

Turbulence Model of Atmospheric Boundary Layer in General Curvilinear Coordinate System

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Modeling atmospheric boundary layer with standard linear models does not sufficiently reproduce wind conditions in complex terrain, especially on leeward sides of terrain slopes. More complex models, based on Reynolds averaged Navier-Stokes equations and two-equation $k - \epsilon$ turbulence models for neutral conditions in atmospheric boundary layer, written in general curvilinear non-orthogonal coordinate system, have been evaluated. In order to quantify the differences and level of accuracy of different turbulence models, investigation has been performed using standard $k - \epsilon$ model without additional production terms and $k - \epsilon$ turbulence models with modified set of model coefficients. The sets of full conservation equations are numerically solved by Computational Fluid Dynamics technique. Numerical calculations of turbulence models are compared to the reference experimental data of Askervein Hill measurements.

Keywords: atmospheric boundary layer, turbulence modelling, stratified atmosphere.

1. INTRODUCTION

In recent years modelling of micro and meso scales of atmospheric boundary layer phenomena has received growing interest. One of the reasons for such situation is the need for better wind field predictions which are connected, along with local climatology and orography, with the procedures of selection of wind farm sites, also known as siting.

One of the basic phenomena associated to air motion is its turbulence nature; for that reason, there have been many attempts to make turbulence models as accurate as acceptable. For the sake of simplicity, most models make use of a simple gradient transfer hypothesis where only a turbulent exchange coefficient has to be defined. At the beginning, this coefficient is often evaluated by a mixing-length hypothesis, where the mixing length is taken as height dependent. However, for air flows over highly irregular terrain, it is not always obvious how to apply a mixing-length with respect to a varying underlying ground surface. The complexity arises when atmosphere conditions are different than neutral ones, when the temperature becomes an active scalar, producing additional force of air motion.

Actually, a few models have tried to circumvent this problem by use of second-order closure model [1]. This is a rather time consuming approach due to many additional equations needed for second order turbulence modelling and is not practicable for most atmospheric boundary layer modellers at present. Two-equation turbulence models, based on the linear eddy-viscosity concept, have been used as a compromise. The most

popular and frequently used two-equation turbulence model is well known $k - \epsilon$ model.

However, the popularity of the $k - \epsilon$ turbulence model in engineering applications raises the question of whether it could be used for micro and meso scales modelling in the atmospheric boundary layer. Applying the standard $k - \epsilon$ turbulence model, used in engineering applications to atmospheric flows, yields unrealistic results. Mostly, it is unable to reproduce the right level of turbulence in the weak shear layer away from the ground, where the turbulent viscosity is over predicted [2]. Some modifications of standard $k - \epsilon$ turbulence model have been proposed, almost modifying the set of model's coefficients based on experimental evidence of open terrain [3]. This paper deals with accuracy investigation of different $k - \epsilon$ turbulence model modification known as: standard model (STKE), boundary layer modification (BLKE), Chen-Kim (CKKE) and renormalization-group (RNG) modification of $k - \epsilon$ turbulence model. All of these simulations are compared to the full-scale experiment performed at Askervein Hill (Scotland) [4,5].

2. MATHEMATICAL MODEL OF WIND RESOURCES

There is a renewed interest in recent times in considering equations of mathematical physics written in general curvilinear coordinates with dependent variables as physical components of tensors. This trend is more apparent in fluid dynamical researches where solutions are sought for arbitrary geometries and thus the needs for general geometries are unavoidable.

A key result is that if λ is a vector and a_i are the base vectors for a coordinate system x^i then in the representation of $\lambda = a_i \lambda^i$, where λ^i are the contravariant components, the vectors $a_1 \lambda^1$ etc. are parallel to the coordinate curves. Thus, one can interpret λ as the diagonal of a parallelepiped. If λ is a small vector, then the magnitudes of the edge vectors are $\sqrt{g_{11}} \lambda^1$,

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$\sqrt{g_{22}}\lambda^2$, $\sqrt{g_{33}}\lambda^3$. In essence, the length $\sqrt{g_{ij}\lambda^i\lambda^j}$ equals the length of the diagonal of a parallelepiped whose sides are parallel to the coordinate curves and whose edges are of length $\sqrt{g_{ii}}\lambda^i$ (no sum).

The above result lies at the foundation of the definition of physical components of tensors. Thus, the analytical definition of the physical components of a vector u (e.g. fluid velocity) is $u_{(i)} = \sqrt{g_{ii}}u^i$ (no sum).

Using the standard result $u^i = g^{ik}u_k$ we also have $u_{(i)} = \sqrt{g_{ii}}g^{ij}u_j$ (sum on j).

Applied mathematical model in this study involves a full set of the differential equations: mass and momentum conservation, coupled with the turbulence $k - \varepsilon$ model transport equations with modified set of turbulence model coefficients. Using contravariant physical components of wind velocity vector $U^{(i)}$ in an arbitrary curvilinear coordinate frame $x^{(i)}$ above selected terrain, these equations can all be expressed in the following general conservation form:

$$\frac{\Delta}{\Delta x^{(j)}} \left(\rho U^{(j)} \Phi - \Gamma_\Phi g^{(jm)} \frac{\partial \Phi}{\partial x^{(m)}} \right) = S_\Phi. \quad (1)$$

The terms and coefficients occurring in this expression depend on the conservation equation under consideration, and have to be specified individually (Tab. 1). In the case of Cartesian coordinate system, previous equation has the standard form; however, writing general conservation equation in an arbitrary curvilinear coordinate system, used tensor differential operators have to be:

$$\frac{\partial}{\partial x^{(j)}} = \frac{1}{\sqrt{g_{jj}}} \frac{\partial}{\partial x^j}, \quad (2)$$

$$\frac{\Delta}{\Delta x^{(j)}} = \sqrt{\frac{g_{ij}}{\det(g_{ij})}} \frac{\partial}{\partial x^{(j)}} \sqrt{\frac{\det(g_{ij})}{g_{jj}}}. \quad (3)$$

In the momentum equations, the physical analogue of the standard Christoffel's symbol of the second kind is given by:

Table 1. Governing equations with summarised standard and boundary layer $k - \varepsilon$ turbulence model

Equation	Φ	Γ_Φ	S_Φ		
Continuity	1	0	0		
Momentum	$U^{(j)}$	μ_{eff}	$-g^{(ji)} \frac{\partial P}{\partial x^{(j)}} - \left[\rho U^{(j)} U^{(i)} - \left(\tau^{(ij)} - \rho \overline{u^{(j)} u^{(i)}} \right) \right] \left(\begin{smallmatrix} i \\ m \end{smallmatrix} \right)$		
Turb. kin. energy	k	μ_{et}/σ_k		$G - \rho \varepsilon$	
Dissipation rate	ε	μ_{et}/σ_k		$\varepsilon / k (C_{\varepsilon 1} G - \rho C_{\varepsilon 2} \varepsilon)$	
			$G = g^{(ik)} \left[\tau_{\text{turb}}^{ij} - \frac{2}{3} g^{(ij)} (\rho k + \mu_{\text{turb}} \nabla_{(m)} U^{(m)}) \right] \nabla_{(j)} U^{(k)}$		
Model coefficients	σ_k	σ_ε	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$	C_μ
STKE	1.0	1.3	1.44	1.92	0.09
BLKE	1.0	1.85	1.44	1.92	0.0324

$$\left(\begin{smallmatrix} i \\ j \\ k \end{smallmatrix} \right) = \sqrt{\frac{g_{ii}}{g_{jj}g_{kk}}} \left(\begin{smallmatrix} i \\ j \\ k \end{smallmatrix} \right) - \delta_i^j \frac{g_{jm}}{g_{jj}} \left(\begin{smallmatrix} m \\ j \\ k \end{smallmatrix} \right). \quad (4)$$

In the turbulence production term (G), the contravariant partial derivation of velocity has the form:

$$\nabla_{(j)} U^{(i)} = \frac{\partial U^{(i)}}{\partial x^{(j)}} + U^{(m)} \left(\begin{smallmatrix} i \\ m \\ k \end{smallmatrix} \right). \quad (5)$$

The constitutive relation of the Newton's stress hypothesis is used to express the normal and viscous stress tensors:

$$P = p + \frac{2}{3} \left(\rho k + \mu_{\text{eff}} \frac{\Delta U^{(m)}}{\Delta x^{(m)}} \right), \quad (6)$$

$$\tau^{(ji)} = \mu_{\text{lam}} \left[g^{(jm)} \nabla_{(m)} U^{(i)} + g^{(im)} \nabla_{(m)} U^{(j)} \right]. \quad (7)$$

The Reynolds turbulent stress tensor is derived by Boussinesq concept of turbulent viscosity:

$$-\rho \overline{u^{(j)} u^{(i)}} = \mu_{\text{turb}} \left[g^{(jm)} \nabla_{(m)} U^{(i)} + g^{(im)} \nabla_{(m)} U^{(j)} \right]. \quad (8)$$

The effective viscosity is derived from the following term:

$$\mu_{\text{eff}} = \mu_{\text{lam}} + \mu_{\text{turb}}. \quad (9)$$

Turbulent viscosity is derived by the algebraic term:

$$\mu_{\text{turb}} = \frac{C_\mu \rho k^2}{\varepsilon} \quad (10)$$

Where k and ε are turbulent kinetic energy and its dissipation rate, respectively, defined as:

$$k = \frac{1}{2} g_{(ij)} \overline{u^{(i)} u^{(j)}}, \quad (11)$$

$$\varepsilon = \nu_{\text{lam}} g_{(im)} g^{(jn)} \overline{\nabla_{(n)} u^{(m)} \nabla_{(j)} u^{(i)}}. \quad (12)$$

Presented mathematical model (Tab. 1) has been solved numerically using BFC option of PHOENICS software.

3. MODIFICATIONS OF $k - \varepsilon$ TURBULENCE MODEL APPLIED TO ABL

Atmospheric boundary layer (ABL) is defined [6] as “the region in which the atmosphere experiences surface effects through vertical exchanges of momentum, heat and moisture”. The traditional approach is to divide the ABL vertically into various layers, each characterized by different “scaling” parameters. The ABL can be divided into three major sublayers:

- The layer near ground up to the height of the roughness length z_0 . This layer has traditionally been referred to as a “laminar sublayer” or “roughness layer”. Actually, in this layer molecular viscosity hardly plays a role and turbulent fluxes still occur, except very close to ground where the motion is primarily laminar. Within this layer, up to height z_0 , turbulence is intermittent or not fully developed, therefore z_0 can be interpreted as the eddy size at the surface;
- The surface layer (SL) from z_0 to z_s , where z_s varies from about 10 m to 200 m. In this layer, the fluxes of momentum, heat and moisture are assumed to be independent of height and the Coriolis effects is generally negligible and
- The transition (or Ekman) layer (TL) from z_s to z_i , where z_i varies from about 100 m to 2 km. In special situations, such as during thunderstorms, the boundary layer can extend into the stratosphere.

From the point of wind power assessment at micro (site area less than 10 km²) and meso (area less 1000 km²) sites, the surface layer is of primary importance. The mean characteristic of surface layer is weak shear flow. By experimental results of Panofsky et al. [3,7], the shear stress to kinetic energy ratio is typically

$-\overline{uw}/k \approx 0.16 \div 0.18$, which implies value of $C_\mu = 0.0256 \div 0.0324$ and consequently, the sets of the rest of model coefficients (Tab. 1). The lower value is recommended to meso scale simulations, where the lower part of surface layer up to 10 \div 50 m is bridged by standard log-wall functions [2], whereas the higher value is recommended to micro scale simulations where the grid resolutions near the surface is sufficiently high [4]. If we focus on micro scale simulations (Askervein case study), then $C_\mu = 0.0324$. The value of $C_{\varepsilon 2}$, determined from experiments with decaying grid turbulence, should be remained unchanged. The diffusion coefficient σ_k (Prandtl number of turbulence kinetic energy), close to unity following Reynolds analogy, should also be remained unchanged. The rest of model coefficients can be deduced by knowing expression valid in the log-law region:

$$\kappa^2 = \sigma_\varepsilon C_\mu^{1/2} (C_{\varepsilon 2} - C_{\varepsilon 1}), \quad (13)$$

where $\kappa = 0.4$ is the von Karman constant. Using above recommendations and also leaving $C_{\varepsilon 1}$ unchanged, the diffusion coefficient σ_ε (Prandtl number of dissipation rate) can be deduced from (13). New set of $k - \varepsilon$ model coefficients is:

$$C_\mu = 0.0324, \sigma_k = 1.0, \sigma_\varepsilon = 1.85, C_{\varepsilon 1} = 1.44, C_{\varepsilon 2} = 1.92. \quad (14)$$

The $k - \varepsilon$ turbulence model, defined in Table 1 can be referred to as “standard $k - \varepsilon$ model of surface layer” (BLKE).

Besides standard $k - \varepsilon$ model and it's atmospheric boundary layer modification, we will also use Chen-Kim modification of $k - \varepsilon$ turbulence model and RNG $k - \varepsilon$ turbulence model. Last two models will be summarised in Table 2 and Table 3, respectively.

Table 2. Summarized Chen-Kim turbulence model

Equation	Φ	Γ_Φ	S_Φ			
Turb. kin. energy	k	μ_{et}/σ_k	$G - \rho\varepsilon$			
Dissipation rate	ε	μ_{et}/σ_k	$\varepsilon / k (C_{\varepsilon 1}G - \rho C_{\varepsilon 2}\varepsilon - \rho\alpha\varepsilon) + \rho C_{\varepsilon 3}G^2 / k$			
			$G = g^{(ik)} \left[\tau_{tur}^{ij} - \frac{2}{3}g^{(ij)}(\rho k + \mu_{tur} \nabla_{(m)} U^{(m)}) \right] \nabla_{(j)} U^{(k)}$			
Model coefficents	σ_k	σ_ε	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$	$C_{\varepsilon 3}$	C_μ
	0.75	1.15	1.15	1.15	0.25	0.09

Table 3. Summarized RNG $k - \varepsilon$ turbulence model

Equation	Φ	Γ_Φ	S_Φ			
Turb. Kin. Energy	k	μ_{et}/σ_k	$G - \rho\varepsilon$			
Dissipation Rate	ε	μ_{et}/σ_k	$\varepsilon / k (C_{\varepsilon 1}G - \rho C_{\varepsilon 2}\varepsilon - \rho\alpha\varepsilon)$			
			$G = g^{(ik)} \left[\tau_{tur}^{ij} - \frac{2}{3}g^{(ij)}(\rho k + \mu_{tur} \nabla_{(m)} U^{(m)}) \right] \nabla_{(j)} U^{(k)}$			
			$\alpha = C_\mu \eta^3 \left(1 - \frac{\eta}{\eta_0} \right) / \left(1 + \beta \eta^3 \right); \eta = Sk/\varepsilon$			
			$S = \sqrt{2\nabla_{(j)} U^{(i)} \nabla_{(i)} U^{(j)}}$			
Model coefficents	σ_k	σ_ε	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$	C_μ	η_0
	0.7194	0.7194	1.42	1.68	0.0845	4.38
						β
						0.012

4. ASKERVEIN CASE STUDY

4.1 Site description

Askervein, or Askernish Hill as it is sometimes referred to locally, is located near the west coast of South Uist, toward the southern end of the Outer Hebrides island chain of Scotland. The hill coordinates are 57°11'N, 7°22'W. It is essentially elliptical in plan form with 1 km minor axis and 2 km major axis. The major axis is oriented along a generally NW-SE line. The predominant wind directions during September and October (the period of the experiments) are from the SW and S. The digital elevation model, a portion of which is shown as Figure 1, shows that the hill is relatively isolated, apart from the hills to the NE and E, i.e., downstream of the prevailing wind direction. The hill top (HT) is at a height of 126 m above sea level. Since HT is somewhat to the NW end of the hill, a second reference location ('centre point' or CP) was chosen as an additional point of reference on hill. During the experiments, most of the masts were deployed in approximately linear arrays through CP or HT. The mean lines chosen were oriented at 043° (grid) and 133° (grid), approximately NE-SW and SE-NW along the minor and major axes of hill, respectively. They are shown in Figure 1 and referred to as lines A, AA and B, as shown.

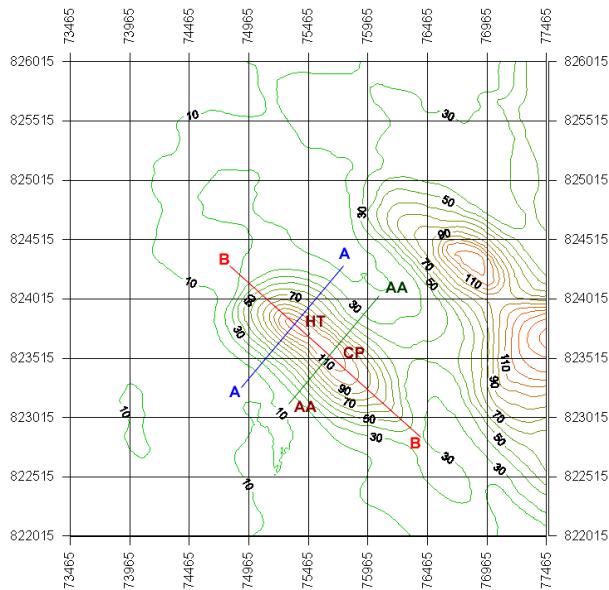


Figure 1. Lines of installed masts at Askervein Hill

4.2 Askervein project

Askervein project was a collaborative study carried out under the auspices of the International Energy Agency Programme of R&D on Wind Energy Conversion System [5]. The experiments were conducted in 1982 and 1983. Askervein is the site of the most complete field experiment to date, with 50 masts deployed, and whose 27 of them were equipped with three component turbulence sensors. Askervein '83 was conducted between September 14 and October 18, with mean observational runs in the period September 25 – October 10. All of the designated runs over the 16-day period

provided good and interesting data covering a range of wind directions. Monday, 3 October was perhaps the 'best' day for data collection with steady, moderate-to-strong winds (10 m/s) from 210° through most of the day. Richardson number was varying from 0.0131 to – 0.011. These results have been used in many verifications and testings of different models, both numerical and experimental (wind tunnel experiments), therefore this case is particularly well documented. This made the Askervein Hill case the most suitable reference to test numerical results versus full-scale experimental data when it comes to micro scale modelling. For this study, three sets of data have been available: a) normalized velocity values at 10 meters height along the A line, b) normalized velocity values at 10 meters height along the AA line and c) fractional speed up at the hill top (HT).

5. NUMERICAL SETUP AND BOUNDARY CONDITIONS

The physical domain is discretised by 100*100 grid cells covering area of 2 km × 2 km with the $x_1 - x_2$ plane. The height of domain is fixed at 1 km, with 30 uniform grid resolution of 20 m in 21 grid cells with variable geometrical distribution. In this study, the recommendation of grid resolution tests [4] has been accepted having minimum 3 cells in the first 10 m above ground surface.

Equilibrium wall-functions have been used as the boundary conditions at the ground surface.

Upper and outlet boundary conditions are specified by von Neumann conditions (zero first derivatives of all variables). Lateral inlet profiles of velocity, turbulence kinetic energy and its dissipation rate are specified by the following analytical expressions [1]:

$$U_1(x_3) = \frac{u_\tau}{\kappa} \ln \left(\frac{x_3}{z_0} \right) \quad \text{for: } x_3 \leq 500 \text{ m}$$

$$U_1(x_3) = 10 \text{ m/s} \quad \text{for: } x_3 > 500 \text{ m} \quad (15)$$

$$k(x_3) = \frac{u_\tau^2}{C_\mu} \left[1 - \left(\frac{x_3}{H} \right)^2 \right] \quad (16)$$

$$\varepsilon(x_3) = \frac{u_\tau^3}{\kappa} \left(\frac{1}{x_3} + \frac{4}{L_{MO}} \right) \quad (17)$$

where friction velocity $u_\tau = 0.4423$ m/s is deduced by (15), H is domain height and L_{MO} is Monin-Obukov length scale taking equal to H .

6. PRESENTATION OF RESULTS

Normalized velocity along the A and AA lines as well as fractional speed up parameter at HT are shown on Figures 2, 3 and 4, respectively. Velocity along the A and AA lines are normalized by reference velocity.

The fractional speed-up parameter is calculated by expression:

$$\Delta S(h) = \frac{V(h) - V_{ref}(h)}{V_{ref}(h)} \quad (18)$$

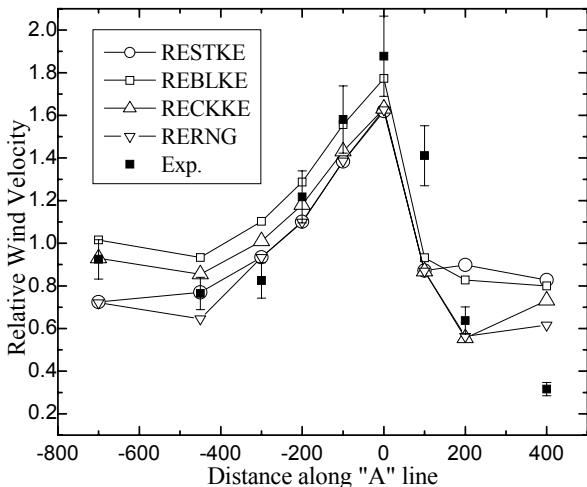


Figure 2. Normalized velocity along A line with different turbulence models

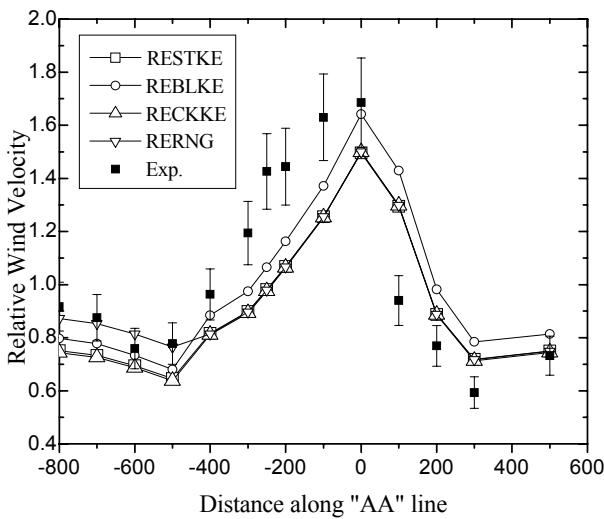


Figure 3. Normalized velocity along AA line with different turbulence models

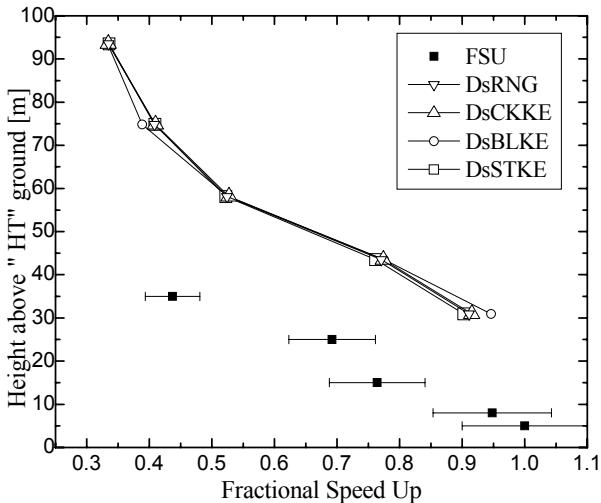


Figure 4. Fractional speed-up above top hill

Generally speaking, based on the shown diagrams, it can be concluded that no significant gain is obtained by CKKE and RNG model, comparable to STKE, whereas the BLKE shows the most promises that should be expected due to model coefficient set up for weak shear flow of atmospheric boundary layer. The relative

velocity at both lee-ward and wind-ward sides along A line is acceptable, whereas at the HT is under-predicted. However, relative velocity is under-predicted at wind-ward side and over predicted at lee-ward side along AA line, but at the CP, it is quite correct. Fractional speed-up parameter at HT is, generally, under-predicted with tendency to be closed to experimental data at the higher height.

7. CONCLUSION

The present paper is intended to provide a comparison of basic results of the Askervein Hill project and the results of numerical simulation in which standard $k - \varepsilon$ turbulence model, as well as three modifications of that model, were used to simulate the flow. Among these numerical results the best predictions were those of BLKE model, which was not surprising, but inspite of that fact an increasing of accuracy is needed, which may be obtained by better tuning of model coefficients.

It is also stressed in this paper that the velocity field on Askervein may be affected by the neighbouring hills downwind. These hills may probably produce some upwind blockage (Fig. 1). Actually, in most cases, the speed on the hill top is under-predicted. This could be related with the fact that the topography around the site is not correctly taken into account with a smaller micro-model.

As in [4], it is pointed out that a lower roughness gives better results for the hill top in a study on the Askervein Hill case; there are still questions about the roughness length that should be adopted. A lower roughness seems to give better results for the hill top.

Further improvements may be accomplished by introducing considerations of unstable atmospheric conditions, where temperature becomes active scalar, producing additional buoyancy driven flow. It implies that there is an additional time scale of buoyancy effects. The fluctuating body force permits the work that must be added as a source term in the budget of turbulence kinetic energy.

Note that as long as wind energy assessment is the matter of interest, the most important point is to predict maximum speed up in the right locations, if possible, with good accuracy. The details of flows on the lee-ward side are not relevant for such an application. However, in the interest of micro scale modelling aimed at other purposes, such as pollutant dispersion, the study of other cases may be desirable.

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NOMENCLATURE

G	volumetric production rate of k
k	turbulence kinetic energy
g_{ij}	metric tensor

Greek symbols

δ_i^j	Kronecker delta operator
ε	turbulence kinetic energy dissipation rate
τ_{ij}	Reynolds or turbulent stress $\tau_{ij} = -\rho \overline{u_i u_j}$

МОДЕЛ ТУРБУЛЕНЦИЈЕ АТМОСФЕРСКОГ ГРАНИЧНОГ СЛОЈА У ГЕНЕРАЛИСАНОМ КРИВОЛИНИЈСКОМ КООРДИНАТНОМ СИСТЕМУ

Никола Мирков, Жана Стевановић, Жарко Стевановић

Моделирање атмосферског граничног слоја стандардним линеарним моделима није у могућности да репродукује у довољној мери стање ветра при сложеним геометријама терена, посебно када су у питању стране брда које су у заветрини. У раду су зато коришћени сложенији модели засновани на Ренолдсовим осредњеним Навије-Стоксовим једначинама и двоједначинским $k - \varepsilon$ моделима турбуленције за атмосферски гранични слој у неутралним условима, написаним у генерализаним криволинијским координатама. Да би испитали разлике и ниво тачности различитих модела турбуленције, коришћени су стандардни двоједначински $k - \varepsilon$ модел без додатних изврних чланова и $k - \varepsilon$ модели са изменјеним скупом коефицијената модела. Добијени нумерички резултати модела турбуленције су упоређени са референтним резултатима мерења на брду *Askervein* у Шкотској.