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New Contributions to Cavitation Erosion Curves Modeling

In the paper a new relation for the cavitation erosion curves is proposed, the cumulated mass curve m(t) and the erosion velocity curve of material mass v(t). In constructing this relation one starts from the experimental and analytical models established by Thiruvengadam [10]*, Hammitt* [2]*, Steller* [7] *and Noskievic* [4]. *The parameters that occur in the new model appear naturally and describe in a very good approximation the experimental data curve. The test of the model is realised on carbon steel* OL 370-3k *and stainless steel* 1H18N9T. *The first material is tested in the magnetostrictiv vibration device of the Hydraulic Machinery Laboratory in Timisoara, and the second is tested by J. Steller in the similar device in the Cavitation Laboratory in Gdansk (Polonia).*

Keywords: *erosion, cavitation, loss curve, cavitation erosion velocity, parameters*

1. INTRODUCTION

Due to appreciation of the material behavior is done, often, on the characteristic curves which give the variation with attack time of the eroded volume $Vol(t)$ or of the eroded mass $m(t)$, respectively of the erosion velocity $v(t) = \mathrm{d}Vol(t)/\mathrm{d}t$, or $v(t) = \mathrm{d}m(t)/\mathrm{d}t$, the constructing of an analytical model of approximation of the experimental points became a necessity. Thus, the establishing of an analytical or experimental model characterizes a studied problem by the experts in this domain such as Thiruvengadam [10], Noskievic [4], K.Steller [8], J. Steller [7] etc. The objective is represented by the construction of a function which is to assure simultaneously an acceptable approximation of the experimental points, both for the cumulated losses and for the erosion velocities. Due to the high complexity of the cavitation erosion process and its dependency of very many hydrodynamic and material specific factors, the obtained models are not satisfactory, because of the impossibility to transpose from one material to another, respectively from a experimental device/station to another. So, the analytical description problem of the characteristic cavitation erosion curves remains open. In this paper an analytical model for construction of the curves which give the variation of the mass losses and the erosion velocity with the cavitational attack time is presented, starting from the experimental and analytical models established by Thiruvengadam [10], Hammitt [2], Steller [5] and Noskievič [7]. For application the carbon steel OL 370-3k and stainless steel 1H18N9T were used. The first material is tested in the magnetostrictiv vibratory device with nickel tube from the Hydraulic Machinery Laboratory in Timisoara, and the second one was tested by J. Steller in the similar device from the Cavitation Laboratory in Gdansk (Polonia) [7].

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2. ABOUT CAVITATION EROSION MODELS

Hammitt [2] and Thiruvengadam [10], on the basis of multiple experimental research done in the laboratory, in the cavitation erosion domain, define the typical shapes of the experimental curves Vol(t) and v(t), presented in Figure 1 :

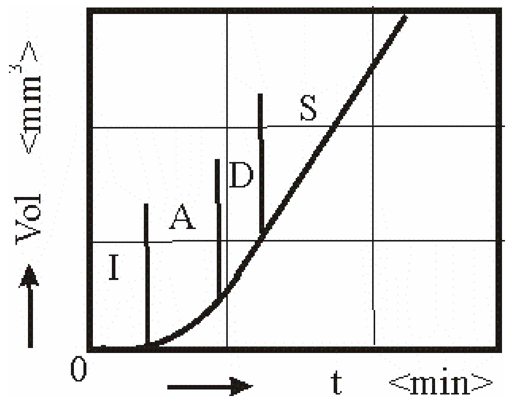

Figure1a. The typical cavitation erosion curves (volumetric losses *Vol*(*t*))

Thiruvengadam divides the erosion velocity curve in four areas:

I - Incubation period;
A - Acceleration period;
D - Deceleration period;
S - Stagnation period (stabilisation).

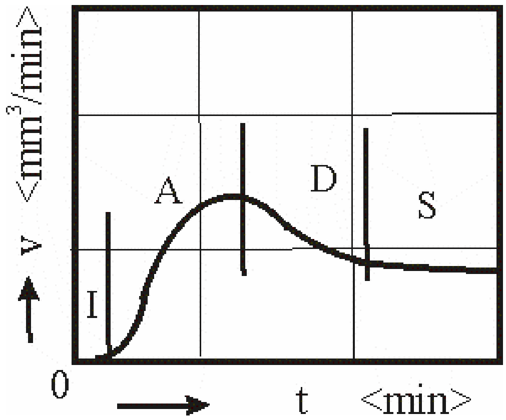

Figure1b. The typical cavitation erosion curves (erosion velocity $v(t)$)

OBSERVATION: These curves′ shapes are specific to eroded cavitation materials in stations/devices with low cavitation intensity, such as the hydrodynamic tunnel and the rotation disc device [1]

Starting from these shapes, J. Steller [6], considers that the eroded volume curve can be described by relations, such as the following:

$$Vol(t) = A\,\Im\,U(k,t)\ , \tag{1}$$

$$Vol(t) = A\,U(k,\Im t)\ , \tag{2}$$

where:
A – eroded area; $\Im$ – the measure of the cavitation erosion intensity; U – a function which results from the phenomenological application of the model; k – a real parameters set (usually three parameters are sufficient) determined from the condition that the erosion curve will give a very good approximation of the experimental data; t – the cumulated exposition period to the cavitation attack.

When applying these relations, the imposing of certain values for some of the parameters is needed, the others resulting from applying the relations in experimental points approximation.

K. Steller [6] proposes a relation based on the cav itation endurance:

$$P_t = R_{\text{cav}} V\ , \tag{3}$$

where P is the absorbed power in material erosion for the erosion of the volume V :

$$R_{\text{cav}} = \frac{\chi + e^{-kt}}{\chi + 1} R_0\ , \tag{4}$$

χ is a factor which describes the final resilience and the initial endurance at cavitation, k is a parameter which defines the decreasing velocity of the cavitation endurance, during the eroding process.

F.J Heymann [3] shows that the erosion velocity of the material on the surface area unit can be described by an integral equation such:

$$Y(t) = f(t) + \int_0^1 f(t-T)Y(T)\mathrm{d}T\ , \tag{5}$$

by which it is assumed that the eroded material consists of layers made of units of layers, $f(t)\mathrm{d}t$ being the moving probability of the element from each layer depending on time $\mathrm{d}t$.

J. Noskievič [4] proposes as a model for the erosion curve, kinetical erosion models, the eroded volume curve being obtained by solving one of the differential equations:

$$\frac{d^2v}{dt^2} + 2\alpha\frac{dv}{dt} + \beta^2 v = I\ , \tag{6}$$

$$\frac{1}{P}\frac{\mathrm{d}^2v}{\mathrm{d}t^2} + \frac{2\alpha}{P}\frac{\mathrm{d}v}{\mathrm{d}t} + \beta^2 v = I = \gamma P\ , \tag{7}$$

where $v = \mathrm{d}Vol/\mathrm{d}t$ is the velocity of the eroded volume, α and β - material properties depending coefficients, I – parameter which characterizes the intensity of the cavitational erosion, P - the power of the energy flux transmitted by the cavitational cloud at material erosion, γ - coefficient resulted from applying equation (7).

For the above presented relations, the authors do not give details about the values of the parameters obtained from applying them in the approximation of the experimental points, nor how one can use them to construct a transposing relation from one device to another, for the same material., or from one material to another, when the test is done on the same device or station. Also, the disadvantage of these relations consists of the changing of the parameters with the curve evolution (incubation, acceleration, etc.), due to fragmentary application on portions of the approximation curves.

3. A NEW RELATION FOR THE ERODED VOLUME CURVE AND FOR THE EROSION VELOCITY

Instead of (1) and (2) formula for the volumetric loss curve, the following formula is proposed:

$$Vol\,(t) = A\,[v_{\text{s}} t - f(t)]\ , \tag{8}$$

where v_{s} is the value of the corosion velocity from the stabilization area, $f(t)$ is a real function which must be determined and A is a proportionality coefficient.

Relation (8) is suggestible of the near linear character of the eroded volume curve (Fig.1), after the incubation period (period in which the eroded volume starts to increase).

The erosion velocity will be:

$$v(t) = \frac{\mathrm{d}Vol(t)}{\mathrm{d}t} = A\left[v_{\text{s}} - \frac{\mathrm{d}f(t)}{\mathrm{d}t}\right], \tag{9}$$

The typical erosion curves (Fig.1) suggest to be accompanied by the following conditions:

a. $f(0) = 0$ (because $\Delta\,Vol\,(0) = 0$)

b. $\lim_{t\to\infty} f(t) = 0$ (due to the near linear character of the eroded volume curve (rel.(8))

c. $\lim\limits_{t \to \infty} \frac{\mathrm{d}f}{\mathrm{d}t}(t) = 0$ (because $\lim\limits_{t\to\infty} v(t) = v_{\mathrm{S}}$).

For the function $\mathrm{d}f / \mathrm{d}t$ it is admitted that it is the solution of the uniform differential equation of second order with constant coefficients:

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + 2\beta \frac{\mathrm{d}y}{\mathrm{d}t} + \beta^2 y = 0, \qquad (10)$$

which describes the damped oscillations with infinite period. So

$$\frac{\mathrm{d}f}{\mathrm{d}t}(t) = \lambda e^{-\beta t} + \delta t e^{-\beta t} = e^{-\beta t}(\lambda + \delta t),$$

respectively

$$f(t) = \int \frac{\mathrm{d}f(t)}{\mathrm{d}t}\mathrm{d}t = \int e^{-\beta t}(\lambda + \delta t)\mathrm{d}t =$$
$$\int (-\frac{1}{\beta} e^{-\beta t})'(\lambda + \delta t)\mathrm{d}t .$$

Integrating by parts, the last integral, it is obtained:

$$f(t) = -\frac{\lambda}{\beta} e^{-\beta t} - \frac{\delta}{\beta} t e^{-\beta t} - \frac{\delta}{\beta^2} e^{-\beta t} + k .$$

Condition **b)** $\lim\limits_{t\to\infty} f(t) = 0$ implies the real parameter $k = 0$, and condition a). $f(0) = 0$ implies $\delta = -\lambda\beta$.

Thus it is obtained:

$$f(t) = \lambda t e^{-\beta t} .$$

In these conditions the eroded volume curve is defined by relation:

$$Vol(t) = A\left[v_{\mathrm{S}} t - \lambda t e^{-\beta t}\right], \qquad (11)$$

respectively the erosion velocity is defined by:

$$v(t) = A\left[v_{\mathrm{S}} - \lambda e^{-\beta t} + \lambda \beta t e^{-\beta t}\right], \qquad (12)$$

The real parameters v_{S}, λ and β can be determined from the approximation of the experimental data condition by the analytical curve (using the ofleast square method or other numerical methods).

Instead of the relations which give the eroded volume or the eroded volume velocity, (11) and (12), the analytical model for the eroded mass curve can be constructed:

$$m(t) = A1\left[a t - b t e^{-ct}\right], \qquad (13)$$

or of the corresponding erosion velocity:

$$v1(t) = \frac{\mathrm{d}m(t)}{\mathrm{d}t} = A1\left[a - b e^{-bt} + b c t e^{-bt}\right], \qquad (14)$$

Between the coefficients from the relations (11), (12) and (13), (14) the following link is established:

$$a = v1_{\mathrm{S}} = \rho V_{\mathrm{S}},$$

$$b = \rho\lambda,$$

$$c = \rho\beta, \qquad (15)$$

where ρ - material density.

The real parameters a , b and c are determined by relation (15) or from the application of relation (13) for the approximation of the experimental points.

4. APPLICATION

In the below applications $A1 = 1$ was considered.

4.1 Below, we show the massic loss obtained in the experimental tests of cavitational attack for OL 370 steel, realised in the vibrating device with nickel tube, from the Hydraulic Machinery Laboratory in Timisoara:

$$T := \begin{pmatrix} 0 \\ 5 \\ 15 \\ 30 \\ 45 \\ 60 \\ 75 \\ 90 \\ 105 \\ 120 \\ 135 \\ 150 \\ 165 \end{pmatrix} \qquad M := \begin{pmatrix} 0 \\ 0.00082 \\ 0.00245 \\ 0.01004 \\ 0.020702 \\ 0.03065 \\ 0.04190 \\ 0.05348 \\ 0.06391 \\ 0.0729 \\ 0.08502 \\ 0.0976 \\ 0.10662 \end{pmatrix}$$

Using the ofleast squares method, the real parameters *a*, *b* and *c*, determined from the approximation of the experimental points by the analytical curve described by the equation (13), for $A1 = 1$, are:

$a := 0.6571 \cdot 10^{-3}$ $b := 0.6709 \cdot 10^{-3}$ $c := 0.2551 \cdot 10^{-1}$.

In the below Fig. 2 the experimental data and the analytical approximation curve m(t) is presented for the cumulated eroded mass, and in Fig.3 the experimental points and the analytical approximation curve are given for the erosion velocity v1(t).

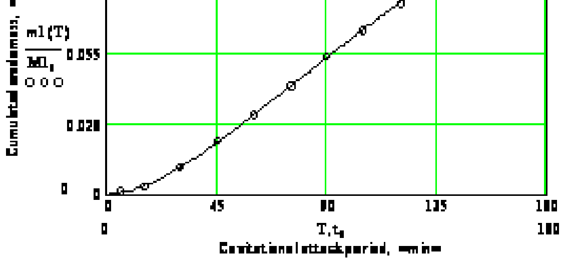

Figure 2. Eroded mass OL 370-3k

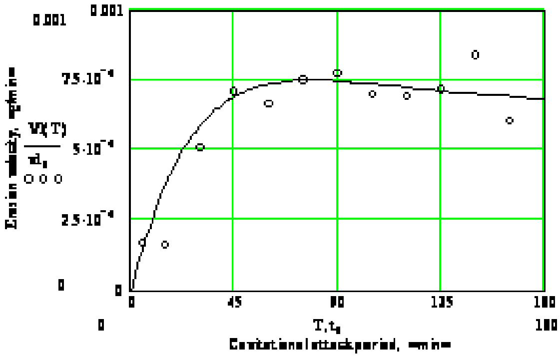

Figure3. Erosion velocity for OL 370-3k

From Fig.2 and 3 one can observe a uniform distribution of the experimental points against the approximation curves, which shows good practicability of the new model. Also, it is observed that the stabilization velocity to which the approximation curve $v1_s = a = 0.6571 \cdot 10^{-3}$ g/min which, compared with the average of the last four values of the experimental velocity $v_{sm} = 0.711 \cdot 10^{-3}$ g/min, presents an error of 7.5%, acceptable for the process such complex as the cavitational erosion is [1].

4.2 For the stainless steel 1H18N9T, tested in the vibrating device of the Cavitation Laboratory in Gdansk, the experiment data obtained in [5] are:

$$T := \begin{pmatrix} 0 \\ 5 \\ 15 \\ 30 \\ 45 \\ 60 \\ 75 \\ 90 \\ 105 \\ 120 \\ 135 \\ 150 \\ 165 \end{pmatrix} \quad M := \begin{pmatrix} 0 \\ 0.0003 \\ 0.00105 \\ 0.00237 \\ 0.00346 \\ 0.00444 \\ 0.00655 \\ 0.00762 \\ 0.00867 \\ 0.00970 \\ 0.010725 \\ 0.0117455 \\ 0.012765 \end{pmatrix}$$

Using again the ofleast squares method, the real parameters a , b and c determined from appliacation of relation (13) in approximation of the experimental points, considering that the eroded area is equal with unit, are:

$$a := 0.7979 \cdot 10^{-4} \quad b := 0.3185 \cdot 10^{-4} \quad c := 0.8268 \cdot 10^{-1}$$

In Fig.4 and 5 are shown the experimental points and the analytical approximation curves described with the proposed models for the cumulated mass $m(t)$ and the erosion velocity $v1(t)$.

Figure 4 also shows the capacity of the new model through the distribution of the experimental points of the massic losses compared with the approximation curve $m(t)$.

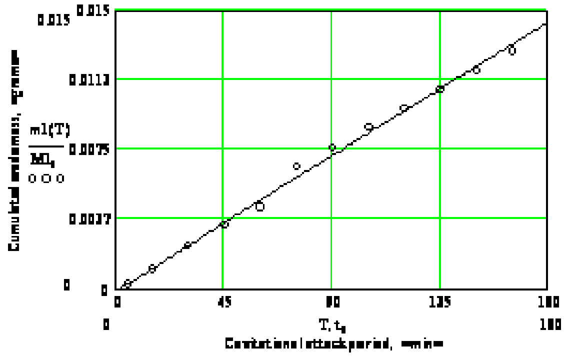

Figure 4. Eroded mass (1H18N9T -Gdansk)

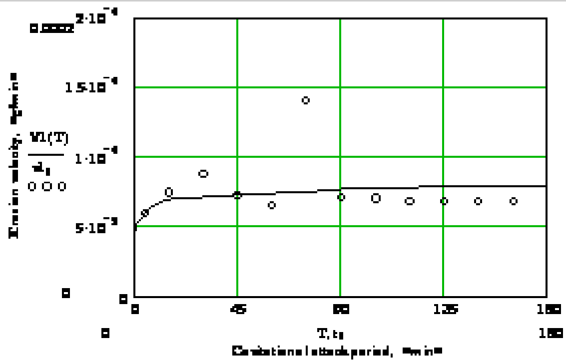

Figure 5. Erosion velocity (1H18N9T – Gdansk)

The approximation curve of the experimental velocities, Fig.5, presents a deviation in the stabilization of the erosion area, explainable due to the fact that the data for the stainless steel 1H18N9T, were worked from the diagram presented by Steller in [5]. This deviation is given by the jump of the velocity at 75 minute and from the fact that the values of the coefficients a, b and c were determined from the approximation of the experimental points of the cumulated eroded mass. Hence, on the bases of Fig.4, we appreciate that also for this material, the new model offers an acceptable approximation of the experimental points.

5. CONCLUSION

a. The proposed model offers, compared with the relations found in the references, the advantage of certain relations with unique coefficients for the approximation curves $m(t)$, respectively $v(t)$, no matter the cavitational attack period is, and also the direct obtaining of the value to which the erosion velocity tends to stabilize.

b. The used parameters in our formulae (11) and (12) appear naturally from the approximation of the experimental points. Also, they lead automatically to the $v1_s$ velocity of stabilization of the erosion.

c. It is not necessary to apriori define the number or parameter, like in the case of using the formulae (1) and (2).

d. The new relations are applicable for the cavitational eroded materials in vibrating devices which have high erosion intensity (in which the incubation period, in fact, does not exist).

e. The new model can be easily modified in order to be extended to the curves that have the incubation period specific to the eroded materials in the hydrodynamical tunnels or rotation disc devices.

REFERENCES

[1] Bordeaşu, I.: Cavitation erosion for materials used for hydraulic machinery runners and ship propellers - Scale effect, Doctoral thesis, Timişoara, 1997, pp.83-99 (in Rumanian),

[2] Hammitt, F.G.: *Cavitation and multiphase floow phenomena*, Mc Graw-Hill, New-York, 1980.

[3] Heymann, F.J.: On the time dependence of the rate of erosion due to impingement or cavitation, A symposium presented at the 69th Annual Meeting, ASTM, pp.70-82, Atlantic City, 1996.

[4] Noskievič, J.: *Some Aspects With regard to cavitation Resistance of Materials*, HIDROTURBO 76, Vol. C7, pp.1-16, Brno, 1976, (in Polish).

[5] Savelyev, I.V.: *Mechanics Molecular Physics*, Mir Publisher, Moskow, 1980.

[6] Steller, J., Kaczmarzyk, P.: Phenomenological Models of Cavitation Erosion Progress, International Cavitation Erosion Test Seminar, Sopot, pp.1-9, 1-2nd June 2000.

[7] Steller, J.: A concept of Fractional Cavitation Erosion Resistance, International Cavitation Erosion Test Deminar, Sopot, pp.F1-F10, 1-2nd June 2000.

[8] Steller, K.: A New Conception Upon the cavitation Erosion of materials Prace IMP, Gdansk, Vol.76, , pp.127-150, 1978, (in Polish).

[9] Silva W.P., Silva C.M.: LAB Fit Curve Fitting Software (Nonlinear Regression and Treatment of Data Program) V 7.2.29 (1999-2004)

[10] Thiruvengadam, A., Preiser, H.S.: On testing materials for cavitation damage resistance, Journal Ship., Res.8, pp. 233-243, 1964.