locomotive or
- pushing-off of single wagon/vehicle.

Emergency braking operation is performed at speed 100, 120, 140, 150 and 160 km/h, i.e. up to the speed for which is vehicle designed. Measured stopping distances are entered on appropriate speed line in UIC 544-1, Appendix 6. Speed and stopping distance measurments have to be accurate and repeated several times in order to determine reliable average value for each specified speed.

At-motion vehicle brake testing is conducted by pushing-off method**,** using gauge vehicle JŽ (Yugoslav railway) 480, that was coupled to locomotive during testing.

Actually achieved speeds at the moment of braking start have to be in permissive limit range $\pm 3\ \text{km/h}$ against reference speed. Pressure in brake cylinder need also to be in permissive limits (if this pressure did not change in the course of stopping process, it can be concluded that wheel-slide protection - WSP did not operate). For the purpose of obtaining the most accurate stopping distance value before engaging emergency braking (breaking of coupling), achieved rolling speed should be kept constant for at least 200 m. Considering that is impossible to commence emergency braking process right at the moment of achieving planned speed, value of stopping distance needs to be updated by means of equation (14) [1]. Consequently, this equation also updates deviation obtained when the testing is conducted on a track which deviates from totally horizontal track for more than $\pm 4\ \text{mm/m}$.

$$s = \frac{\varphi V^2}{1{,}09375\lambda + 0{,}126 - 0{,}235\, i\, \varphi}\ ;\ s' = s\,\frac{4{,}24 V_0^2}{4{,}24 V^2 \pm i\, s}\ , \quad (14)$$

where:

s' [m] - updated stopping distance,
s [m] - travelled stopping distance (measured),
V_0 [km/h] - start reference speed,
V [km/h]- achieved start speed,
φ - coefficient dependent upon speed (table [1]),
λ - brake weight persentage and
i [%.] - declination of track (in equation, plus sign means descending, and minus sign means ascending of declination of track).

3.2. Measuring of brake transmission speed

Measuring of train transmission speed of need to be carried out for both continuous brake (freight train and passenger train). Transmission speed is determined when we divide length of brake pipe circuit that runs from driver's brake valve on locomotive upto air shut-off cock at the end of the train (spur not included), with time that ticks from the moment when driver has put the handle of a driver's brake valve in braking position till the moment when air begins to enter into brake cylinder of the last vehicle circuit.

Transmission speed of a train consisting of 8 passenger stock Ba type Y was measured in place and at motion. Measuring of train not-running is carried out by recording the diagram of pressure of the last cylinders. Entering time in the last cylinder in train circuit length, from the moment of applying emergency braking, amounts 0,81 s, which represents transmission speed of $210\,\text{m}/0{,}81\,\text{s} = 260\,\text{m/s}$. Time elapsed from entering of air in the first cylinder of the first vehicle circuit till entering of air in the last train cylinder is 0,64 s which represents transmission speed of $190\,\text{m}/0{,}64\,\text{s} = 296\,\text{m/s}$. Achieved transmission speeds are more than minimal expected - 250 m/s. Fig. 2 shows function of transmission speed in brake pipe in dependance upon train length, namely upon brake pipe length and percentages of braked vehicle in train on temperature of 12°C [7].

Analysing results of stopping distance obtained by braking of moving train consisting from 8 passenger stock type Y, and measured values of transmission speed for the same train [8], we can determine effect of response time on stopping distance of this train. A distance that train passes from the moment of giving the signal for braking till beginning of braking force s_{ok} development is, actually, multiplication of braking start speed V_0 with transmission time 0,81 s (table 3).

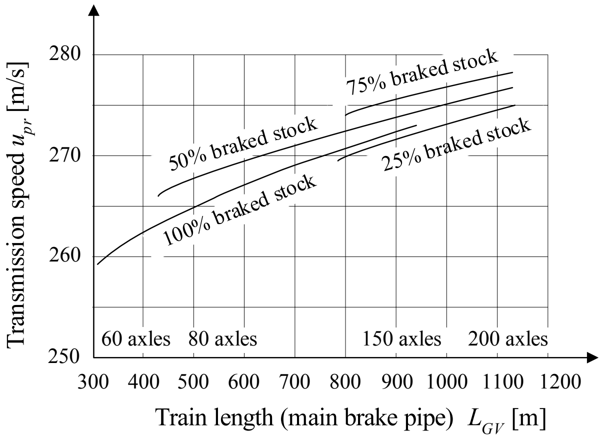

Figure 2. Transmission speed in brake pipe in dependance upon train length [7]

Ratio of distance, which unbraked train travels from the moment of applying braking until braking starts, and the mean value of stopping distance s_{sr} (measured) for adequate speed, shows what the influence of pneumatic brake control is. Thereby, we consider that this influence of pneumatic brake control is different for substantially different brake design and train length.

Table 3. The influence of pneumatic brake control [8]

Regime	Speed [km/h]	s_{ok} [m]	s_{sr} [m]	s_{ok} / s_{sr} [%]
R $p \approx 3{,}63$ bar	120	27	763,8	3,53
	130	29,2	856	3,41
	140	31,5	977	3,22
	150	33,7	1165,5	2,89
R $p \approx 3{,}54$ bar	120	27	821,8	3,29
	130	29,2	909,5	3,21
	140	31,5	1043,8	3,02
	150	33,7	1198	2,81
RIC $p \approx 2{,}83$ bar	100	22,5	683	3,29
	120	27	931,6	2,90

Transmission speed in brake pipe depends on train length, temperature, starting pressure, number of braked vehicles etc. If we assume that certain transmission speed does not change its value for a respective train by same test conditions, the following conclusions are to be established:

- for the same train response part of stopping distance increases with rise of speed whereat we start the braking,
- percentual response time has less influence on stopping distance by increasing of speed whereat we start the braking.

In table 4 are shown values (percentages of braked weight λ and braked weight *B*) obtained by testing of single stock type Y through pushing-off of stock and by testing the same stock in train, with almost indentical brake.

Fig. 3 shows known diagram of stopping distance function and percentage of braked weight (UIC 544-1, appendix 5). It can be noticed that the curves for single stock and for train consisting of 60 axles are dislocated almost parallel one compared to another, for the same speed by which braking started.

Table 4. Values obtained by testing stock type Y [8]

Regime	*V* [km/h]	Vehicle	λ [%]	*B* [t]
R	100÷150 $p \approx 3{,}72$ bar	wagon	153,5	61,4
		train	122,6	491,4
	100÷150 $p \approx 3{,}55$ bar	wagon	141,5	56,6
		train	113	452,9
RIC	100	wagon	97,6	39,1
		train	78	312,6
	120	wagon	109	43,6
		train	87	348,7

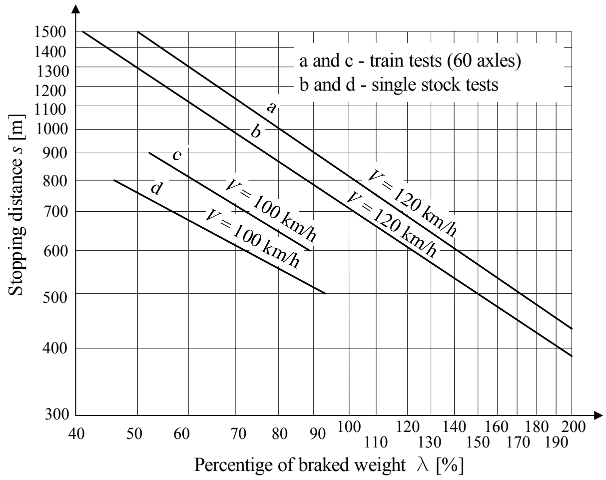

Figure 3. Diagram of stopping distance and percentage of braked weight (UIC 544-1, appendix 5 [6])

4. CONCLUSION

Ratio of response time and stopping distance, i.e. ratio of distance which train passes from the moment of giving braking demand till beginning of braking, and stopping distance which train ranges from the moment of giving braking signal till stopping, has shown impact on pneumatic control braking. Thereby, it should be taken into account that impact of pneumatic brake control on stopping distance depends upon brake design (continuous brake - freight and passenger train P/G, application of brake accelerator, electro pneumatic brake, etc.) and train length.

By comparing results of response and stopping distance, which are obtained by analytical and empirical approach, for, as much as that was possible, similar conditions, we can conclude that effect of pneumatic brake control in operation (~3%, table 3) is to some extent smaller than the result gained by calculation (~4%, table 1). However, it should be also considered that the train of medium length in continuous braking (passenger train) consisting of only 8 four-axle wagons (32 axles) was tested.

The longest part of stopping distance vehicle/train covers partly or totally braked (almost around 97% of stopping distance) and for that reason up to now the most of the effort was put on decreasing this part of a distance, namely on application of friction pairs (e.g. brake block /wheel, brake lining/disc etc.) that would enable larger coefficient of friction.

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УТИЦАЈ ВРЕМЕНА ОДЗИВА НА ЗАУСТАВНИ ПУТ

Марија Вукшић

Циљ овог рада је одређивање времена које је потребно за одвијање пнеуматских процеса кочења, посебно преноса сигнала падом притиска у пнеуматској инсталацији кочнице возила, и његов утицај на зауставни пут при кочењу. При томе ће се процеси од тренутка достизања максималне кочне силе у кочним цилиндрима сматрати познатим и за њихово описивање ће се користити познати обрасци. Досадашње одређивање зауставног пута имало је тежиште на процесу кочења трењем (при максималној кочној сили), при чему су процеси кочења пре постизања максималне кочне силе сматрани мање битним, будући да мало утичу на укупан зауставни пут и одређивани су приближно. На овај део процеса кочења битно утиче људски фактор, брзина преноса сигнала кочења и кочење од почетка развоја кочне силе до њене максималне вредности. Тачнијим одређивањем дела пута који воз пређе од тренутка давања команде за кочење до почетка кочења максималном кочном силом утврдиће се колико је одступање при досадашњем приближном одређивању ове вредности на основу искуства у експлоатацији, и колико ово време утиче на зауставни пут.

Table 2. Equations for determination of stopping distance for different methods of braking and times required [1, 2, 5].

<table>
<tr><td colspan="2" rowspan="3">Braking</td><td colspan="4">Distance which train travels at braking</td></tr>
<tr><td>s_{ps}</td><td>s_{ok}</td><td>s_{rk}</td><td>s_{mk}</td></tr>
<tr><td>Human or device effect</td><td>Response of brake system</td><td>Pressure force development to 95% maximum value in BC</td><td>Braking with maximum brake force until train stops</td></tr>
<tr><td colspan="2">Rapidly</td><td rowspan="5">$V_0 t_{ps}$</td><td rowspan="8">$V_0 t_{ok}$
or
$V_0 t_{ok} - \frac{1}{2}\zeta w t_{ok}^2$
or
$\rho\left[\frac{1}{c}\ln\left(\frac{V_0^2 + pV_0 + q}{V_1^2 + pV_1 + q}\right) - \frac{p}{c}\frac{1}{\sqrt{\frac{4q-p^2}{4}}}\left[\arctan\left(\frac{V_0 + \frac{p}{2}}{\sqrt{\frac{4q+p^2}{4}}}\right) - \arctan\left(\frac{V_1 + \frac{p}{2}}{\sqrt{\frac{4q+p^2}{4}}}\right)\right]\right]$</td><td rowspan="9">$V_{sr}\left(10 - \frac{12\,i}{1000\,\varphi_{kp}\,\vartheta_p}\right)$
or
$\rho\int_{V_2}^{V_1}\frac{V}{F_k(V,t) + F_r(V)}\,dV$</td><td rowspan="9">$\rho\, m\int_0^{V_0}\frac{V}{F_k(V,t) + F_r(V)}\,dV$
or
$\frac{1}{2}\frac{\rho V^2}{\frac{7}{10}\lambda\mu + w_{sr}}$
or
$\rho\left[\frac{1}{c}\ln\left(\frac{V_0^2 + pV_0 + q}{q}\right) - \frac{p}{c}\frac{1}{\sqrt{\frac{4q-p^2}{4}}}\left[\arctan\left(\frac{V_0 + \frac{p}{2}}{\sqrt{\frac{4q+p^2}{4}}}\right) - \arctan\left(\frac{\frac{p}{2}}{\sqrt{\frac{4q+p^2}{4}}}\right)\right]\right]$</td></tr>
<tr><td colspan="2">Fully</td></tr>
<tr><td colspan="2">Graduated</td></tr>
<tr><td rowspan="2">Emergency</td><td>Brake</td></tr>
<tr><td>Valve or air shut-off cock</td></tr>
<tr><td rowspan="3">Perforce</td><td>DSD</td><td>$\sim V_0$ 30 s</td></tr>
<tr><td>automatic application of the brakes</td><td>$V_0\ (4 \div 20\text{s})$</td></tr>
<tr><td>breaking-loose of a train or breaking of BP</td><td>0</td></tr>
<tr><td colspan="2">Brake accelerator of BP</td><td>$V_0 t_{ps}$</td><td>$V_0 t_{br}$</td></tr>
<tr><td colspan="2">Electro-pneumatic</td><td>$V_0 t_{ps}$</td><td>~ 0</td><td></td><td></td></tr>
<tr><td colspan="2">Effect of WSP</td><td>-</td><td>-</td><td>-</td><td>decrease F_k, prolongation s_{mk}</td></tr>
</table>

Abbreviations: BC – brake cylinder, BP – main brake pipe, WSP – wheel-slide protection, DSD – driver`s safety device.