

High-Bay Warehouse Analysis Based on Influence of Stochastic Parameters

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A group of high-bay warehouses with one servicing device is analysed. Working cycle of servicing device is calculated upon FEM regulation, using optimal change of velocity in time. Different theoretical distributions for arrival rate, servicing flow and failure were used, in this paper, for determining stochastic influence on warehouse system performances. The other influence on warehouse system performances is taken through offered load of the system. Combination of both influences and variety of each brought up the conclusion about stochastic behaviour of observed system performances presented by theoretical distributions. Developed simulation model was used for analysis of warehouse system work.

Keywords: Monte Carlo simulation, mathematical modelling, high-bay warehouse, working cycle.

1. INTRODUCTION

This paper deals with material flow in warehouse systems. Design of transport and warehouse systems is of great importance for the work of the whole production system where the subsystem of material flow is sometimes integrated (factories, workshops etc.). Its main function is to connect working processes in the production system or to represent the whole system by itself (distribution centres, unloading terminals, stand alone warehouse systems).

This approach underlines the work of warehouse systems which are the one of main representatives of material flow systems. Due to the fact that the warehouse systems exist in different industrial surroundings, they can be analysed as "black box" using system approach. On the basis of the appropriate arrival flow to the system, which covers outside influences, and working cycle defined by servicing flow (rate), appropriate output of the system can be obtained with adequate system performances.

The influence of the system surroundings and determining of the system boundaries can be undertaken through difference of arrival flow i.e. its stochastic character.

Elementary subsystem in the warehouse system can be defined as work of each and any servicing device (fork lift truck, AS/RS machine etc.). This elementary subsystem is the starting point for the design of the warehouse system and it must be analysed and defined in details. Using this elementary subsystem, for analysis of the whole warehouse system, is possible only if all relevant parameters, both deterministic and stochastic, are defined.

In both cases, integrated or stand alone warehouse system, the starting point for design process is elementary subsystem which can be also defined as a "knot point" and it is represented by work of servicing

device, based on working cycle [1].

Warehouse system defined in this way assumes the existing of deterministic and stochastic parameters. Knowing these parameters and their statistical distributions, where it is possible, as well as the quantification of their influences, we can get the output performances of the system. Changes (variation) of this output performances enable forming of the different warehouse systems.

Deterministic parameters which are significant are: length (L) and height (H) of the warehouse, ratio between length and height ($L:H$), velocity (V) and acceleration (a) in horizontal (x) and vertical (y) direction of the servicing device (elementary subsystem) and position of entrance and exit of warehouse, while stochastic significant parameters are: arrival flow, servicing flow and failure with their statistical distributions.

The main idea of this work is that parameters which describe behaviour of the warehouse system can be quantified. Quantification of their single or group influence on the work of the warehouse system can also be done. The defined parameters and their quantification, single or group, influence the change of warehouse system performances. This influence quantification is used for better and swifter design process which has to fulfil practical demands, on one side, and optimal theoretical demands of the warehouse system, on the other side.

2. WAREHOUSE SYSTEM PARAMETERS

A group of high bay warehouses with one servicing device (AS/RS machine) is analysed. Starting with warehouse with 5 pallet levels and going up to max. 24 pallet levels in height, gives us a system of 20 different warehouse configurations. In other words the height of these warehouses is from 7 to 29.8 meters and appropriate length is from 31 to 133 meters, taking into consideration that dimensions of the pallet place are 800×1200 mm which gives us 31 to 133 pallet places along the length of the warehouse. This group of 20 warehouses is adequate to represent most of the existing warehouse systems in industry and economy (Fig. 1).

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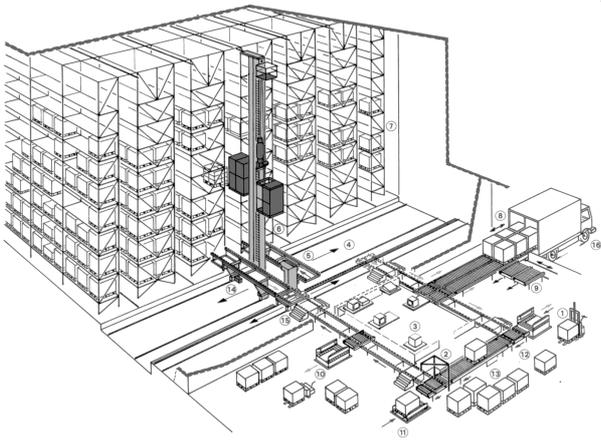


Figure 1. Warehouse system with AS/RS sub-system

Working cycle (T_c) of the servicing device is calculated upon FEM regulations. Input data used for calculation of the working cycle such as: velocities, accelerations, extra times etc., standard euro pallet, was generated according to data of the major European manufacturers of warehouse devices and equipment and design experience of authors. Working cycle (T_c) is calculated only upon deterministic parameters [1].

For the defined group of warehouses, the arrival flow is defined with: quantity and type of units which are generated and brought to the system with demand for servicing and distribution of time between units arrival. Servicing rate is defined with quantity and way of servicing and distribution of servicing time. Failure in the work of the system assumes influences of all parameters which can lead to the system failure. Failure can be defined with: moment of appearance, time distribution of failure duration and average number of failure appearances in pre-defined time interval.

Those three parameters present the basics of stochastic behaviour of the system. Their knowing and evaluation with defined mutual influences means determination of stochastic parameters influence on the warehouse system.

The system is analysed using, for this purpose, specially developed simulation model. The model always uses the same values of deterministic parameters, so that influence of stochastic parameters variation on the work of warehouse system can be seen and underlined.

Deterministic parameters used for experiment simulation, given in next section (2.1 Average working cycle of S/R machines), are used for calculation of average working cycle i.e. average servicing rate μ . On the basis of average arrival rate, offered load of the system ρ is changed with values of 0.1 – 0.95. For adopted values of offered load, values for arrival rate λ are calculated.

Arrival flow, servicing flow and failure in simulation model are represented using Erlang distribution of order k (E1, E5, and E10) and Normal distribution. Using those distributions, arrival and servicing flow and failure are defined for the case of strong stochastic behaviour (represented with E1), over stochastic changes through E5 and E10 up to minimal stochastic behaviour represented with N.

When speaking only about failure some other parameters had to be defined such as: probability of failure ($P_f = 0.005$) defined upon design practice, time interval for failure check ($t_{cf} = 3600$ s) and average duration of failure ($t_f = 9000$ s) distributed by Exponential distribution (E1) [2].

Variation of stochastic parameters enables the analysis of the following system performances:

- P_{ser} – probability of servicing,
- P_{oc} – probability that channel is occupied,
- P_{eq} – probability of existing a queue,
- N_w – average number of units in the queue,
- N_{ws} – average number of units in the system,
- t_w – average time that unit spends in the queue,
- t_{ws} – average time that unit spends in the system and
- t_c – time that channel waits for servicing.

2.1 Average working cycle of S/R machines

The design of high-bay warehouses requires the implementation of Storage/Retrieval (S/R) machines as the best and most sophisticated solution. Generally, the input and output of such warehouse system strongly depends on the technical and working capabilities of the main elements – S/R machines. The main characteristic of those machines is the working cycle. An analysis of the working cycle according to the influent parameters on the basis of the existing FEM (Federation Europeenne de la Manutention – FEM, No. 9.851) regulation is given.

The review of indications used for calculation of average working cycle of S/R machines:

- H – height of the rack in warehouse [m],
- H_p – pallet place height [m],
- L – length of the rack in warehouse [m],
- L_p – pallet place length [m],
- $I(x,y)$ – position and coordinates of input – entrance of the rack,
- $O(x,y)$ – position and coordinates of output – exit from the rack,
- $P_i(x_i,y_i)$ – coordinates of the pallet place location in warehouse,
- S – path between two locations in x or in y direction [m],
- S_p – path of positioning [m],
- V_c – maximal constant velocity of S/R machine either in x (V_x) or y (V_y) direction [m/s],
- V_p – velocity of positioning [m/s],
- a – acceleration either in x (a_x) or y (a_y) direction [m/s²],
- T – time of movement between two locations either in x or y direction without time of positioning [s],
- $T(P_i,P_j)$ – lead time of the S/R machine between start (P_i) and end (P_j) point,
- T_t – total time of movement between two locations either in x (T_{tx}) or y (T_{ty}) direction [s],
- t_w – waste time [s] and
- T_c – cycle time according to FEM [s].

Change of velocity in time of S/R machine in horizontal direction (x -axis) and vertical direction (y -

axis) is given in Figure. 2. Movement in x -direction assumes the movement of whole S/R machine while movement in y -direction assumes hoisting and lowering of the cabin with forklift table and weight on the pallet.

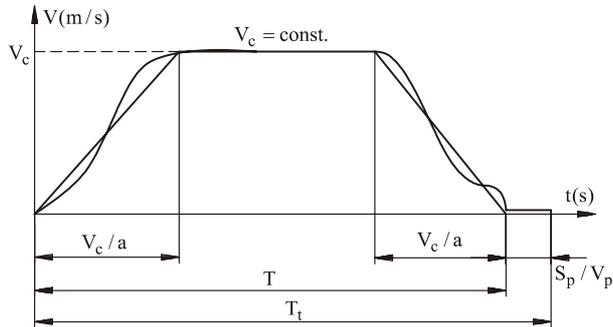


Figure 2. Change of velocity in time

Velocity of S/R machine as a function of time is given by (1):

$$V(t) = \begin{cases} a \cdot t, & 0 < t < \frac{V_c}{a} \\ V_c, & \frac{V_c}{a} \leq t \leq T - \frac{V_c}{a} \\ a \cdot (T - t), & T - \frac{V_c}{a} < t < T \\ V_p, & T < t < T_t. \end{cases} \quad (1)$$

The path between two locations in x or y direction made by S/R machine in period of time T_t is given by (2) and (3).

$$S(T_t) = V_c \cdot T - \frac{V_c^2}{a} + S_p, \quad T \geq 2 \cdot \frac{V_c}{a}, \quad (2)$$

$$S(T_t) = \frac{1}{4} \cdot a \cdot T^2 + S_p, \quad T \leq 2 \cdot \frac{V_c}{a}. \quad (3)$$

Equation (2) is used when S/R machine moves according to change of velocity shown in Figure 2. Equation (3) is used when the path between two locations is such that the maximal velocity can not be reached, meaning that the S/R machine is moving in non stationary regime in the whole period, as it is shown in

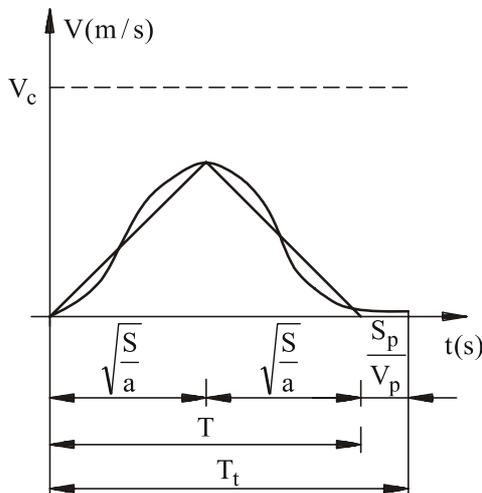


Figure 3. Change of velocity in non-stationary regime

Figure 3. Positioning is used for adjusting S/R machine according to pallet place, and can never be neglected.

Needed time for S/R machine to move the weight (pallet) from the starting point to the end point of the path S , in x or y direction is given by (4).

$$T_t = \begin{cases} \frac{S}{V_c} + \frac{V_c}{a} + \frac{S_p}{V_p}, & S \geq \frac{V_c^2}{a} \\ 2 \cdot \sqrt{\frac{S}{a}} + \frac{S_p}{V_p}, & S \leq \frac{V_c^2}{a}. \end{cases} \quad (4)$$

The lead-time of the S/R machine while moving the weight (pallet) from the starting point $P_1(x_1, y_1)$ to the end point $P_2(x_2, y_2)$ of the path is given by (5).

$$T(P_1, P_2) = \max\{T_{tx}(P_1, P_2); T_{ty}(P_1, P_2)\}. \quad (5)$$

Average single cycle time of S/R machine according to FEM is calculated using (6).

$$T_c = \frac{1}{2} [2T(I, P_1) + 2T(I, P_2)] + 2t_w. \quad (6)$$

Symbols in (6) represent:

- $2T(I, P_1)$ – single cycle time of S/R machine when moving from input $I(0,0)$ to the reference point $P_1(\frac{2}{3}L, \frac{1}{5}H)$ and back to point $I(0,0)$.

Input and output have the same position $I(0,0) = O(0,0)$,

- $2T(I, P_2)$ – single cycle time of S/R machine when moving from input $I(0,0)$ to the reference point $P_2(\frac{1}{5}L, \frac{2}{3}H)$ and back to point $I(0,0)$.

Input and output have the same position $I(0,0) = O(0,0)$ and

- t_w – waste time, covers the time needed for starting the working mechanisms, finding the location of pallets, moving of forks while fetching and leaving the pallet etc.

Equation (6) can be used for calculating the average single cycle time of S/R machine only if condition given by (7) is fulfilled.

$$0.5 < \frac{H}{L} \cdot \frac{V_x}{V_y} < 2. \quad (7)$$

The characteristic deterministic parameters of the warehouse system, used for calculation of average working cycle of S/R – machine are shown in Table 1.

Table 1. Deterministic parameters of the warehouse system

No. of pal. places per height	No. of pal. places per length	V_x [m/min]	V_y [m/min]	a_x [m/s ²]	a_y [m/s ²]
8 ($H_p = 1.2$ m)	32 ($L_p = 1$ m)	80	40	0.4	0.4
$S_{px} = 0.04$ m; $S_{py} = 0.081$ m; $V_p = 2$ m/min; $t_w = 20$ s					

Finally, average working cycle of servicing device i.e. S/R – machine, for deterministic parameters of warehouse system, is $T_c = 72.88$ s.

3. SIMULATION MODEL

The purpose of developed simulation model is to analyse the work of single channel queuing system with finite or infinite queue. Simulation model also covers failure in the work of servicing channel. States of servicing channel and transitions between states are shown in Figure 4. Possible states of servicing channel are: **Wo** – channel is working, **Wa** – channel is waiting for unit (system is empty) and **Re** – channel is under repairing (failure).

Input values, which define the system and have to be known before the beginning of the simulation experiment, are:

- number of places in the queue (finite – m or infinite),
- distribution of time between units arrival,
- distribution of servicing time,
- distribution of failure duration,
- probability of servicing channel failure,
- time interval for failure check,
- duration of simulation experiment and
- starting conditions for system.

Initial conditions for the system at the starting moment are defined with:

- number of units in the queue (system) and
- state of servicing channel.

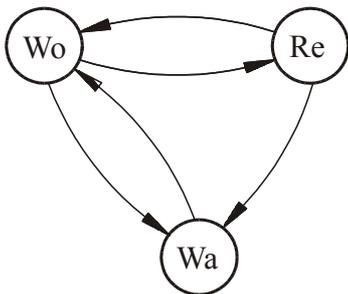


Figure 4. State transition diagram for servicing channel

All stochastic variables which are used for system definition can be generated according to the following distributions: Erlang distribution, Normal distribution, empirical continual and discrete distribution and as constant value [3].

System performances which are obtained as an output from simulation model are: P_{ops} , P_{oc} , P_{pr} , N_w , N_{ws} , t_w , t_{ws} and:

- system state probabilities (total of $m + 2$ probabilities),
- state probabilities of servicing channel,
- values for t_w and t_{ws} , for each unit which is accepted to the system, are written into file for further processing (χ^2 – test),
- values for N_w and N_{ws} , for each change of system state, are written into file for further processing (χ^2 – test),
- values which define time that channel is waiting for unit arrival are also observed (channel is idle, there are no units in the system) and they are also written into file for further processing (χ^2 – test) [4].

The work of the simulation model assumes the existing of the current (internal) time in the system. Internal time begins with starting of simulation the experiment and lasts until the end of simulation experiment, and is defined as input data. Current time is increasing for one discrete time unit for each executed cycle of simulation experiment. It is adopted that discrete time unit is one second, but it can also be some other value. Critical time of servicing channel is the moment of time when channel changes its state.

When the current time is equal to the moment of when unit arrives to the system, it is assumed that unit is in the system and the number of arrived units is increased by one. Depending on servicing channel state, the arrived unit can stand in a queue, directly be accepted for servicing or can be cancelled.

When the channel is in the state of waiting (Wa), then the arrived unit goes directly to the servicing where servicing time for this unit is generated according to previously defined servicing distribution. After that, the time that channel waits for the beginning of servicing is written into file and statistics which defines the number of channel waiting and average waiting time is updated. At that moment the channel changes its state from waiting (Wa) to work (Wo) and a new critical time is generated (time when channel changes state) as current time plus generated servicing time.

When the channel is in the state of work (Wo) or in the state of repair (Re), depending on number of units in a queue, the newly arrived unit can be accepted to the system and put in a queue (statistics for the number of units in a queue is updated – increased by one, and time of unit arrival is written into file) or can be cancelled (statistics for number of cancelled units is updated – increased by one).

Independently from the channel state, the moment of time when next unit will come to the system is generated according to previously defined arrival distribution.

When current time is not equal to unit arrival time, or when procedure of unit arrival to the system is finished, the system checks if the current time is equal to channel critical time.

In the case that current time is equal to channel critical time, check of channel state is performed. If channel is in the state of work (Wo), it means that servicing of unit is finished. After that failure check is performed i.e. current time is correlated with previously defined moment of time for failure check. If the current time is greater, first the new time for failure check is generated (current time + time interval of failure check). After that, the channel is tested to failure in the following way: if random number generated according to Uniform distribution (in interval 0 – 1) is less than probability of channel failure, it is assumed that channel has failure. Then channel changes its state to state of repair (Re). The next channel critical time is generated as current time + time of failure duration (obtained from failure duration time distribution). In the case that probability of channel failure is less than generated random number or current time is less than time moment of failure check, two cases can occur depending on instant number of units in a queue. First

case, there are no units in a queue, which means that channel should change the state to waiting (Wa) and current time is recorded as the beginning of channel waiting. Next change of channel state is defined with arrival of the next unit to the system. Second case, there is some number of units in a queue, means that first unit in a queue (FIFO strategy) is going to servicing. At that moment the number of units in a queue is decreased by one and servicing time is generated according to previously defined service distribution. The time which unit spent in a queue and time which unit will spend in a system are calculated and written in appropriate files.

Statistics for average time that unit spends in a queue is now updated and the next moment of channel state change is generated (current time + generated servicing time for this unit).

In the case that current time is equal to channel critical time and if the channel does not work it has to be in the state of repair (Re). It is the same case like case when the channel is finished servicing and procedure, when probability of failure is less than random number generated according to Uniform distribution or current time is less than time moment defined for failure check, is repeated depending on instant number of units in a queue.

In the case that current time is not equal to channel critical time the update of appropriate statistics is performed. It assumes update of statistics for: average number of units in a queue and in the system, probability that channel is occupied, system state probabilities, channel state probabilities and instant number of units in a queue and in the system are written to appropriate files.

With this one cycle of simulation model work is finished. After that, current time in the system is increased by one discrete time unit and described procedure is repeated until current time in the system becomes equal to time of simulation experiment duration.

When simulation experiment is finished final values for system performances (P_{ops} , P_{oc} , P_{pr} , N_w , N_{ws} , t_w , t_{ws}), state probabilities, channel state probabilities as well as average time that unit spends in a queue and in the system and average time that the channel waits for servicing are calculated.

Duration time for simulation experiment is defined upon design practice and it is chosen to be 750,000 seconds, which is approximately 26 shifts of one month of system work.

4. SYSTEM PERFORMANCE BEHAVIOUR

Behaviour of system performances can be obtained and shown through the change of their statistical distributions due to variation of arrival and servicing flow.

The analysis of system performance behaviour covers the change of arrival and servicing flow described by Erlang distributions E1, E5 and E10. The combination of this distributions as arrival and service flows is realized through the following servicing systems: E1/E1/1/∞, E5/E5/1/∞ and E10/E10/1/∞.

The analysis is given for servicing system with infinite queue as a universal model which covers all the other cases with finite number of places in a queue. Due to restricted number of events, which can be realized when servicing systems with finite number of places in a queue are in question, it is not possible to establish rules of parameters change. Those rules can only be obtained when the number of realized events is big enough, meaning that queue is infinite. The obtained results can be used in both models with finite and infinite queues.

Further analysis is applied to the following continuous system performances:

- t_w – average time that unit spends in the queue,
- t_{ws} – average time that unit spends in the system and
- t_c – time that channel waits for servicing,

as well as on discrete system performances:

- N_w – average number of units in the queue and
- N_{ws} – average number of units in the system.

The analysis covers three characteristic values of the system offered load ρ : 0.2 – low utilisation, 0.6 – normal utilisation and 0.9 – high utilisation or overloading.

Testing and verification of sample belongings, obtained by simulation model, to statistical distributions is done by χ^2 – test. The program for χ^2 – test was specially developed for this purpose [4].

5. RESULTS OF ANALYSIS

The results of analysis for the system performances are given in the tables 2, 3, 4, 5 and 6 [2],

Table 2. Behaviour of t_w

Model	ρ		
	0.2	0.6	0.9
E1/E1/1/∞	÷	E1	E1
E5/E5/1/∞	÷	E1/÷	E1
E10/E10/1/∞	÷	÷	E1

Table 3. Behaviour of t_c

Model	ρ		
	0.2	0.6	0.9
E1/E1/1/∞	E1	E1	E1
E5/E5/1/∞	E4	E2	E2
E10/E10/1/∞	E8	E2	E1

Table 4. Behaviour of N_w

Model	ρ		
	0.2	0.6	0.9
E1/E1/1/∞	G	G	G
E5/E5/1/∞	÷	$P_n = 15$	$P_n = 10$
E10/E10/1/∞	÷	÷	$P_n = 10$

where: B – Bernoulli distribution, P_n – Pascal distribution, G – Geometrical distribution, E_k – Erlang distribution of the order k and ÷ – distribution can not be obtained.

For all five system performances, according to general aspects of increasing of stochastic values in arrival and servicing flow, on one side, and increasing of system offered load, on the other side, a certain conclusion for each performance can be underlined.

The stochastic influence on t_w – average time that unit spends in the queue is always extremely strong for all values of arrival and servicing flow and system offered load.

The stochastic influence of arrival and servicing flow on t_c – time that channel waits for servicing, is bigger than stochastic influence system offered load.

For low stochastically influence of arrival and service flow with low system offered load there are no units in a queue, and therefore there is no N_w – average number of units in the queue.

Following two system performances (t_{ws} , N_{ws}) are most important, because they are directly connected the with total cost of system performance [5]. So, the results of their behaviour are given in expanded form i.e. diagrams of obtained theoretical probability distributions, which can easily give probability that unit will stay in the system less than arbitrarily given time.

For t_{ws} – average time that unit spends in the system, stochastic influence through arrival and service flow is bigger than the stochastic influence of the system offered load.

For N_{ws} – average number of units in the system, the stochastic influence rises with the rise of offered load of the system, while influence of arrival and servicing flow is less significant.

Table 5. Behaviour of t_{ws}

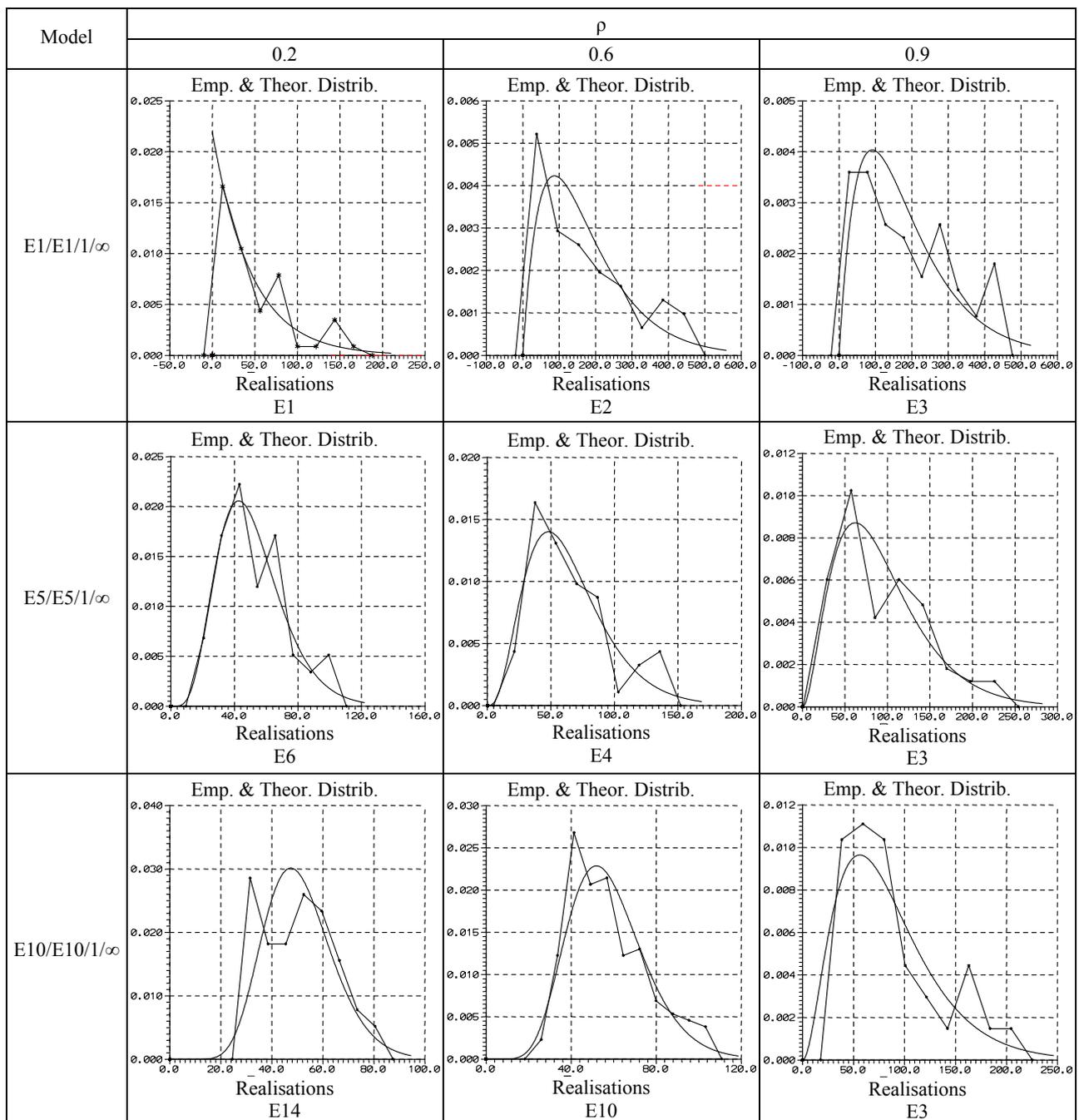
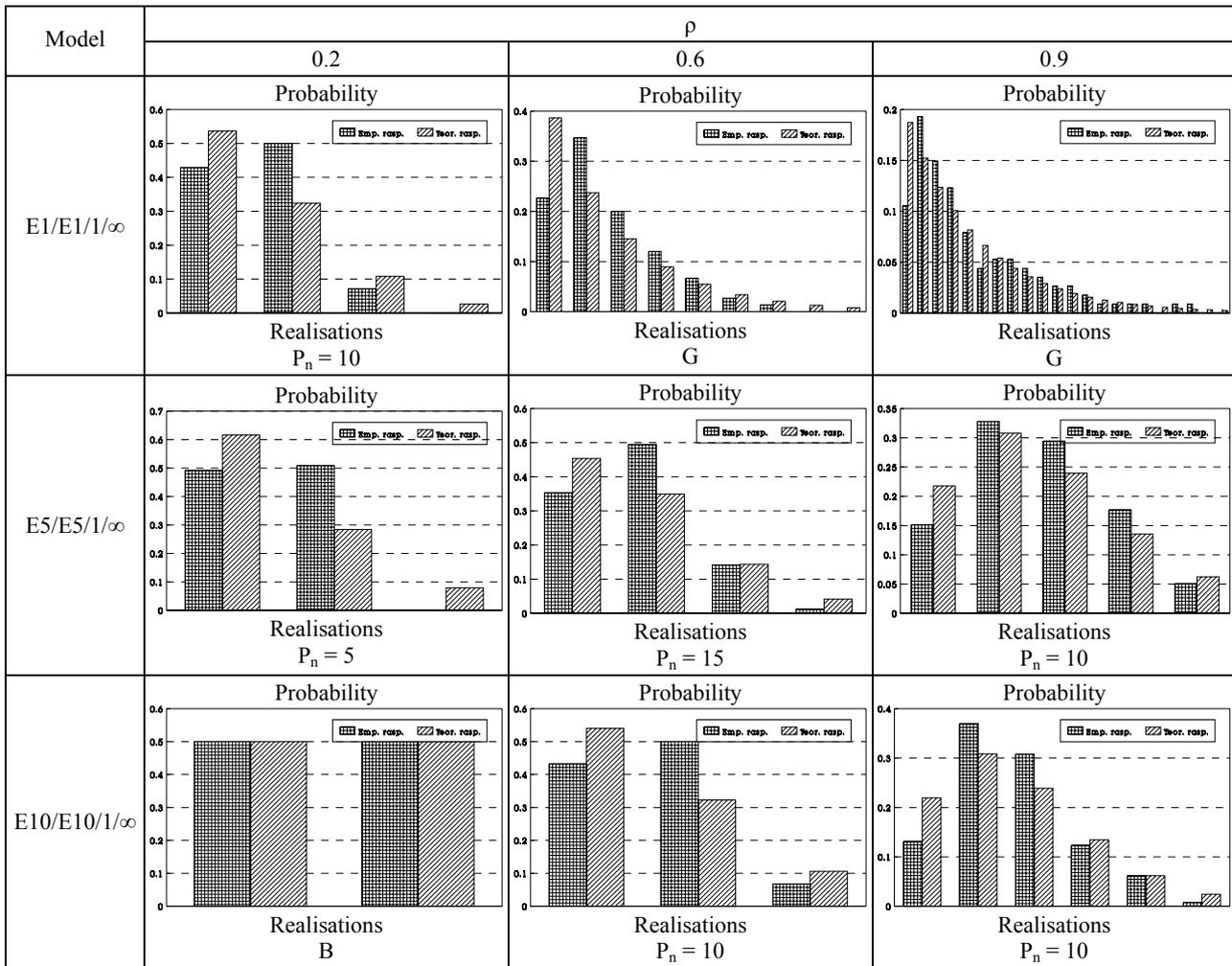


Table 6. Behaviour of N_{ws}



6. CONCLUSION

The work of warehouse system is under a strong influence of stochastic arrival and servicing flow. Due to this fact, simulation modelling is one of the possible ways of analysing such system.

Only part of simulation modelling results is presented in this paper. The focus is on determining the statistical distributions of observed system performances. The stochastic influence was taken into consideration using different statistical distributions for arrival and servicing flow and failure. The other influence on system performances is given through the offered load of the system. Combination of both influences and variety of each brought up the conclusion about stochastic behaviour of the observed system performances presented by statistical distributions.

The application of obtained statistical distributions is in warehouse system design specially in part of mathematical modelling of system behaviour. Those conclusions can be used in the design of new or for re-designing of the existing warehouse systems.

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**АНАЛИЗА ВИСОКОРЕГАЛНИХ СКЛАДИШТА
БАЗИРАНА НА УТИЦАЈУ СТОХАСТИЧКИХ
ПАРАМЕТАРА**

Душан Б. Петровић, Угљеша С. Бугарић, Зоран М. Петровић

У раду је анализирана фамилија високорегалних складишта са једним уређајем за опслуживање. Циклус рада, као основни елемент система, прорачунат је према *FEM* пропису са посебним освртом на промену брзине кретања уређаја за опслуживање у времену. Различите теоријске расподеле за моделирање долазног тока,

опслуживања и отказа су коришћене у раду за одређивање стохастичког утицаја на перформансе складишног система. Остали утицаји на перформансе складишног система су обухваћени преко промене оптерећења система. Комбинација оба утицаја и њихова промена доводе до закључка о стохастичком понашању посматраних перформанси система приказаних преко различитих теоријских расподела. Посебно је развијен симулациони модел који је коришћен за анализу рада складишних система.