Circles and Ellipses Fitting to Measured Data

Miloš Glavonjić

Associate Professor

University of Belgrade
Faculty of Mechanical Engineering

Algebraic and geometric fitting methods for circle and ellipse to measured data were studied and implemented in a low-cost but functional laboratory measuring system for standardized circular tests for numerically controlled machine tools. The measuring device was made using the existing measuring components for registration of small displacement. The system was verified by testing one horizontal machining center prototype. Using the developed programs, some useful results were obtained in error sources analysis of some characteristic trace patterns of motion. The used ellipse fitting methods were mutually compared and with some of the other existing ones. For circular tests of numerically controlled machine tools both of the developed algebraic and geometric fitting of circles and ellipses methods have the same efficiency. In this paper, the ellipse fitting was analyzed in details because the circle fitting is standardized for machine tools testing.

Keywords: curve fitting, circular test.

1. INTRODUCTION

Fitting a circle or an ellipse to a set of noisy data points is an old problem that has motivated a large amount of often-duplicated literature in various fields [8]. In general, the fitting of geometric features to given 2D/3D points is desired in various fields of science and engineering: astronomy, physics, biology, quality control, image processing, metrology [1] etc. The measurement of motion errors of CNC machine tools in standard circular tests for numerically controlled machine tools are herein examined [6], [11]. Measuring motion errors of CNC machine tools in this way is more accurate and faster than the conventional one that requires the circular plate cutting on the machine to measure. Standard [11] does not regulate measuring resources, but only experimental data acquisition and processing methods. A long time ago, two types of measuring instruments for testing coordinate measuring machines and machine tools were developed [2]. The first version, the Fixed Magnetic Ball Bar (FMBB) was used to test the accuracy of a manually actuated, low friction coordinate measuring machine as well as computer controlled point-to-point measuring machines. The second version, the Telescoping Magnetic Ball Bar (TMBB) was used to test the accuracy of numerically controlled lathes, milling machines, robots and servo driven contouring measuring machines. It was the beginning of development of circular tests for numerically controlled machine tools. Then, motion errors of various NC machine tools were actually measured [6].

The system that uses standard disc was developed later [7]. The standard disc is measured in different positions in the working area of the machine. These developed techniques for mapping the volumetric positioning error of machine tools were time and labor intensive. To address these shortcomings, a linear displacement measuring device was introduced to rapidly and easily determine tool positions via trilateration [14]. It was the laser ball bar (LBB), a new instrument for machine tool metrology. The LBB consists of a laser interferometer aligned within a telescoping ball bar. Another system, based on the laser, was a noncontact laser technique for circular contouring accuracy measurement, especially for high-speed machine tools, to deliver contouring accuracy in the order of a few micrometers, while moving at relatively high-feed rates [12]. This technique is based on a single-aperture laser Doppler displacement meter with a flat mirror as the target. Using this instrument, the actual radius, feed rate, velocity, and acceleration profiles can also be determined. Today there exist a couple of industrial systems used in circular tests for CNC machine tools. There are also many resources for low cost but functional measuring system prototype development for the machine tools testing in laboratory conditions. In that way a Double Rotational Joints Telescoping Bar (DRJTB) for the machine tools testing in a single coordinate plane only is manufactured. However, mathematical software, particularly curve fitting routines, is a critical component of one DRJTB system. For standardized circular test, a circle fitting to measured data is quite enough [11]. The standard specifies methods of testing and evaluating the bi-directional circular deviation, the mean bi-directional radial deviation, the circular deviation and the radial deviation of circular paths produced by the contouring performance of a numerically controlled machine tool. It leads to finding the centre of least squares circle of the two actual paths, one in clockwise (CW), and the

other in anticlockwise (counter-clockwise CCW) contouring. All possible error origins of CNC machine tools were classified into several groups based on the motion error traces obtained by a number of circular tests [6], [11]. Ellipses fitted to measured data in circular tests are also used in that analysis. However, the method for fitting an ellipse to measured data is not explicitly defined. Only the inclination of the main axis of the identified ellipse is often used. Characteristic trace patterns of motion errors origins are usually formed. Care must be taken at that time since some error origins have similar trace patterns. Possible origins of motion errors could be position dependent errors and feed motion dependent errors. Geometric error of guide ways and error of positioning system are two typical position dependent errors [6] etc. So, the measuring system for standardized circular tests for numerically controlled machine tools incorporates four core modules: a measuring device, a data acquisition system, a software for circle (and ellipse) fitting to measured data with the presentation of the results, and analysis of achieved results by using different characteristic trace patterns of motion and other errors origins. The present paper focuses on the development of one low- cost measuring device and on the ellipse (and circle) fitting to measured data only. The paper is organized as follows: Section 2 describes fitting algorithms and developed programs for circle and ellipse fitting to measured data from circular tests of machine tools. Section 3 describes the developed measuring device and some of the experiments made for verification of the developed programs for circle and ellipse fitting in circular tests for CNC machine tools and completely developed system.

2. METHODS FOR CIRCLE AND ELLIPSE FITTING AND DEVELOPED PROGRAMS

Essentially, two classes of methods have been applied. One class has been referred to as algebraic fitting, where the implicit form of the conic section is used and the residual is minimized [4], [10], [13]. This gives rise to linear least squares problems subject to constraints and these can be usually solved efficiently as eigen value or singular value problems. The other is geometric fitting, where the errors in the data are minimized. Here, the implicit form of the conic may be used and the problem posed as a constrained least squares problem in the pa- rameters. The minimum error methods are all variants of the more general technique known as orthogonal dis- tance regression.

Data from the circular tests of CNC machine tools and measuring device are characterized by:

- Many measured positions, usually a couple of thou- sand.
- Actual path is usually a closed path in at least one contouring direction, CW, or CCW.
- Actual paths are the type of the circle and/or ellipse by nature of the experiment – originated from cir- cular interpolation made on CNC machine tool dur- ing the circular tests.
- Measured data are not so noisy when measuring system and tested machine tool are stable.

- The reference dimensions of the measuring device are usually known.

Because of that, the problem of circle and ellipse fit- ting to measuring data obtained during circular tests of CNC machine tools becomes simplified.

2.1 Circle fitting and developed programs

A model for circle fitting to experimental data is shown in Figure 1. Base coordinate frame is $O(X, Y, Z)$. The circle is centered to its center $C(x_C, y_C, z_C = 0)$ whose position vector is $\vec{r}_{C,O} = [x_C, y_C, z_C = 0]^T$. Coordinate frame $O'(X', Y', Z')$ is connected to the circle. The position vector of one point P on the circle is defined in frame O as $\vec{r}_{P,O} = [x, y, z = 0]^T$ and in frame O' as $\vec{r}_{P,O'} = [x', y', z' = 0]^T$. Position vector of one measured point E is known in frame O only and given as $\vec{r}_{E,O} = [x_i, y_i, z_i = 0]^T$, $i = 1, ..., N$, where N is the number of measured points on the actual contouring paths in one circular test. Coordinate frames O and O' are always mutually parallel due to circle's nature. Radius of the circle is R. Vector $\vec{n}$ is a unit vector of the main normal to the circle at the point P. The distance of the point E to the circle is d_i.
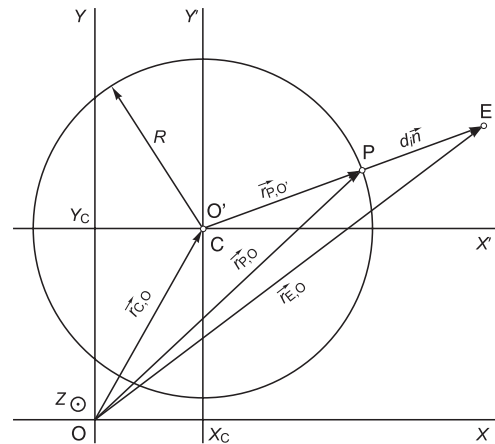

Figure 1. A model of geometric circle fitting

As there are two main categories of fitting problem for geometric features, algebraic and geometric fitting, these are differentiated by their respective definition of the error distances involved [1]. By algebraic fitting, the circle is described by implicit equation $f_C(\vec{p}_o, \vec{r}_{P,O})$, where $\vec{p}_o = [a, b = 0, c = a, d, e, f]^T$ is the circle parameters vector, i.e:

$$f_{C1}(\vec{p}_o, \vec{r}_{P,O}) = ax^2 + ay^2 + dx + ey + f = 0 \quad (1)$$

The equation of the circle is also

$$f_{C2}(\vec{p}, \vec{r}_{P,O}) = (x - x_C)^2 + (y - y_C)^2 - R^2 = 0 \quad (2)$$

where it is now a circle parameters vec- tor $\vec{p} = [x_C, y_C, R]^T$. Algebraic distance of measured point E, shown in Figure 1 could be expressed using (1) as

$$e_i = f_{C1}(\vec{p}_o, \vec{r}_{E,O}) = ax_i^2 + ay_i^2 + dx_i + ey_i + f \quad (3)$$

or, using (2) and circle parameters vector $\vec{p}$ as

$$e_i = f_{C2}(\vec{p}, \vec{r}_{E,O}) = (x_i - x_c)^2 + (y_i - y_c)^2 - R^2 \quad (4)$$

In this paper, the algebraic circle fitting is concerned with the sum of squares S_{CA} of ε_i given in (4):

$$S_{CA} = \sum_{i=1}^{i=N} [(x_i - x_C)^2 + (y_i - y_C)^2 - R^2]^2 \quad (5)$$

It is necessary to find circle parameters vector $\vec{p}$ such that S_{CA} is minimal. So the achieved three equations are as follows:

$$f_1 = \frac{\partial S_{CA}}{\partial x_c} = 0; f_2 = \frac{\partial S_{CA}}{\partial y_c} = 0; f_3 = \frac{\partial S_{CA}}{\partial R} = 0 \quad (6)$$

For that manner, the program FCA was developed for algebraic circle fitting, using the IMSL® routine *neqnj* for solving nonlinear equations by applying modified Powel hybrid algorithm with a user supplied Jacobian. The program FCA also presents the results according to the standard ISO 230-4 [11]. In the program, there are two routines implemented: *fcnc* for calculating functions given in (6) and *fjacc* for calculating Jacobian matrix $[J]$ as follows:

$$[J] = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_C} & \dfrac{\partial f_1}{\partial y_C} & \dfrac{\partial f_1}{\partial R} \\[2mm] \dfrac{\partial f_2}{\partial x_C} & \dfrac{\partial f_2}{\partial y_C} & \dfrac{\partial f_2}{\partial R} \\[2mm] \dfrac{\partial f_3}{\partial x_C} & \dfrac{\partial f_3}{\partial y_C} & \dfrac{\partial f_3}{\partial R} \end{bmatrix} \quad (7)$$

For geometric circle fitting the distance equation

$$d_i = f_{C3}(\vec{p}, \vec{r}_{E,O}) = \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2} - R \quad (8)$$

was used. Sum of squares S_{SG} S_{CG} of geometric distances d_i given in (8) is

$$S_{CG} = \sum_{i=1}^{i=N} d_i^2 \quad (9)$$

It is necessary to find circle parameters vector $\vec{p}$ such that S_{CG} is minimal. For that purpose, the program FCG was developed for geometric fitting the circle by using simplex method and Nelder-Mead's algorithm [9] .

2.2 Ellipse fitting and programs developed for it

A model for ellipse fitting to experimental data is shown in Figure 2. The same notation for coordinate frames and related vectors as for circle fitting is used.

Parameters vector of the ellipse is now $\vec{p} = [p_x, p_y, \gamma, x_C, y_C]^T$. Coordinate frame $O''(X'', Y'', Z'')$ is an intermediate one which has the same orientation as frame O', but whose origin is coincident with the origin of frame O. The equation of an ellipse in frame O' is
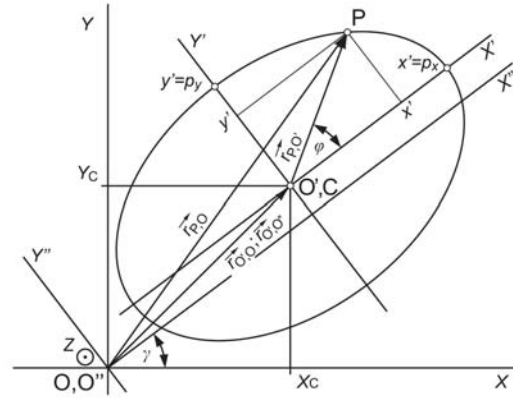

Figure 2. An ellipse centered at C, with axis lengths p_x and p_y and angle γ

$$f_{E1}(\vec{p}, \vec{r}_{P,O'}) = p_y^2 x'^2 + p_x^2 y'^2 - p_x^2 p_y^2 = 0 \quad (10)$$

In general, experimental point E is not on the ellipse, as shown in Figure 3. These points are known in frame O only. It is necessary to determine vectors $\vec{r}_{E,O'} = [x_i', y_i', z_i' = 0]^T$ and $\vec{r}_{P,O'} = [x', y', z' = 0]^T$ to enable the using of function given in (10). For example, $\vec{r}_{E,O'} = [M]\vec{r}_{E,O}$, where $[M]$ is the matrix of that transformation.
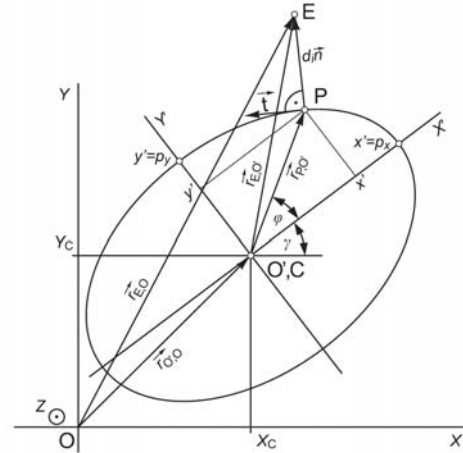

Figure 3. A model of geometric ellipse fitting

We know the description of a vector $\vec{r}_{E,O} = [x_i, y_i, z_i = 0]^T$ with respect to frame O and we would like to know its description with respect to frame O', $\vec{r}_{E,O'} = [x_i', y_i', z_i' = 0]^T$. We can first calculate $\vec{r}_{E,O''} = [R_{O',O}]\vec{r}_{E,O'}$, where $[R_{O',O}] = [R_{O'',O}]$ is the orthogonal rotation matrix about Z axis:

$$[R_{O',O}] = \begin{bmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{bmatrix}. \quad (11)$$

Now O'' is translated into O' along vector $\vec{r}_{O',O} = [x_c, y_c, z_c = 0]^T$. The matrix of total transformation $[M]$ and the transformation itself are:

$$[M] = \begin{bmatrix} R_{O',O} & \vec{r}_{O',O} \\ 0\;0 & 1 \end{bmatrix}, \; \vec{r}_{E,O} = [M]\vec{r}_{E,O'} \quad (12)$$

By inverse transformation in (12), the needed transformation for calculating $\vec{r}_{E,O'} = [x_i{}', y_i{}', z_i{}' = 0]^T$ is achieved:

$$\vec{r}_{E,O'} = [R_{O',O}]^T (\vec{r}_{E,O} - \vec{r}_{O',O})\ \ (13)$$

or

$$\begin{aligned} x_i{}' &= (x_i - x_C)\cos\gamma + (y_i - y_C)\sin\gamma \\ y_i{}' &= -(x_i - x_C)\sin\gamma + (y_i - y_C)\cos\gamma \end{aligned}\ \ (14)$$

Algebraic distance e_i of measured point E can be calculated now in coordinate frame O' by substituting (x', y') in frame O' with (x_i', y_i') from (14):

$$e_i = f_{E1}(\vec{p}, \vec{r}_{E,O'}) = p_y{}^2 x_i{}'^2 + p_x{}^2 y_i{}'^2 - p_x{}^2 p_y{}^2\ \ (15)$$

It is necessary to find ellipse parameters vector $\vec{p}$ such that the sum of squares S_{EA} of algebraic distances is minimal, where is

$$S_{EA} = \sum_{i=1}^{i=N} e_i^2\ \ (16)$$

For that reason, the program FEA was developed for algebraic fitting the ellipse by using simplex method and Nelder-Mead's algorithm [9].

Geometric ellipse fitting, as shown in Figure 3, is much more complicated. Geometric distance is d_i. Three of five ellipse parameters (γ, x_C and y_C) in parameters vector $\vec{p}$ disappear in the coordinate frame O', as shown in (10), and the ellipse is described with the other two parameters (p_x and p_y). For a measured point E, referenced to O, it is necessary to find the closest point P on ellipse, Figure 3. These two points are in the same quadrant in frame O'. Parametric equations of the ellipse in frame O' are:

$$x' = p_x \cos\varphi, \quad y' = p_y \sin\varphi\ \ (17)$$

The unit tangent vector $\vec{t}$ to ellipse in P is:

$$\vec{t} = [t_x, t_y]^T$$
$$t_x = -\frac{p_x \sin\varphi}{\sqrt{p_x{}^2 + p_y{}^2}}; \ t_y = \frac{p_y \cos\varphi}{\sqrt{p_x{}^2 + p_y{}^2}}\ \ (18)$$

Unit normal vector $\vec{n}$ to ellipse in P is orthogonal to tangent vector and could be achieved by rotating of tangent vector about Z' axis by -90º, as shown in Figure 3:

$$\vec{n} = [n_x, n_y]^T = [t_y, -t_x]^T\ \ (19)$$

Now is $\vec{r}_{E,O'} = \vec{r}_{P,O'} + d_i \vec{n}$, as shown in Figure 3. From there is

$$\frac{x_i{}' - x'}{t_y} = \frac{-y_i{}' + y'}{t_x}\ \ (20)$$

That is the second equation, with (10), for finding point P on ellipse closest to measured point E. By substituting (17) and (18) in (19) we achieve the following, more suitable form of the second equation:

$$f_{E2}(\vec{p}, \vec{r}_{P,O'}) = $$
$$= (p_x{}^2 - p_y{}^2)x'y' - p_x{}^2 x_i{}'y' + p_y{}^2 y_i{}'x'\ \ (21)$$

For geometric ellipse fitting the distance equation

$$d_i = \overline{EP}\ \ (22)$$

is solved iteratively. For every measured point E, we achieve the closest point P on the ellipse. After that, we calculate the distance d_i. After repeating that procedure for every measured point, we calculate the sum of squares S_{EG} of geometric distances as in (9). For that purpose, the program FEG was developed for geometric ellipse fitting by using simplex method and Nelder-Mead's algorithm [9] for minimizing S_{EG}. It is called subroutine in each iteration, which for every measured point E finds the closest point on ellipse by using the IMSL® routine *neqnj* for solving the system of nonlinear equations (10) and (21). Initial estimate for the point P on the ellipse is a measured point E. In this case routine *fcne* is used for calculating two functions given in (10) and (21) for each iteration, and routine *fjace* for calculating Jacobian matrix.

For the complete analysis of ellipse fitting, direct least squares fitting of ellipses [3], [5] is also included. This is an algebraic fitting of one ellipse given with its parameters vector $\vec{p}_o = [a, b, c, d, e, f]^T$ with ellipse-specific constraint $b^2 - 4ac < 0$. Original method [3] and its modification [5] are implemented using Matlab®. Both of them are used to develop the program FED using Matlab® function *eig*. The result is two-ellipse parameters vectors, $\vec{p}_{oo} = [a_o, b_o, c_o, d_o, e_o, f_o]^T$ and $\vec{p}_{oM} = [a_M, b_M, c_M, d_M, e_M, f_M]^T$. The first of them is obtained by using original method [3], the second one by using modified method [5]. However, we need an ellipse parameters vector $\vec{p}$ as a more convenient. First, the following calculation is necessary:

$$x_C = \frac{be - 2cd}{4ac - b^2}, \ \ y_C = \frac{bd - 2ae}{4ac - b^2}, \ \ \gamma = \frac{\text{atan2}(b, a - c)}{2}\ \ (23)$$

The other two ellipse parameters in vector $\vec{p}$ are calculated in coordinate frame O''. The fitted ellipse parameters vector $\vec{p}'' = [a'', b'' \equiv 0, c'', d'', e'', f'']^T$ is referenced to frame O'', i.e.

$$a'' = \frac{(a+c) + b\sin 2\gamma + (a-c)\cos 2\gamma}{2}\ ; \ b'' = 0$$

$$c'' = \frac{(a+c) - b\sin 2\gamma - (a-c)\cos 2\gamma}{2}\ ; f'' = f\ \ (24)$$

$$d'' = d\cos\gamma + e\sin\gamma\ ; \ e'' = -d\sin\gamma + e\cos\gamma$$

Now is:

$$p_x = \frac{1}{2a''}\sqrt{d''^2 + \frac{a''e''^2}{c''} - 4a''f''}$$
$$p_y = \frac{1}{2c''}\sqrt{e''^2 + \frac{c''d''^2}{a''} - 4c''f''}\ \ (25)$$

During the test, we measure deviation from nominal circular path: d_{G2} in CW (G02) and d_{G3} in CCW (G03) contouring directions, as shown in Figure 4. The start and the end of actual paths are at the point E_{SF}. Contouring times are t_{SG2} in CW (G02) and t_{SG3} in CCW (G03) directions. At positions S_i, $i = 1,2,3,4$ on nominal path one of the machine's axis changes its motion direction. For each of the developed programs, it is necessary to choose adequate initial estimates and other parameters that will make the fitting efficient.
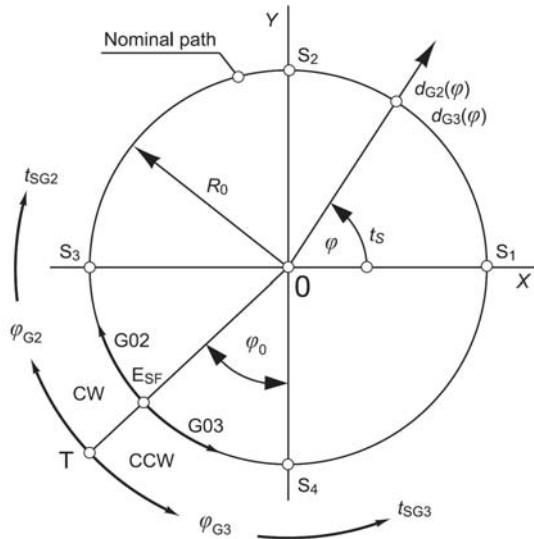

Figure 4. Details on nominal path

3. EXPERIMENTAL VERIFICATION

Developed programs for circle and ellipse fitting to measured data are used with a developed low cost measuring device for render monitoring the accuracy of CNC machine tools, using contouring tests, even more cost effective. The software incorporates an extensive range of features, completely written in a Windows® environment to run on PC.

3.1 Developed measuring instrument

The instrument is shown in Figure 5. The assembly of main parts of the instrument is shown in Figure 5.a. During the testing, the base 1 was fixed to machining center table. The slider 2 was attached to main spindle. Measuring cell 3 is connected to amplifier 8. Proximity sensor activating tip 4 is used for generating a trigger signal during the actual path sampling process.

Preparing of measuring instrument is shown in Figure 5.b. First are determined coordinates of rotating joint located on the base 1. Radius R_0 of nominal path is chosen. Then, the main spindle 7 in position S_1 is attached to the slider 3, as shown in Figure 5.b.

We thus get the second rotational joint of the measuring device. Dial test indicator 5 is used to control the cell 3 set-up procedure and to monitor a measuring process. The measuring device in work is shown in Figure 5.c. It is the position before E_{SF} for CCW direction contouring, as shown in Figure 4. After that position the
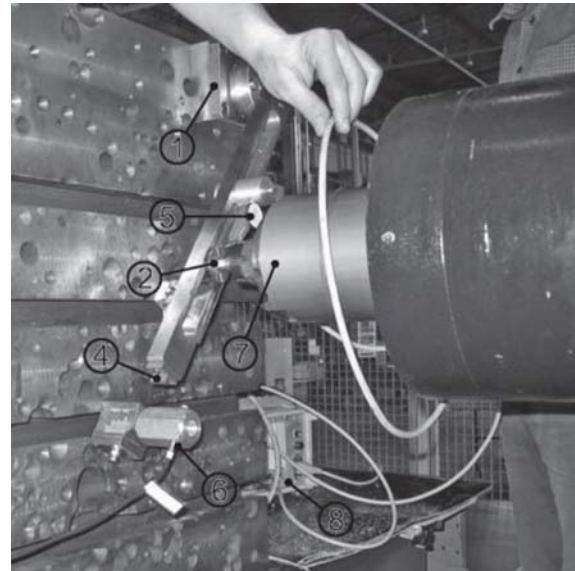
triggering signal T will be activated, crossing the activating tip 4 near proximity sensor 6. Measuring cell 3 is based on two differential ended proximity sensors. Linear domain of the cell is ±0.16 mm. That was quite enough for measuring deviation d_{G2} and d_{G3} on actual contouring paths.
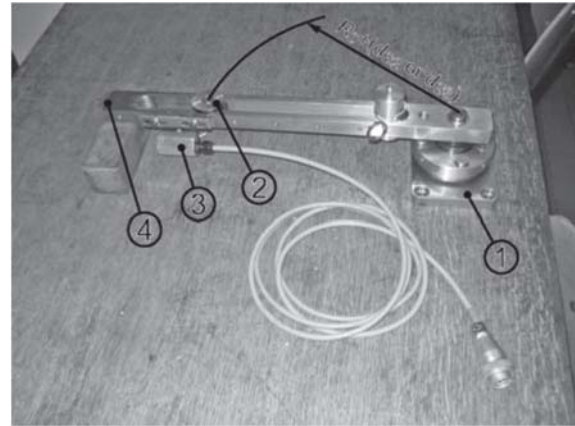
a) Main parts of measuring instrument

b) Adjusting radius R_0 of nominal path

c) Measuring instrument near the starting point E_{SF} in CCW direction

1 – Base; 2 – Slider; 3 – Measuring cell Tr 10, Hottinger;
4 – Proximity sensor activating tip; 5 – Dial test indicator;
6 – Proximity sensor, Institute M. Pupin; 7 – Main spindle;
8 – Amplifier, KWS 3082A, Hottinger

Figure 5. Measuring instrument set-up on the machining center LOLA HBG 130

The tested machining center was a prototype completely assembled and operational at a moment of its testing, but its fundament was not completed. Example of one measured deviation d_{G2} of the actual path in CW direction is shown in Figure 6. Also was shown the triggering signal T. This deviation and deviation d_{G3} (not shown in Figure 6.) were used for experimental verification of developed programs described in Section 2. There are marked positions S_2 and S_4 with bigger deviations in measured signal d_{G2}.
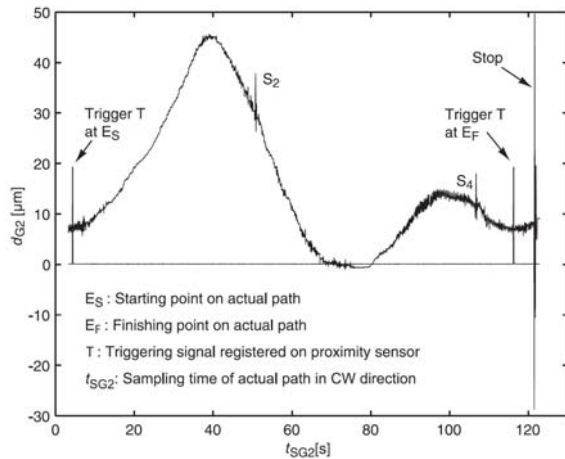

Figure 6. Experimental data for actual path in CW direction

There were not registered peaks in measured signals in positions S_1 and S_3. A stronger vibration is visible in the measured signal during finishing the actual path in CW contouring direction (position named as Stop in Figure 6).

3.2 Results obtained by developed programs

Radial deviation d_{G2} as a function of sampling time t_{SG2} in CW direction, shown in Figure 6, and d_{G3} d_{G3} as a function of sampling time t_{SG3} in CCW direction relative to nominal circle path, are shown in Figure 7. Actual paths are scaled by a noted scale factor. It can be seen in details in D1 and D2 that the tested machining center has bigger radial deviations in positions S_2 and S_4 respectively. It does not appear in positions S_1 and S_3.

Two actual paths from Figure 7 were processed by using the developed program FCA. The results are given in Table 1 and shown in Figure 8.

Here $\vec{p}_s$ is the initial estimate for the circle parameters vector. The stopping criterion is ε: the result is accepted if the relative error between two successive approximations to circle parameters vector is less than ε.

Identified least squares circle is drawn with the same scaling factor as the actual paths. Radius of the fitted circle is R and its center is C. The scale for decoding actual distances in Figure 8 is given in detail. As can be seen, the fitted circle deviates from actual paths, or, the actual paths are of the elliptical type. The details in positions S_2 and S_4 are noted again.

Table 1. Some numerical results for the fitted circle obtained by program FCA

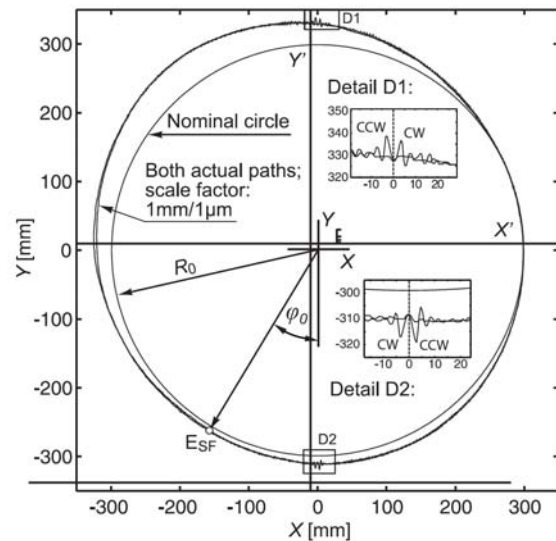| Actual paths	Circle parameters vector $\vec{p} = [x_C, y_C, R]^T$		
	x_C [μm]	y_C [μm]	R [mm]
d_{G2} and d_{G3}	−11.84	9.81	299.0132
d_{G2} only	−11.67	9.50	299.0131
d_{G3} only	−12.00	10.11	299.0134
$\vec{p}_s = [0.0, 0.0, 299.0]^T , \varepsilon = 1\cdot10^{-8}$			

Figure 7. Nominal and both full circle actual paths for given scale factor
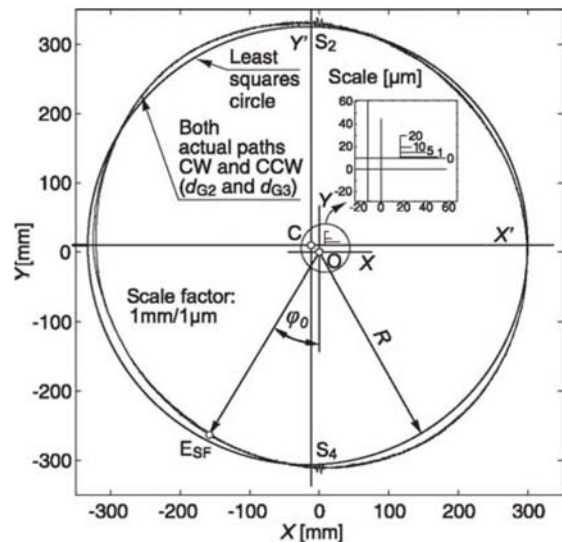

Figure 8. Least squares circle for both full circle actual paths

The results obtained by the program FCG are given in Table 2. We can see that both programs produce similar results. Both of them were efficient and can be used independently for fitting a circle in circular tests of CNC machine tools.

Actual paths	Circle parameters vector $\vec{p} = [x_C, y_C, R]^T$		
	x_C [μm]	y_C [μm]	R [mm]
d_{G2} and d_{G3}	-11.83	9.79	299.0132
d_{G2} only	-11.72	9.53	299.0131
d_{G3} only	-12.02	10.06	299.0134
$\vec{s}_{IS} = [0.0, 0.0, 299.0]$, $\varepsilon = 1 \cdot 10^{-9}$, $\vec{\lambda} = [0.001, 0.001, 0.001]$			

Here $\vec{s}_{IS}$ is the first vertex of the initial simplex, $\vec{\lambda}$ is parameters vector for generating the other three simplex vertices, ε is maximal error accepted.

For the ellipse fitting the developed program FEA was first used. The results are shown in Figure 9 and given in Table 3. Rotation angle γ of the fitted ellipse is given in two calculated values, γ_1 and γ_2. Final results for the identified ellipse are shown in Figure 9 for $\gamma = \gamma_1$ and with the scale for decoding the actual distances. Actual paths near position S_2 are given in detail D. There can be seen again a little vibration of tested machining center Y axis.
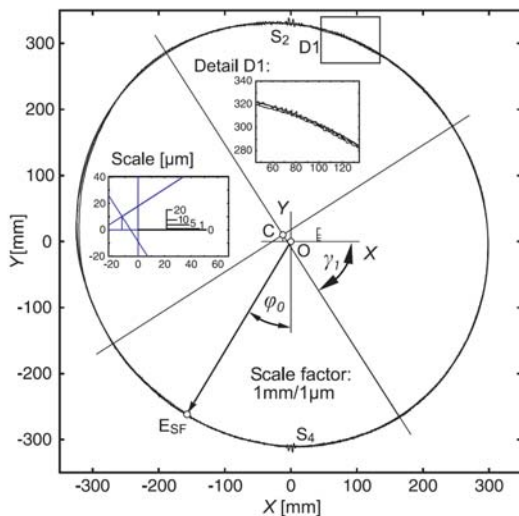

Figure 9. Fitted ellipse for both full circle actual paths for given scale factor

Similar results are obtained and given in Table 4 by using the developed program FEG for geometric ellipse fitting with the same first vertex of the initial simplex, as for the program FEA, given in Table 3.

Table 3. Some numerical results for the fitted ellipse obtained by program FEA

Actual paths	Ellipse parameters vector $\vec{p} = [p_x, p_y, \gamma, x_C, y_C]^T$		
	p_x [mm]	p_y [mm]	γ_1 [°]
	γ_2 [°]	x_C [μm]	y_C [μm]
d_{G2} and d_{G3}	299.0243	299.0022	-57.96
	32.04	-11.85	9.81
d_{G2} only	299.0243	299.0019	-58.27
	31.47	-11.66	9.50
d_{G3} only	299.0243	299.0025	-57.36
	32.64	-12.00	10.11
$\vec{s}_{IS} = [299.0, 299.0, 0.0, 0.0, 0.0]^T$, $\varepsilon = 1 \cdot 10^{-8}$, $\gamma = \gamma_1$			

Table 4. Some numerical results for the fitted ellipse obtained by program FEG

Actual paths	Ellipse parameters vector		
	p_x [mm]	p_y [mm]	γ_1 [°]
	x_C [μm]	y_C [μm]	γ_2 [°]
d_{G2} and d_{G3}	299.0241	299.0024	-57.62
	32.38	-11.97	9.74
d_{G2} only	299.0241	299.0020	-57.96
	32.04	-11.97	9.34
d_{G3} only	299.0242	299.0027	-57.35
	32.65	-12.10	10.08
$\vec{s}_{IS} = [299.0, 299.0, 0.0, 0.0, 0.0]^T$, $\varepsilon = 1 \cdot 10^{-8}$, $\gamma = \gamma_1$			

Table 5. Control results obtained by program FED

Actual paths	Method	Result	a	b	c
			d	e	f
			p_x [mm]	p_y [mm]	γ_1 [°]
			x_C [μm]	y_C [μm]	γ_2 [°]
d_{G2} and d_{G3}	Original method	Vectors p_{oo} and p	-1.1185e-5	-1.4895e-9	-1.1184e-5
			-2.6501e-7	2.1917e-7	1.0000
			299.0243	299.0022	-57.94
			-11.85	9.80	32.06
	Modified method	Vectors p_{oM} and p	-7.0713e-5	-9.4171e-9	-7.0708e-5
			-1.6755e-6	1.3856e-6	6.322165
			299.0243	299.0022	-57.94
			-11.85	9.80	32.06
The rounded values are given. -1.234e-5 ≡ -1.234 · 10⁻⁵.					

Finally, the ellipse fitting is tested by using the developed program FED. The results are given in Table 5 for both actual paths only. The first solution obtained by the program FED is for $\gamma = \gamma_1$ and these results are given in Table 5. As can be seen, both versions of ellipse fitting in program FED produce the same results.

3.3 Some standardized results

A graphical method of presenting results is preferred in standardized presentation of results with the following test results data specified numerically [11]:

Bi-directional circular deviation, the mean bi-directional radial deviation, the circular deviation and the radial deviation of circular paths that are produced by the simultaneous movements of two linear axes. The part of one such report for one experiment, modified to standard ISO 230-4/2005, is as follows:

Machine: Machining center LOLA HBG 130
Test date: 2004/15/09/
Plane: X/Y
Nominal radius: 298.997 mm
Contouring feed: 1000 mm/min
Direction: Bi-directional
A/D conversion frequency: 30 Hz
Number of measured points, CW/CCW: 3358/3358
Start angle: 239°
Temperature Environment/Instrument/Machine: 20 °C
Centre of nominal circle (X/Y): 1200/650 mm
Positions of axes not under test: Z-200 mm, W-700 mm
G, circular deviation (CW/CCW): 27.0/26.3 μm
G(b), bidirectional circular deviation: 27.4 μm
H, circular hysteresis: 9 μm (ISO 230-4/1996)
Some of graphically presented results are given in Figures 6-9.

4. CONCLUSION

Developed programs with a developed measuring device are low-cost but functional laboratory measuring system for circular tests for numerically controlled machine tools in laboratory conditions. Special care is dedicated to ellipse fitting to measured data because the ellipses are used to analyze the characteristic trace patterns of motion errors origins. Algebraic and geometric fitting methods are used with the same efficiency. The developed programs for ellipse fitting are verified by comparing them with two existing algebraic methods for ellipse fitting implemented in program FED. Actual paths in all experiments are of elliptic type. There was a little instability during motion direction change of the tested machining center Y axis. During all the experiments, there were some small vibrations of Y-axis with frequency near 4 Hz. Some of the obtained results were used for the later tested machining center prototype development.

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УПИСИВАЊЕ КРУГА И ЕЛИПСЕ У ЕКСПЕРИМЕНТАЛНЕ ПОДАТКЕ

Милош Главоњић

Анализирани су методи алгебарског и геометријског уписивања круга и елипсе у експерименталне податке и имплементирани у један јефтин функционалан лабораторијски мерни систем за стандардизоване тестове кружне интерполације за нумерички управљане машине алатке. Мерни уређај је направљен коришћењем расположивих мерних компонената за регистровање малих померања. Систем је верификован у испитивању прототипа једног хоризонталног обрадног центра. Неки од резултата, добијених коришћењем развијених програма, били су од користи за анализу узрока кинематичких грешака у типичним скуповима остварених путања. Коришћени методи фитинга упоређени су узајамно и са неким од других постојећих. За тестирање кружне интерполације нумерички управљане машине алатке обе групе метода за алгебарски и геометријски фитинг круга и елипсе имају исту ефикасност. У овом раду је посебно анализиран фитинг елипсе јер је фитинг круга обухваћен стандардима за испитивање машина алатки.