Mechanics of Human Locomotor System

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(Bio)mechanical models of human body are important tools in understanding the functional principles of human movement and coordination as well as they have widespread applications for the industrial, scientific and medical purposes. In this paper (bio)mechanical models of the upper human limb (arm, forearm and hand, 7 degree-of–freedoms (DOFs)), upper torso and right arm (15 DOFs) and of the leg with (2DOFs) are presented, where model of upper human limb is discussed in detail. Also, multi-chain (bio)mechanical model of a human body, anthropomorphic locomotion configuration, is introduced. At last, simulations in MATLAB enviroment are performed and the results of kinematical and dynamical model of an anthropomorphic arm (5 DOFs) in the task of writing are presented.

Keywords: *biomechanics, locomotion, musculoskeletal models.*

1. INTRODUCTION

Biological (human) systems have great capabilities, which attained perfection in their specializied functions through long evolutionary processes. Advances in biomechanical assessment in the last 20 years have been considerable, [1]. The applications of human motion analysis are limitless. Researchers in the fields of biomechanics, medicine, sports, and rehabilitation study human locomotion for evaluating joint forces and moments that control motion and posture, [1,2]. Such studies are fundamental to understanding the mechanics of normal and pathological movement and for the diagnosis and treatment of patients with motor deficiencies, [3]. Many of these systems are possible to emulate by "intelligent" devices which can be utilized for the industrial, scientific and medical purposes, for example: walking machines, prosth./orthotic devices, aids for improving human manual dexterity and sensory ability, systems for safe extension of the human domain into unreachable and hazardous environments.

Biomechanics uses laws of physics and engineering concepts to describe motion undergone by the various body segments and the forces acting on these body parts during normal daily activities, [4].The study of human locomotion can be described as the interdiscipline that describes, analyzes, and assesses human movement, [2,3].The study of the biomechanics of locomotion provides very extensive and interesting material for investigating the physiological processes and neural mechanisms involved in controlling the system and for designing different aids for the handicapped. Also, one goal in biomechanics is to obtain mathematical models for the dynamical properties of the skeleton as well as for the electro-chemical and cell-physiological processes of the force generation in the muscles, [5,6]. The musculoskeletal system provides support and generate movement, which allows us to interact with our enviroment. Moreover, musculoskeletal system is a mechanically orientated biological entity whose composition, function, and organization reflect the functional demands of the whole body. The skeleton, with its appropriate mechanical properties, can fulfill all its biomechanical and supportive functions, such as to protect internal organs, provide rigid kinematic links and muscle attachment sites, and facilitate muscle action and body movement. Structural support and movement can be described in terms of mechanical principles.

On the other hand, dynamic systems representing humans/animals are very complex and difficult to model, identify, and control due to inertia couplings, gravitation forces, visco-elastic effects, and highly nonlinear actuator characteristics, [7]. In general, for a simulation technique to be efficient, some optimal tradeoffs between efforts/time spent on full dynamic modeling, accurate system identification, and motion optimization have to be found.

The puprose of this paper is to provide fundamentals of mechanics of human locomotor system as it applies to the (musculo)skeletal system. Mechanical models of the upper limb,leg, upper-body, whole body are presented while a model of human arm for writing task has been developed and discussed in detail. At last, simulations in a MATLAB enviroment of a planar anthropomorphic arm (5 DOFs) in the task of writing are performed.

2. HUMAN MOTION ANALYSIS AND CONTROL: FUNDAMENTAL ISSUES

The original concept of the proposed framework can be understood by considering a simplified illustration of how the human neuromuscular system controls its motion (see Fig. 1), [8]. Motion planning is initiated in the central nervous system (CNS). The sensorimotor control system in our brain generates a sequence of

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neural activations that innervate the muscles (known as ElectroMyoGraphic–EMG activity), causing them to contract and generate the forces required to drive a skeletal system to a desired position. Sensory information such as muscle length, and skeleton motion parameters are fed back to the sensorimotor control and skeletal muscles.

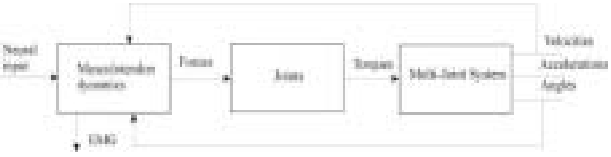

Figure 1. Natural flow of dynamic and kinematic events of the human neuromusculoskeletal system

Also, control theory offers ways to recognize optimal solutions although it cannot describe the exact form of the criteria governing the execution of functional motions.Trajectories of motions of mechanical systems also obey the general principle of minimum effort. One is thus tempted to conclude that minimum effort criterion governs the dynamics of both inorganic and organic systems, [3,9]. Instead of a variational calculus, value judgements are instrumental in the decision processes pertaining to large systems. Heuristics, creativity, domain oriented experience are the main approaches used by man in running multilevel optimizations.The review of analytical concepts used in control theory led to the conclusion that the low levels of sensory-motor control rely on the simplest but most general methods like look-up tables and functional relations.For nonanalyticical control finite automata have interesting properties which are relevant to the understanding of pattern generators in motor control, [10,11].

3. MODEL DESCRIPTION OF BIOMECHANICAL SYSTEM

3.1 Human kinematic modeling

In contrast to Denavit-Hartenberg's method (DH), the Rodrigueze method is proposed for modeling human biomechanics, kinematics and dynamics, [12]. In this study, the proposed method is used to describe a multi-segmental relationship in a serial chain(s) that leads,for example,from the body to the hand (upper limb) and closed chains,for instance, posture of the whole body. This is done by embedding coordinate systems at each segment in a systematic manner.A mechanical model of arm is considered as an open linkage consisting of $n+1$ rigid bodies interconnected by n one-degree-of–freedom (DOF) joints (Fig.2). The joints are modeled as kinematic pairs of V`s class and so arm has n degrees of freedom.

Moreover, if a model contains cinematic pair of fourth class and third class, they can be modeled in the following manner: two segments connected with cinematic pair of fourth class can be presented as open loop chain with three segments (one fictional and two real) and cinematic pairs of fifth class, [13]. Also, cinematic pair of third class can be presented as three cinematic pairs of fifth class and two fictional segments.

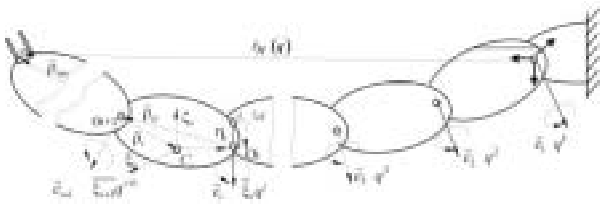

Figure 2. Define the position vector $\vec{r}_H$

Geometry of the system has been defined by unit vectors $\vec{e}_i$ as well as vectors $\vec{\rho}_i$ and $\vec{\rho}_{ii}$ expressed in local coordinate systems connected with bodies. For the entire determination of this mechanical system in the matter of dynamics, it is necessary to specify masses m_i and tensors of inertia J_{Ci} expressed in local coordinate systems, [14]. Configuration of the mechanical model can be defined by the vector joint of (internal) generalized coordinates q of dimension n, $\{q\}=\left(q^1,q^2,...,q^n\right)^T$. The vector of global (external) coordinates of dimension defines the position of the point of interest $m<n$, $m\le 6$, $\{\overline{q}\}=\left(q^1,q^2,...,q^m\right)^T$. The kinematic model of the kinematic chain is given by the following expression:

$$\overline{q}^i(t)=f^i(q^1,q^2,...,q^n),\ i=1,2,...,m\ . \quad (1)$$

Equation (1) is well known as the *direct kinematic problem* (calculation of $\overline{q}^i(t)$ for given $q(t)$) and has a unique solution. Also, Eq. (1) can be presented in the first and the second-order Jacobian forms:

$$\dot{\overline{q}}^i=\sum_{\alpha=1}^{n}\frac{\partial f^i}{\partial q^\alpha}\dot{q}^\alpha\Rightarrow\{\dot{\overline{q}}\}=[J]\{\dot{q}\}$$

$$\{\ddot{\overline{q}}\}=[J]\{\ddot{q}\}+[\dot{J}]\{\dot{q}\}=[J]\{\ddot{q}\}+[A(q,\dot{q})], \quad (2)$$

where $[J]$ is the Jacobian matrix of dimension $m\times n$ and $\left[A(q,\dot{q})\right]$ is the $m\times 1$ adjoint vector. However, an *inverse kinematics* (calculation of $q(t)$ for given $\overline{q}^i(t)$) has an infinite number of solutions since equation (1) represents a set of m equations with n variables if exists the redundancy. In that case, the dimension of ther redundancy is $n_r=n-m$. In other words, a mechanism is called kinetically redundant if it has more DOFs than required for the realization of a prescribed task in a task space. The main difficulty of redundant mechanisms is that the task cannot define the joint motions uniquely. From a mechanical point of view, any human or animal represents a redundant mechanism, [9].The main role of redundancy is to provide the flexibility of maneuvering space. In some tasks, the redundancy is not needed, but it exists and should be compensated, for [15].

The position vector of a point of interest $\vec{r}_H$ is written as a multiplication of matrices of trasformation $\left[A_{j-1,j}\right]$ and position vectors $\vec{\rho}_{ii}$ and $\xi_i q^i \vec{e}_i$ is expressed by:

$$\{\vec{r}_H(q)\} = \begin{Bmatrix} x_H(q) \\ y_H(q) \\ z_H(q) \end{Bmatrix} =$$

$$= \sum_{i=1}^{n} \left(\prod_{j=1}^{i} \left[A_{j-1,j}\right] \right) \left(\left\{ \rho_{ii}^{(i)} \right\} + \xi_i q^i \left\{ e_i^{(i)} \right\} \right), \tag{3}$$

where appropriate Rodrigo's matrices of transformation are, [13,14]:

$$\left[A_{j-1,j}\right] =$$

$$= [I] + \left[e_j^{d(j)}\right]^2 \left(1 - \cos q^j\right) + \left[e_j^{d(j)}\right] \sin\left(q^j\right) \tag{4}$$

and

$$\left\{e_j^{(j)}\right\} = \left(e_{\xi j}, e_{\eta j}, e_{\zeta j}\right)^T,$$

$$\left[e_j^{d(j)}\right] = \begin{bmatrix} 0 & -e_{\zeta j} & e_{\eta j} \\ e_{\zeta j} & 0 & -e_{\xi j} \\ -e_{\eta j} & e_{\xi j} & 0 \end{bmatrix}. \tag{5}$$

The parameter ξ_α, $\bar{\xi}_\alpha = 1 - \xi_\alpha$ is the parameter for recognizing the joint of link: $\xi_\alpha = 0$ if joint is revolute and $\xi_\alpha = 1$ if joint is prismatic. Also, angular velocities vectors of the rigid bodies $i, i = 1, 2, ..., n$ with respect to coordinate *i* (6) and linear velocities vectors of the mass centers of the rigid bodies with reference to coordinate *i*, (7) can be obtained as follows, [14]:

$$\vec{\omega}_i = \sum_{\alpha=1}^{i} \bar{\xi}_\alpha \vec{e}_\alpha \dot{q}^\alpha, \tag{6}$$

$$\vec{v}_{Ci} = \sum_{\beta=1}^{i} \frac{\partial \vec{r}_i}{\partial q^\beta} \dot{q}^\beta = \sum_{\beta=1}^{n} \vec{T}_{\beta(i)} \dot{q}^\beta, \tag{7}$$

$$\vec{T}_{\beta(i)} = \begin{cases} \bar{\xi}_\beta \vec{e}_\beta \times \vec{R}_{\beta(i)} + \xi_\beta \vec{e}_\beta, & \forall \beta \le i \\ 0, & \forall \beta > i \end{cases}, \tag{8a}$$

$$\vec{R}_{\beta(i)} = \sum_{\alpha=\beta}^{i} \left(\vec{\rho}_{\alpha\alpha} + \xi_\alpha \vec{e}_\alpha q^\alpha \right) + \vec{\rho}_i, \tag{8b}$$

where $\dot{q}^\alpha, \alpha = 1, 2, ..., n$ are generalized velocities.

3.2 Human dynamic modeling

It is shown that whatever theoretical approach we choose, dynamical model of considered mechanism will be always expressed in the same covariant form, [14,15].

$$\sum_{\alpha=1}^{n} a_{\alpha\gamma}(q) \ddot{q}^\alpha + \sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} \Gamma_{\alpha\beta,\gamma}(q) \dot{q}^\alpha \dot{q}^\beta = Q_\gamma^a + Q_\gamma^u, \quad \gamma = 1, 2, ..., n \tag{9}$$

where kinetic energy of mechanical model is given by:

$$E_k = \frac{1}{2} \sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} a_{\alpha\beta} \dot{q}^\alpha \dot{q}^\beta = \frac{1}{2} (\dot{q}) \left[a_{\alpha\beta}\right] \{\dot{q}\}, \quad \alpha, \beta = 1, 2, ..., n, \tag{10}$$

Coefficients $a_{\alpha\beta} = a_{\beta\alpha}$ of square form are covariant coordinates of basic metric tensor $\left[a_{\alpha\beta}\right] \in R^{n \times n}$ and $\Gamma_{\alpha\beta,\gamma} = \Gamma_{\beta\alpha,\gamma}$, $\alpha, \beta, \gamma = 1, 2, ..., n$ pres-ent Christoffel symbols of first kind defined as follows:

$$\Gamma_{\alpha\beta,\gamma} = \frac{1}{2} \left(\frac{\partial a_{\beta\gamma}}{\partial q^\alpha} + \frac{\partial a_{\gamma_\alpha}}{\partial q^\beta} - \frac{\partial a_{\alpha\beta}}{\partial q^\gamma} \right), \tag{11}$$

Also, Q_γ^u control part of generalized forces Q and Q_γ^a non-control part of Q are given such as, [14]:

$$Q_\gamma^a = \sum_{\gamma=1}^{n} \left(\vec{F}_{R(i)} \cdot \vec{T}_{\gamma(i)} + \vec{M}_{CiR(i)} \cdot \vec{\Omega}_{\gamma(i)} \right), \tag{12}$$

$$Q_\gamma^u = \left(\bar{\xi}_\gamma \vec{M}_\gamma + \vec{P}_\gamma \xi_\gamma \right) \cdot \vec{e}_\gamma, \quad \gamma = 1, 2, ..., n. \tag{13}$$

In prismatic joints forces are acting, while in revolute joints torques are acting. For some special problems and purposes (e.g the inverse task of dynamics) it might be necessary to transform equations in covariant form into the contravariant form, [14], [15]:

$$\ddot{q}^\alpha + \Gamma_{\alpha\beta}^\gamma(q) \dot{q}^\alpha \dot{q}^\beta = a^{\alpha\gamma} \left(Q_\gamma^a + Q_\gamma^u \right). \quad \alpha, \beta, \gamma = 1, 2, ..., n \tag{14}$$

Where a contravariant coordinates $a^{\alpha\gamma}$ of the basic metric tensor and Christoffel's symbols of the second kind $\Gamma_{\alpha\beta}^\gamma$ can be obtained from the following relations:

$$\Gamma_{\alpha\beta}^\gamma = a^{\gamma\delta} \Gamma_{\alpha\beta,\delta}, \quad \left[a^{\alpha\gamma}\right] = \left[a_{\alpha\gamma}\right]^{-1}. \tag{15}$$

Also, complex (closed) kinematic chains have additional considerations. A specification of complex kinematic chains is that there exists at least one mechanism link participating in more than two kinematic pairs. Such links are nominated *branching links,* (Fig.3). Additional equations are presented as follows, where $\vec{p} = \vec{e}_{r+1}, \vec{q} \perp \vec{p}, |\vec{p}| = 1, \vec{r} = \vec{p} \times \vec{q}$:

$$\left(\vec{p}_{(0)}\right) \left[A_{j,r}\right] \left\{\vec{p}_{(0)}\right\} = 1, \quad \left(\vec{p}_{(0)}\right) \left[A_{j,r}\right] \left\{\vec{q}_{(0)}\right\} = 0,$$

$$\left(\vec{p}_{(0)}\right) \sum_{k=j+1}^{r} \left[A_{j,k}\right] \left\{ \vec{\rho}_{kk}^{(k)} + \xi_k q^k \vec{e}_k^{(k)} \right\} + \left(\vec{p}_{(0)}\right) \left\{\vec{d}\right\} = 0,$$

$$\left(\vec{q}_{(0)}\right)\sum_{k=j+1}^{r}\left[A_{j,k}\right]\left\{\vec{\rho}_{kk}^{(k)}+\xi_k q^k \vec{e}_k^{(k)}\right\}+\left(\vec{q}_{(0)}\right)\left\{\vec{d}\right\}=0\,,$$

$$\left(\vec{r}_{(0)}\right)\sum_{k=j+1}^{r}\left[A_{j,k}\right]\left\{\vec{\rho}_{kk}^{(k)}+\xi_k q^k \vec{e}_k^{(k)}\right\}+\left(\vec{r}_{(0)}\right)\left\{\vec{d}\right\}=0\,. \quad (16)$$

Maximal number of independent equations is equal to 5.The model of anthropomorphic mechanism dynamics is obtained in the form, [14,16]:

$$\sum_{\alpha=1}^{n} a_{\alpha\gamma}(q)\ddot{q}^{\alpha}+\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\Gamma_{\alpha\beta,\gamma}(q)\dot{q}^{\alpha}\dot{q}^{\beta}=$$

$$=Q_{\gamma}+\sum_{\sigma=1}^{l}\lambda_{\sigma}b_{\gamma}^{\sigma},\ (\sigma=1,2,...,l),\ \gamma=1,2,...,n\,. \quad (17)$$

with additional constraints which follow from set a of additional independent equations (16):

$$\sum_{k=1}^{n} b_k^{\sigma}(q)\dot{q}^k=0\,. \quad (18)$$

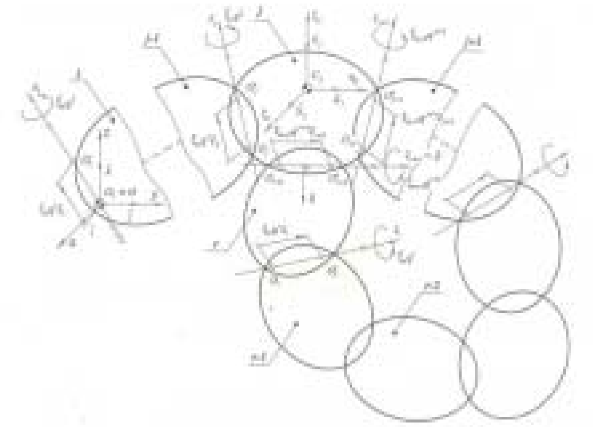

Figure 3. Mechanical model with one closed kinematic chain, [16]

4. (BIO)MECHANICAL MODEL OF HUMAN BODY

(Bio)mechanical models of human body are important tools in understanding the functional principles of human movement and coordination. From the beginning of the development, the aim was to create a system with the following characteristics. It should be a modeling system, i.e. a tool that allows users to construct models from scratch or use or modify existing models to suit different purposes. Also, the system should facilitate model exchange and allow models to be scrutinized.If possible, it should have sufficient numerical efficiency to allow ergonomic design optimization on inexpensive computers. At last, the system should be capable of handling body models with a realistic level of complexity such as the example of Figure 4, [17].

The multibody system approach is widely used for the analysis of the dynamic behavior of systems with large motions where the deformations of the bodies may be neglected. The bodies are connected by arbitrary ideal joints and/or force/torque elements. The bones are considered to be rigid and the joints are modeled as ideal joints. Both assumptions are justified simplifications since, for the investigated motions, the deformation of the bones may be certainly neglected and a more detailed description of the extremely complicated real behavior of human joints would only be reasonable within a very fine modeling context, also including detailed tissue effects. The forces and moments applied to the bones are created by 'actively controlled' muscles described by appropriate muscle models, (Fig.5). Many models of the whole muscle in the literature are based on the work of Hill [2,3].

Figure 4. A full body model comprising more than 300 individual muscles

These models represent muscle dynamics by a contractile component and nonlinear series and parallel viscoelastic elements. The force generated by the nonlinear contractile component depends on neural activation, and is also length and velocity dependent.

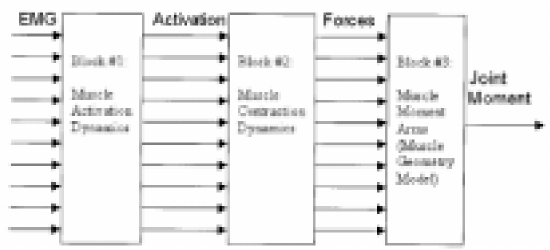

Figure 5. A block diagram of the transformation from EMG to joint movement

Three-dimensional simulation models have also received considerable attention. Many researchers have formulated mathematical expressions to define the dynamics of human motion. Dynamic analysis of the biomechanical model of locomotion is closely related to, and evolved with, design of the bipped robot [10, 11], and [18]. A general approach for studying a human body model using equations based on d`Alambert`s principle was proposed by Huston, [5]. A seven link planar model was developed by Onyshko and Winter, [6] where equations of motion were formulated using Lagrangian mechanics. Hatze [7] used the traditional Lagrangian approach to define a mathematical model of the total human musculoskeletal system. The model comprised a linked mechanical and musculo-mechanical set of ordinary first-order differential equations, which describe the dynamics of the segment model and muscle model respectively. Also, Marshall [19] used a general

Newtonian approach to simulate an N-segment open-chain model of human body. The model simulated planar movement using data for joint torques and initial absolute angular displacements and velocities for each body segment.

The mechanism locomotion of the whole body is presented as configuration with head frame and is given below (Fig. 6), [20]. The legs have to comprise at least 6 powered joints each, allowing to manipulate the feet in 6 DOF relative to torso. The mechanism is divided into three kinematic chains, and the modelling procedure is applied on each chain. During anthropomorphic walk, this chain switches from single-support (open) to double-support (closed chain) phase.

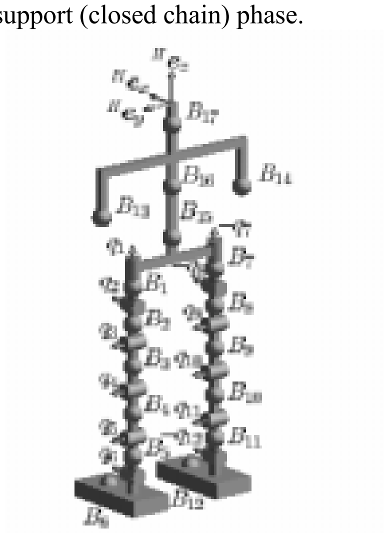

Figure 6. Kinematic scheme of mechanism locomotion of the whole body

4.1 (Bio)mechanical model of the leg

The dynamic equations of a planar two-link pendulum are found in many textbooks, for example [21]. The mechanical model of the controlled leg with two degrees of freedom was adopted as two rigid bodies connected with two hinge joints to a moving point namely the end of the thigh, allowing knee extension and flexion, and ankle plantar- and dorsi-flexion, (Fig.7a), [3].This chained structure is attached to the biological system,(Fig.7b). The model is given by:

$$A_1 \ddot{\phi}_S + A_2 \ddot{\phi}_T \cos(\phi_T - \phi_S) + A_3 \dot{\phi}^2{}_T \sin(\phi_S - \phi_T)$$
$$-A_4 \ddot{x}_{HT} \sin(\phi_S) - A_5 (\ddot{y}_H + g)\cos(\phi_S) -$$
$$-X_G L_S \sin\phi_S + Y_G L_S \cos\phi_S = M_S , \qquad (19)$$

$$B_1 \ddot{\phi}_T + B_2 \ddot{\phi}_S \cos(\phi_T - \phi_S) + B_3 \dot{\phi}^2{}_S \sin(\phi_T - \phi_S) -$$
$$-B_4 \ddot{x}_{HT} \sin(\phi_T) - B_5 (\ddot{y}_H + g)\cos(\phi_T) -$$
$$-X_G L_T \sin\phi_T + Y_G L_T \cos\phi_T = M_T . \qquad (20)$$

Parameters A_i, B_i, $i = 1,2,3,4,5$ are determined by geometrical and inertial properties of the leg segments, [3].

Also, humans and animals use their legs to move with great mobility, but we do not yet have a full understanding of how they do so. Bipedal walking, an example of a basic human motion, might be largely understood as a passive mechanical process. McGeer [22] demonstrated by both computer simulation and physical-model construction that some anthropomorphic legged mechanisms can exhibit stable, human-like walking on a range of shallow slopes with no actuation and no control (energy lost in friction and collisions is recovered from gravity).Three walking phases are usually considered to make up the pattern of the lower-limbs: doble support, swing and contact phase, (Fig.8), [23].

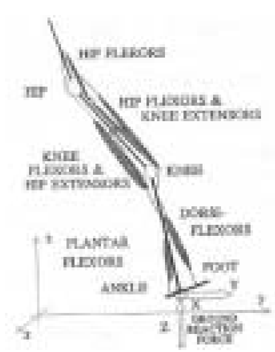

Figure 7a. Model of the two-link musculoskeleton

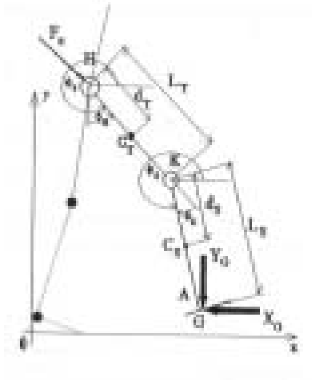

Figure 7b. Model of the two-link skeleton

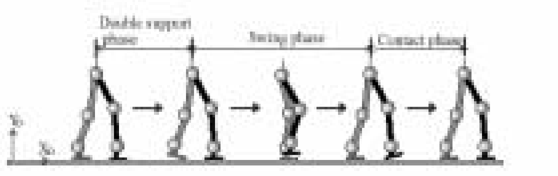

Figure 8. Three walking phases

Also, it is very important to know what are the independent state variables describing best the dynamic performance in a specific motion task. For example, during human locomotion, the set of the generalized coordinates has to be changed four times when performing a step to describe the biped motion during each phase: double-support, taking-off, single-support, and landing. In this case, the dynamics of human locomotion changes its structure at least four times, and, accordingly, a control system should have different structures.

4.2 (Bio)mechanical model of the upper human limb

Musckuloskeletal model of the human arm incorporates the clavicle, the scapula, the arm, the forearm and the hand which are connected together by 34 muscles, [2,4]. The articulation of the wrist is simulated in the model but the hand is treated as a single rigid-body. Although the scapula, the clavicle and the torax have been included in the model, they are treated as ground and arfe used only to establish the locations of the arm and forearm muscles that originate in these segments. The overall configuration is established by defining 14 angles which describe the motions of all the segments.

Human hand is an intricate and complex system capable of a multitude of sensory and actuation functions.A comprehensive three-dimensional model is given as folows: except for the thumb, which has only two phalanges, all the fingers are modeled by three phalanges and the entire hand is modeled as a collection of twenty-one individual segments. The eight carpal or wrist bones can be lumped together as a single segment articulating with the five metacarpals distally and the ulma-radius proximally, [4]. The finger has been considered as an open chain of rigid bodies (three phalanxes and the metacarpal) connected through different joints which characterize the kinematic behaviour of chain.Distal interphalangeal (DIP) and proximal interphalangeal (PIP) joints connect distal to medial phalanx and medial to proximal phalanx, respectively. All major muscles of the fingers are considered and are modeled by their individual lines of action. Approximately 20 degrees of freedom for the hand alone and 7 for the arm and wrist.Since the primary mechanical function of the hand is its ability to grasp and pinch different types of objects, not all the degrees of freedom are necesarry for every anthropomorphic hand. Consequently, the minimum number of degrees of freedom should be determined for each application. A representative example is the human finger that has four DOFs where each joint can move separately. But, in practical operation the tip joints work together forming one-DOF subsystem, so in many manipulation problems the complete hand behaves as a three-DOFs system.The following modes are possible: *pinch grip*, *lateral grip* and *key grip*, [3,4].

Biomechanical model is obtained using following the assumptions as follows: Bones and the soft tissues belonging to them are rigid bodies and their centre of gravity is fixed at a point, unchangeable in the process of motion; the joints are ideal kinematic joints (without friction) with strictly fixed axes or centres of rotation.Moving from an anatomical object to a kinematic scheme, the upper human limb is juxtaposed to a kinematic chain by the number of rigid bodies connected with rotational pairs. The modeling of the muscle force direction by 'segments' straight and curvilinear sections can be applied for the muscle force computation under quasistatic and dynamic conditions, [2, 3].

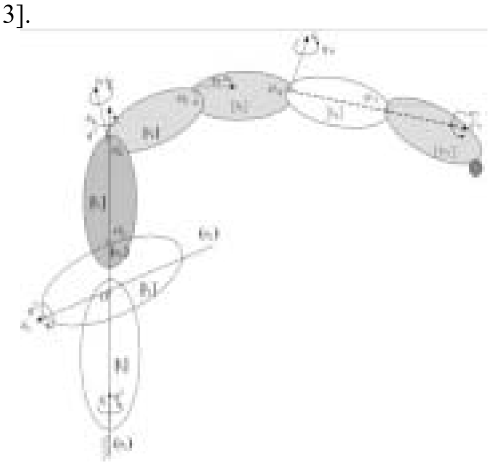

Figure 9. Upper limb model as open linkage with 7 DOF-s joints

Mechanical model of the upper limb consists of three rigid bodies (arm, forearm and hand) connected in between and with an immobile body (trunk) by the rotational kinematic pairs: shoulder joint-a spherical joint which has three degrees of freedom,the elbow joint- two pin joints which have one degree of freedom each, wrist joint- a Hook's joint which has two degrees of freedom. The corresponding upper limb kinematic model can be obtained as a kinematic chain with 7 DOFs (see Fig. 9)

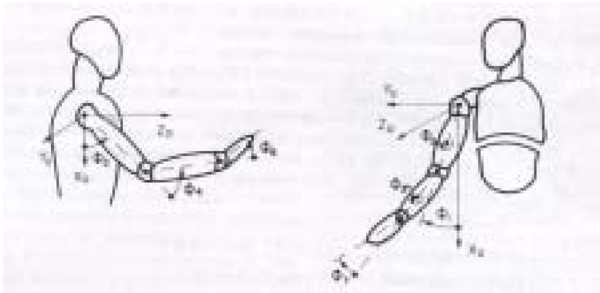

Figure 10. The general coordinates q_i associated with the terms for motions in the gross joints accepted in anatomy

General coordinates q_i can be associated with the terms Φ_i for motions in the gross joints accepted in anatomy by choosing an appropriate initial coordinates system and initial values and directions of the angles q_i, [24]. In Fig. 10 one can observe: Φ_1, Φ_2, Φ_3, -abduction, flexion and external rotation, respectively, in the shoulder joint; Φ_4 and Φ_5 -flexion and supination in the elbow joint;and Φ_6 and Φ_7 -flexion and abduction in the wrist joint;

$$q_1 = \Phi_1,\ q_2 = \Phi_2 + 90^\circ,\ q_3 = 360^\circ - \Phi_3$$
$$q_4 = 180^\circ - \Phi_4,\ q_5 = \Phi_5,\ q_6 = \Phi_6 + 90^\circ$$
$$q_7 = 360^\circ - \Phi_7 . \quad (21)$$

Another model of the upper torso and right arm is extended to a 15 DOFs model, [25] is shown in Fig. 11. Each joint is represented by one or more DOFs. For instance, the shoulder has five DOFs: three rotational joints and two translational joints, (DH method). The joint limits based on the experiments on three human subjects are:

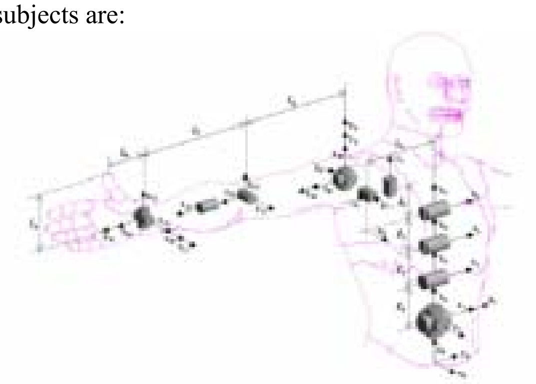

Figure 11. Modeling of the torso-right shoulder arm

$$-\pi/6 \le q_1 \le \pi/6, \quad -\pi/12 \le q_2 \le \pi/12,$$
$$-\pi/18 \le q_3 \le \pi/6, \quad -\pi/18 \le q_4 \le \pi/6,$$
$$-\pi/6 \le q_5 \le \pi/6, \quad -\pi/18 \le q_6 \le \pi/6,$$
$$-3.81 \le q_7 \le 3.81, \quad -3.81 \le q_8 \le 3.81,$$
$$-\pi/2 \le q_9 \le \pi/2, \quad -2\pi/3 \le q_{10} \le 11\pi/18,$$
$$-\pi/3 \le q_{11} \le 2\pi/3, \quad -5\pi/6 \le q_{12} \le 0,$$
$$-\pi \le q_{13} \le 0, \quad -\pi/3 \le q_{14} \le \pi/3,$$
$$-\pi/9 \le q_{15} \le \pi/9. \qquad (22)$$

5. MODEL OF THE ARM-HAND COMPLEX IN WRITING

Human arm movements are considered to be stable, fast and accurate. As we know, writing is coordinated multijoint movement and it is performed by a highly redundant system. Thus, redundancy and coordination are the two sides of the same activity. The overall number of degrees of freedom (DOF) for the hand is 19, and the arm adds another 7 DOFs. For the writing task, the complex model of the arm-hand system (26 DOFs) can be reduced to only 9 DOFs (3 for shoulder, 2 for elbow, 2 for wrist and only 2 for fingers), as shown in [26]. This problem can be reduced by assuming that one shoulder DOF is not active in writing, [26], Fig. 12.

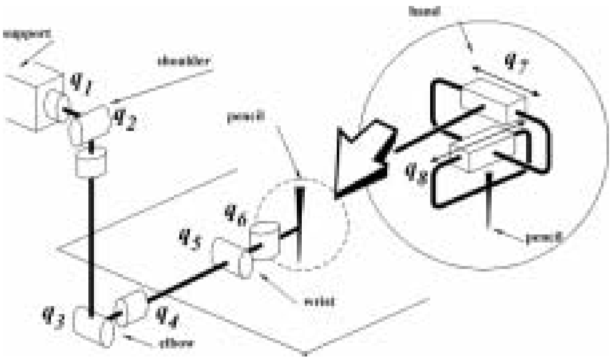

Figure 12. Eight –DOF arm-hand complex

In humans, the high-inertia arm joints (shoulder and elbow) provide the smooth global motion, and the low-inertia hand joints (fingers) perform the fast and precise local motion. The supplementary action of these motions should maintain effective, fast, legible, and nonfatigued writing. The redundancy and DP concept (distributed positioning) [26,27] could be used for solving the trajectory that has problems with increased dynamic requirements. The concept of DP allows us to separate the smooth and accelerated components of required motions by applying appropriate smoothing technique. However, some problems are involved. Fast and precise demands for finger motions are rather in conflict for both humans and robotic systems. As a result, writing legibility problems arise. In humans, these problems occur due to fatigue appearance. Further problems appear when writing some letter like d, t, l, j, g (i.e., whenever the maximal allowed fingers extensions are less than actually needed). This calls for more involvement of the shoulder and the elbow. These high inertia joints have to move faster and assist the fingers, [27, 28]. As a consequence, the energy consumption and muscle fatigue are increased, and the level of legibility is reduced, [29]. With humans, the accelerated motion of the complete arm is not optimal, but it is still workable.

This problem is solved here by following the biological analog DP concept and optimization procedure, [30]. Applying DP concept on kinematic and dynamic model of arm-hand complex, one can obtain separate generalized forces in two parts: first part corresponding to smooth motion and these forces are known, and second part of forces due to accelerated ones are not known, [31]. Minimizing kinematic criterion, that is, function of redundant set of joint accelerations, it is possible to obtain an unknown part of vector-generalized forces and vector of redundancy.

6. EXAMPLE

Consider a planar mechanism with $n = 5$ degrees of freedom (DOFs), [27], [30] performing writing (Fig. 13). Such an arm has three revolute and two prismatic joints. The revolute joints are: shoulder, elbow and wrist. Two prismatic joints replace the fingers (in writing). For this task, the dimensions of the arm-hand complex are $l_1 = 0.2$ m, $l_2 = 0.25$ m, $l_3 = 0.12$ m and motions of "fingers" (translation's q_4, q_5) are within the corresponding range. The mechanism has $n = 5$ DOFs and accordingly the internal position is defined by means of $n = 5$ joint coordinates: $q = \left[q_1\, q_2\, q_3\, q_4\, q_5 \right]^T$. The external position understands $n_e = 2$ coordinates: $X = \left[xy \right]^T$.

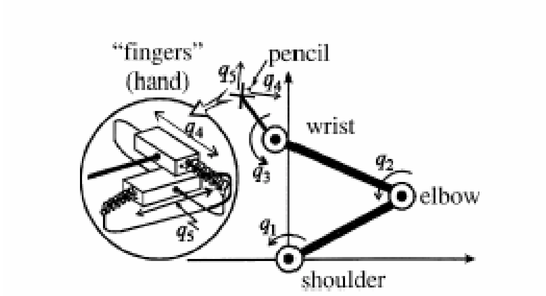

Figure 13. Configuration of the writing mechanism: five-DOFs planar arm

The system starts from the rest position defined by $q(0) = (\pi/4,\ \pi/2,\ 0,\ 0,\ 0)$. The redundant mechanism ($n = 5$ DOF) is now separated into two subsystems. The subsystem with ($n_e = 2$ DOF), with greatest inertia, is called *the basic configuration.* The other subsystem is *the redundancy* having (n_r=3 DOF). It holds that $n = n_e + n_r$. The basic nonredundant configuration consists of the shoulder and the elbow, and redundancy involves the wrist and the "fingers". The internal coordinates vector is now separated into two subvectors: $q = [q_b\ q_r]^T$ where n_e- dimensional q_b corresponds to the basic configuration and n_r - dimensional q_r corresponds to the redundancy. The DP concept solves the inverse kinematics of kinematic mechanism in two steps. At the first step the motion of basic configuration is calculated (q_b) from kinematical model for $q_r = \text{const}$ and at the second step the motion of redundancy (q_r) may be determined, [27, 30]. The task is writing a desired letter m which is shown in Fig.14 where it is also shown the same letter (*) which is obtained using only basic configuration of proposed mechanism.

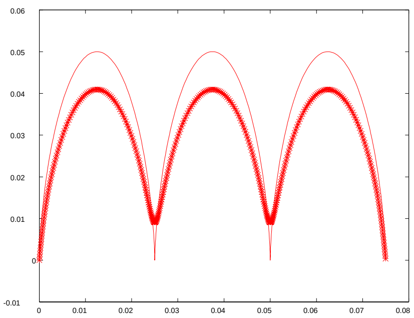

Figure 14. Prescribed letter *m*

Also, motion $q_b(t)$ will be used forward for obtaining vector $Q_b(t)$ where one can see that dynamical model can be represented in a condensed form:

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = Q\,, \tag{23}$$

where the control vector Q can be expressed as a sum of two controls subvectors i.e., [30]

$$Q(t) = Q_b(t) + Q_r(t) \tag{24}$$

and:

$$M_b\ddot{q}_b + M_r\ddot{q}_r + C(q,\dot{q}) + G(q) = Q\,, \tag{25}$$

Here, Q_r denotes a part of vector of generalized forces Q, further "redundant" control vector. Also, Q_b denotes a second subvector of generalized forces, further "basic" control vector corresponding to motions of basic configuration. It can be determined for the case where $q_r = \text{const}$ e.g. one can get *basic dynamical model*:

$$M_b\left(q_b, q_r = \text{const}\right)\ddot{q}_b + C(q_b,\dot{q}_b,q_r = \text{const},\dot{q}_r = 0) + \\ +G(q_b,q_r = \text{const}) = Q_b\;. \tag{26}$$

Substituting $\ddot{q}_b(t)$ in (26) one can obtain the expression for $Q_b(t)$:

$$M_b(q_b,q_r = \text{const})\ddot{q}_b + C(q_b,\dot{q}_b,q_r = \text{const},\dot{q}_r = 0) + \\ +G(q_b,q_r = \text{const}) = Q_b \tag{27}$$

or in a condensed form:

$$M_b^*\ddot{q}_b + Q_0^* = Q_b\,, \tag{28}$$

where basic dynamical model is completely obtained. Separation of control vector into two components is justified and it is based on the fact that in that way it is possible to realize a clear insight into dynamical coupling between segments of chain structure. Minimizing criterion of optimality that is function of $\ddot{q}_r$ in respect to Q_r, one can optimize involving Q_r and q_r. So, one can realize the tendency to obtain a control vector with less possible participation in a proposed model. It offers possibility of taking actuators with fewer dimensions, and consequently, it ensures minimal modification of inertial properties of a given model of arm-hand complex. On a low tactical level (dynamical level) of hierarchical control, a quadratic criterion of optimality is suggested in the following way:

$$I_l = \frac{1}{2}\ddot{q}_r W \ddot{q}_r \to \underset{Q_r}{\text{ext}\,(\text{min})} \tag{29}$$

After replacement the vector Q_b in dynamical model (25) one can obtain the relation between $\ddot{q}_r$ and Q_r such as:

$$M_b\ddot{q}_b + M_r\ddot{q}_r + \underbrace{C(q,\dot{q}) + G(q)}_{Q_o} = Q_b + Q_r \Rightarrow$$

$$M_r\ddot{q}_r = \underbrace{\left[M_b^* - M_b\right]\ddot{q}_b + \left[Q_o^* - Q_o\right]}_{\bar{Q}_o} + Q_r = \bar{Q}_o + Q_r\,, \tag{30}$$

$$\ddot{q}_r = M_r^{\#}\left(\bar{Q}_o + Q_r\right), \tag{31}$$

where $M_r^{\#} = \left(M_r^T M_r\right)^{-1} M_r$ represents pseudoinverse matrix M_r. Taking into account $W = M_r^T M_r$ with kinematic constraint

$$\ddot{r} - A - JM^{-1}M_b\ddot{q}_b = JM^{-1}\left(Q_r + \bar{Q}_0\right) \tag{32}$$

one can obtain augmented criterion I_L. Necessary conditions of optimality are:

$$\frac{\partial I_L}{\partial Q_r} = 0 \qquad \frac{\partial I_L}{\partial \lambda} = 0 \tag{33}$$

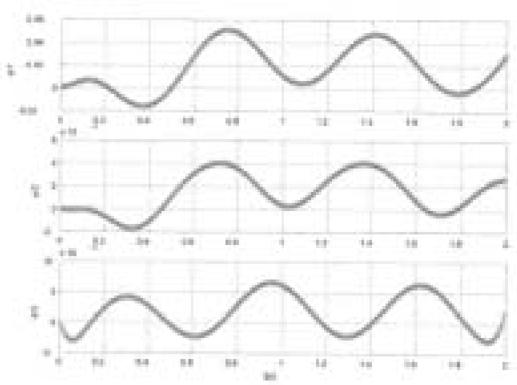

Figure 15. Time histories of redundant joints

It yields:

$$Q_{ropt} = -\bar{Q}_0 + J_{M^{-1}}^{\#}\left\{\ddot{r} - A - JM^{-1}M_b\ddot{q}_b\right\}, \tag{34}$$

where $J_{M^{-1}}^{\#}$ is pseudoinverse of JM^{-1} i.e.:

$$J_{M^{-1}}^{\#} = \left(JM^{-1}\right)^T\left\{\left(JM^{-1}\right)\left(JM^{-1}\right)^T\right\}^{-1}. \tag{35}$$

and

$$\ddot{q}_{ropt} = M_r^{\#}\left(\bar{Q}_0 + Q_{ropt}\right). \tag{36}$$

Using known values $q_r(0)$ and $\dot{q}_r(0)$ one can obtain by applying the procedure of numerical integration, $q_r(t)$, time histories of vector redundancy which are shown in Fig. 15.

5. CONCLUSION

This paper givens fundamentals of mechanics of human locomotor system which is applied to the (musculo)skeletal system. (Bio)mechanical models of the upper limb,leg, upper-body, whole body are introduced and presented. The Rodrigueze method is proposed for modeling human biomechanics, kinematics and dynamics in contrast to Denavit-Hartenberg's method (DH). The proposed method is used to describe a multi-segmental relationship in a serial chain(s) that leads, for example,from the body to the hand (upper limb) and closed chains, for instance, posture of the whole body. First, multi-chain (bio)mechanical model of a human body, anthropomorphic locomotion configuration, is introduced. Then, the mechanical model of the controlled leg with two degrees of freedom was adopted as two rigid bodies connected with two hinge joints to a moving point. Also, a model of the upper torso and right arm extended to a 15 DOFs model is introduced with corresponding joint limits. At last, a model of arm-hand complex for the writing task has been developed and discussed in detail. Also, simulations in MATLAB enviroment of a planar anthropomorphic arm (5DOFs) in the task of writing are performed.

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МЕХАНИКА ЛОКОМОТОРНОГ СИСТЕМА ЧОВЕКА

Михаило Лазаревић

(Био)механички модели људског тела су важна оруђа у разумевању основних принципа човековог покрета и координације, при чему,истовремено модели имају широку примену за индустријске, научне и медицинске сврхе. У овом раду су представљени и разматрани (био)механички модели људске руке (7 СС), горњег дела тела и десне руке (15 СС) и ноге (2 СС).Такође је приказан један (био)механички модел целог људског тела.На крају је спроведена симулација раванског механичког модела руке (5СС) у задатку писања у МАТЛАБ окружењу.