Fragment Mass Distribution of Naturally Fragmenting Warheads

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The paper considers statistical aspects of high explosive warhead fragmentation. The modeling of fragment mass distribution is of great importance for determination of fragmenting warhead efficiency. Seven relevant theoretical fragment mass distribution models are reviewed: the Mott, the generalized Mott, the Grady, the generalized Grady, the lognormal, the Weibull and the Held distribution. Comparison of these models with representative experimental database of 30 fragmenting projectiles has shown, generally, a very good correspondence between theoretical models and experimental data. The goodness of fit analysis has indicated that the generalized Mott, the generalized Grady and the Weibull distribution enable the best description of experimental fragment mass distribution data. Further comparison of these models based on the median analysis prefers the generalized Grady distribution, and its bimodal characteristic can be physically justified. The suggested theoretical fragment mass distribution law can be applied in a complex fragmenting projectile efficiency simulation model.

Keywords: *high-explosive warhead, fragmentation, fragment mass distribution, statistical analysis.*

1. INTRODUCTION

The modeling of fragmentation process is of the utmost importance for design, redesign and efficiency analysis of high-explosive (HE) projectiles. The fragment mass distribution along with the initial fragment velocity, the spatial and the shape distribution of fragments, enables the complete characterization of a fragmentation process. Natural fragmentation of HE projectile is the result of complex processes of explosive detonation, gas products expansion and behavior of the casing material under the intensive impulse loads [1]. The final character and distribution of cracks in the projectile casing determine the shape, the size and the mass of formed fragments. There are several approaches to the fragmentation problem – probabilistic [2-4], energetic [5], approach based on fracture mechanics [6], etc. Having in mind the complexity of underlying physics, the semi-empirical approach based on the mentioned theoretical results, as well as experimental data, seems to be a promising approach to the fragment mass distribution problem [7].

There are a number of concurrent models that define the fragment mass distribution law, without consensus which of them is the most suitable for description of fragments generated by the HE projectiles. The idea is to analyze these models and, through comparison with experimental results, suggest the "optimal" law that can be used in a complex HE projectile efficiency simulation model.

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2. FRAGMENT MASS DISTRIBUTION LAWS

Fragment mass distribution is usually described by a cumulative distribution function, rather than a probability density function (histogram), which is more sensitive to the scatter of the fragment masses data. The cumulative number of fragments $N_T(m) = N_T(>m)$ is the total number of fragments with the mass greater than m, and alternatively, the cumulative fragment mass $M_T(m) = M_T(>m)$ is the total mass of all fragments with individual mass greater than m.

In this paper, relative (normalized) cumulative distributions $N(m)$ and $M(m)$ will be used

$$N(m) = \frac{N_T(m)}{N_0}, \quad M(m) = \frac{M_T(m)}{M_0}, \tag{1}$$

where N_0 is the total fragment number and M_0 is the total mass of fragments. The relation between the cumulative distributions is

$$\frac{\mathrm{d}M}{\mathrm{d}m} = \frac{m}{\bar{m}} \frac{\mathrm{d}N}{\mathrm{d}m}. \tag{2}$$

The average fragment mass (distribution mean), which is the most important characteristic of the distribution, is determined by

$$\bar{m} = \frac{M_0}{N_0} = \int_0^\infty N(m)\mathrm{d}m . \tag{3}$$

A very useful numerical property of the distribution is the median, which is defined by

$$N(\tilde{m}_N) = \tfrac{1}{2}, \quad M(\tilde{m}_M) = \tfrac{1}{2}. \tag{4}$$

There are numerous distribution laws that are used to describe a real distribution of the HE projectile fragments. The most relevant distribution laws will be briefly outlined.

Mott distribution. In his classic works [8], based on the two-dimensional geometric statistics, Mott had formulated the well-known fragment distribution law in the form

$$N(m) = \exp\left[-\left(\frac{m}{\mu}\right)^{\frac{1}{2}} \right]. \tag{5}$$

Generalized Mott distribution. Mott had argued that in three-dimensional fragmentation of thick-walled cylinder, where fragments do not retain the inner and outer surface of original cylinder, exponent ⅓ instead ½ in (5) would be more appropriate. Introducing exponent λ in (5), we get the generalized Mott distribution (e.g. [9,10]) as

$$N(m) = \exp\left[-\left(\frac{m}{\mu}\right)^{\lambda} \right]. \tag{6}$$

This distribution corresponds to the two-parametric Weibull distribution.

Grady distribution. Following Mott's approach based on the Poisson distribution of fracture points, Grady and Kipp [2] established an alternative paradigm, defined also in [11], and proposed the simple linear exponential distribution

$$N(m) = \exp\left(-\frac{m}{\mu} \right). \tag{7}$$

This distribution law is a special case of the generalized Mott distribution law, (6), for $\lambda = 1$.

Generalized Grady distribution. Considering statistically inhomogeneous fragmentation, Grady and Kipp [2] analyzed the three-parametric generalization of distribution defined by (7) as follows:

$$N(m) = f\exp\left(-\frac{m}{\mu_1} \right) + \left(1 - f\right)\exp\left(-\frac{m}{\mu_2} \right). \tag{8}$$

Lognormal distribution. Observing multiplicative nature of fragmentation process, several authors (e.g. [12]) suggested the lognormal distribution for describing the fragment mass distribution:

$$N(m) = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{\ln m - \mu}{\sqrt{2}\sigma} \right) \right], \tag{9}$$

where erf(•) is the error function $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \mathrm{e}^{-t^2} \mathrm{d}t$.

Weibull distribution. The two-parametric Weibull distribution (also known as the Rosin-Ramler distribution), originally used for the description of the grain size distribution in grinding processes, defines the normalized cumulative mass as

$$M(m) = \exp\left[-(\tfrac{m}{\mu})^{\lambda} \right]. \tag{10}$$

Using (2), one gets the relative cumulative number of fragments:

$$N(m) = \frac{1}{\Gamma\left(1 - \frac{1}{\lambda}\right)} \Gamma\left(1 - \frac{1}{\lambda}, \left(\frac{m}{\mu}\right)^{\lambda} \right), \ \lambda > 1 . \tag{11}$$

In (11), $\Gamma(a, x)$ is the upper incomplete gamma function $\Gamma(a, x) = \int_x^{\infty} t^{a-1} \mathrm{e}^{-t} \mathrm{d}t$.

Held distribution. Held [13] introduced the relation between the cumulative mass and the cumulative fragment number in the form

$$M(n) = M_0 \left[1 - \exp\left(-Bn^{\lambda} \right) \right], \tag{12}$$

where n is the cumulative number of fragments sorted in descending order, and $M(n)$ is the total mass of these fragments. The mass of each particular fragment can be calculated by

$$m_n = M(n) - M(n-1) . \tag{13}$$

Transformation of (12), using (2) leads to the implicit form of the cumulative number distribution:

$$m = M_0 B \lambda N_T^{\lambda-1}(m) \exp\left[-B N_T^{\lambda-1}(m) \right] . \tag{14}$$

The review of the fragment mass distribution laws is given in Table 1. The medians can be easily calculated from (4).

In the earlier paper [14], it had been shown that the Strømsøe-Ingebrigtsen distribution [15] does not represent substantial improvement of the Mott law. It is also discussed that the widely applicable power-law distribution (e.g. [16]) cannot successfully describe the HE projectile fragmentation. Finally, the Gilvarry distribution [17] and the Lin distribution [18] has not been analyzed here, regarding a four-parameter fit impractical.

3. COMPARISONS WITH EXPERIMENTS

In order to validate the presented theoretical distribution models, the comparison with experimental data has been performed. Experimental data from [19] (20 projectiles), [13] (3 projectiles), [15] (3 projectiles), [10] (2 projectiles) and [11] (2 projectiles) have been used.

From the aspect of characterization of fragment mass distribution and the analysis of considered models, it would be desirable that the mass of each fragment is exactly known, i.e. experimental results could be presented as an ascendant sequence of fragment masses m_j, j = 1,2,…N_0. However, because of simpler measurement and manipulation with collected fragments, experimental results are usually given in a somewhat less accurate way – in the form of tables with the number of fragments and their total mass in arbitrary defined mass groups. It is therefore clear that selection of too wide ranges of mass groups can distort the real fragment mass distribution. Although in some fragmentation analysis the fragments with the greatest masses are neglected (supposed as the result of irregular fragmentation), all collected fragments have been taken into account in this research.

Table 1. Fragment mass distribution laws and their properties

Distribution	Relative cumulative number of fragments	Relative cumulative mass of fragments	Distribution mean
	$N(m) = N(>m)$	$M(m) = M(>m)$	$\bar{m}$
Mott	$e^{-\left(\frac{m}{\mu}\right)^{\frac{1}{2}}}$	$\frac{1}{2}\Gamma\left(3,\left(\frac{m}{\mu}\right)^{\frac{1}{2}}\right)$	2μ
Generalized Mott	$e^{-\left(\frac{m}{\mu}\right)^{\lambda}}$	$\frac{1}{\Gamma\left(1+\frac{1}{\lambda}\right)}\Gamma\left(1+\frac{1}{\lambda},\left(\frac{m}{\mu}\right)^{\lambda}\right)$	$\Gamma\left(1+\frac{1}{\lambda}\right)\mu$
Grady	$e^{-\frac{m}{\mu}}$	$\left(1+\frac{m}{\mu}\right)e^{-\frac{m}{\mu}}$	μ
Generalized Grady	$fe^{-\frac{m}{\mu_1}}+\left(1-f\right)e^{-\frac{m}{\mu_2}}$	$\frac{1}{\bar{m}}\left[f(\mu_1+m)e^{-\frac{m}{\mu_1}}+(1-f)(\mu_2+m)e^{-\frac{m}{\mu_2}}\right]$	$f\mu_1+(1-f)\mu_2$
Log-normal	$\frac{1}{2}\left[1-\mathrm{erf}\left(\frac{\ln m-\mu}{\sqrt{2}\sigma}\right)\right]$	$\frac{1}{2}\left[1-\mathrm{erf}\left(\frac{\ln m-(\mu+\sigma^2)}{\sqrt{2}\sigma}\right)\right]$	$e^{\mu+\sigma^2/2}$
Weibull	$\frac{1}{\Gamma\left(1-\frac{1}{\lambda}\right)}\Gamma\left(1-\frac{1}{\lambda},\left(\frac{m}{\mu}\right)^{\lambda}\right),\ \lambda>1$	$e^{-\left(\frac{m}{\mu}\right)^{\lambda}}$	$\frac{\mu}{\Gamma\left(1-\frac{1}{\lambda}\right)}$
Held	$M(m)=1-e^{-BN_T^{\lambda}(m)},\ m=M_0B\lambda N_T^{\lambda-1}(m)e^{-BN_T^{\lambda-1}(m)}$		n/a

The parameters in the theoretical distribution laws are calculated by minimizing the deviation of the theoretical from the real distribution in the sense of the least squares method:

$$\min\sum_{i=1}^{n}\left[F_t(m_i,p_1,p_2,...,p_k)-F_e(m_i)\right]^2 , \quad (15)$$

where $F_t(m,p_1,p_2,...,p_k)$ is the theoretical distribution function, p_i are the parameters in distribution functions (depending on model, the function has one, two or three free parameters that should be optimized), and $F_e(m)$ is the experimentally determined distribution function.

Characteristic diagrams of experimental data and the corresponding optimized theoretical models for the typical experimental projectile are given in Figure 1.

In order to evaluate and compare the goodness of fits, the degrees of freedom adjusted coefficient [20] of determination is used:

$$\bar{R}^2=1-\frac{S_D}{S_T}\frac{n-1}{n-k-1}, \quad (16)$$

where S_D and S_T are

$$S_D=\sum_{j=1}^{n}(F_t(m_j)-F_e(m_j))^2,$$

$$S_T=\sum_{j=1}^{n}(F_t(m_j)-\bar{F}_e)^2 . \quad (17)$$

In (17), $\bar{F}_e$ is the mean of an experimental distribution, n is the number of mass groups and k is the number of adjustable parameters in a distribution law. The statistic $\bar{R}^2$ is defined as a proportion of variability in experimental data explained by a statistical model, taking into account the number of free parameters. Therefore, this statistics enables the determination of fitting degree among the observed and the theoretical data.

The survey of coefficient of determination values for the analyzed theoretical distribution models applied on the experimental data for 30 projectiles is given in Table 2 (target function is $N(m)$).

Generally, all models (except the Grady's) are good approximation of experimental data, but the generalized Mott, the generalized Grady and the Weibull distribution have the highest coefficients of determination. Similar results are obtained from the cumulative fragment mass $M(m)$ optimization (Fig. 2).

In addition to the analysis of compatibility of theoretical distribution laws and experimentally determined distribution, the consideration of correspondence between appropriate measures of distribution central tendencies is also important. From the definition of these parameters, (3) and (4), it is obvious that the average fragment mass $\bar{m}$ and the median $\tilde{m}_N$ are dominantly dependent on the total number of generated fragments N_0. The experimental determination of the overall number of fragments N_0, especially in the case of 3D fragmentation and numerous fragments ($N_0 > 1000$), can be unreliable.

Namely, despite the advanced fragment recovery techniques, a certain number of fragments from the smallest mass group can not be collected after the experiment. These “missing” fragments can significantly influence the evaluation of the total number of fragments N_0, but their impact on the total mass of generated fragments M_0 is negligible. Therefore, the mentioned experimental error has

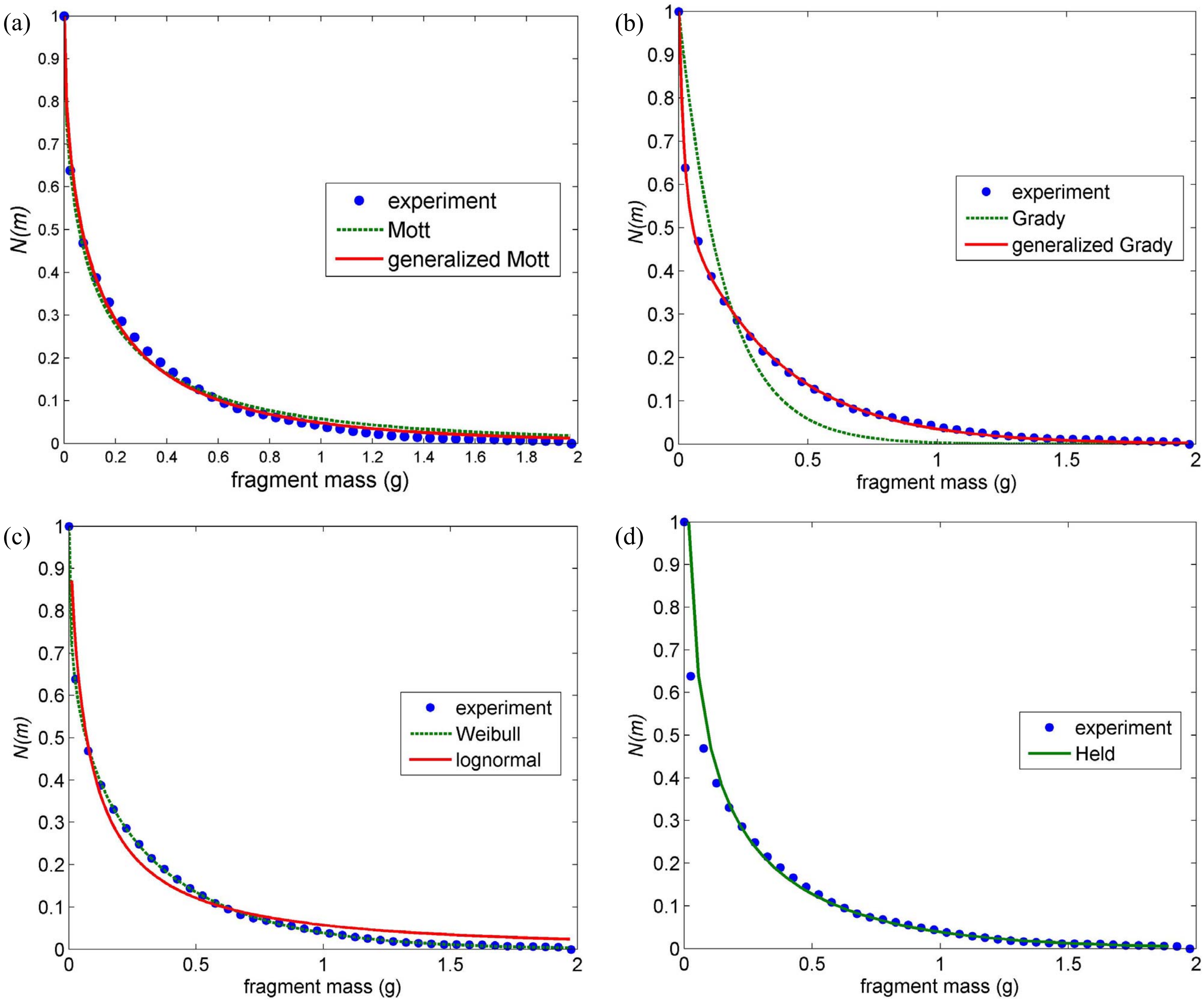

Figure 1. Comparison of the relative cumulative fragment number experimental data for projectile 1 [19] with: (a) the Mott and the generalized Mott, (b) the Grady and the generalized Grady, (c) the Weibull and the lognormal and (d) the Held fragment mass distribution model

minimal influence on the median $\tilde{m}_M$, and this measure will be used as the criterion for further comparison of experimental data and theoretical results for three distributions.

The relative errors of the median $\tilde{m}_M$ for three considered theoretical models and 30 experimental projectiles are compared in Table 3. The analysis of the results presented in Table 3 indicates a satisfactory level of compatibility between the experimentally determined and the theoretically predicted values of the median $\tilde{m}_M$. This criterion prefers the generalized Grady distribution comparing to other two empirical models.

Thus, the conclusion from the performed statistical analysis is that the generalized Grady model provides the best description of the mass distribution of fragments generated by the HE projectiles. Having in mind the main property of this distribution law, this means that fragments have two characteristic sizes (masses), which is shown in Figure 3.

Physically, the finer and coarser fragments can be related to the central cylindrical and the residual portion of the projectile, respectively. Another explanation is that different fragment formation mechanisms in the inner and the outer section of the projectile casing influence the bimodal distribution [1].

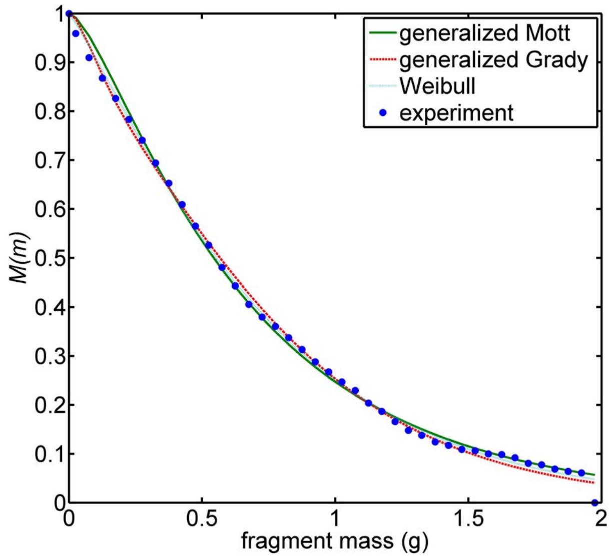

Figure 2. Relative cumulative fragment mass distribution: comparison of the experimental data (projectile 1 from [19]) and the generalized Mott, the generalized Grady and the Weibull distribution

Table 2. The coefficient of determination calculated for different fragment mass distribution laws and 30 experiments. Relative cumulative fragment number *N*(*m*) has been fitted

	Coefficient of determination, $\bar{R}^2$						
	Mott	Gen. Mott	Grady	Gen. Grady	Lognormal	Weibull	Held
1	0.9919	0.9960	0.9271	0.9994	0.9817	0.9997	0.9975
2	0.9946	0.9946	0.9160	0.9999	0.9846	0.9995	0.9899
3	0.9936	0.9936	0.9025	0.9992	0.9787	0.9997	0.9981
4	0.9948	0.9957	0.9252	0.9995	0.9897	0.9994	0.9944
5	0.9896	0.9913	0.8720	0.9994	0.9766	0.9993	0.9988
6	0.9122	0.9951	0.9909	0.9987	0.9807	0.9980	0.9926
7	0.4326	0.9462	0.9026	0.8959	0.9229	0.9521	0.9687
8	0.9984	0.9985	0.9337	0.9973	0.9937	0.9996	0.9606
9	0.9952	0.9984	0.9222	0.9970	0.9946	0.9994	0.9723
10	0.9941	0.9984	0.9120	0.9958	0.9941	0.9994	0.9855
11	0.9990	0.9992	0.9521	0.9985	0.9961	0.9990	0.9805
12	0.9991	0.9991	0.9554	0.9994	0.9964	0.9995	0.9775
13	0.9750	0.9988	0.9599	0.9997	0.9909	0.9961	0.9885
14	0.9884	0.9996	0.9521	0.9990	0.9931	0.9959	0.9967
15	0.9995	0.9997	0.9848	0.9998	0.9990	0.9999	0.9853
16	0.9986	0.9986	0.9667	0.9995	0.9964	0.9999	0.9812
17	0.8879	0.9990	0.9985	0.9990	0.9940	0.9972	0.9760
18	0.9959	0.9971	0.8911	0.9943	0.9951	0.9931	0.9908
19	0.9995	0.9997	0.9553	0.9999	0.9975	0.9991	0.9944
20	0.9990	0.9992	0.9461	0.9996	0.9963	0.9998	0.9899
21	0.9923	0.9996	0.9757	0.9991	0.9991	0.9999	0.9866
22	0.9992	0.9999	0.9933	0.9998	0.9998	0.9999	0.9774
23	0.9960	0.9996	0.9283	0.9930	0.9966	0.9961	0.9937
24	0.9910	0.9980	0.9574	0.9952	0.9893	0.9964	0.9613
25	0.9985	0.9985	0.9351	0.9960	0.9891	0.9988	0.9608
26	0.9561	0.9894	0.8026	0.9966	0.9799	0.9964	0.9916
27	0.9948	0.9995	0.9721	0.9979	0.9967	0.9979	0.9923
28	0.9787	0.9886	0.9483	0.9988	0.9752	0.9961	0.9905
29	0.9104	0.9987	0.8382	0.9707	0.9985	0.9948	0.9986
30	0.9738	0.9992	0.9429	0.9831	0.9992	0.9961	0.9925

Note: data for projectiles 1-20 are taken from [19] (average values for minimum 5 tests for each projectile); results for projectiles 21-23 are from [13], projectiles 24-26 are from [15], 27-28 from [10] and 29-30 from [11] (test cylinders).

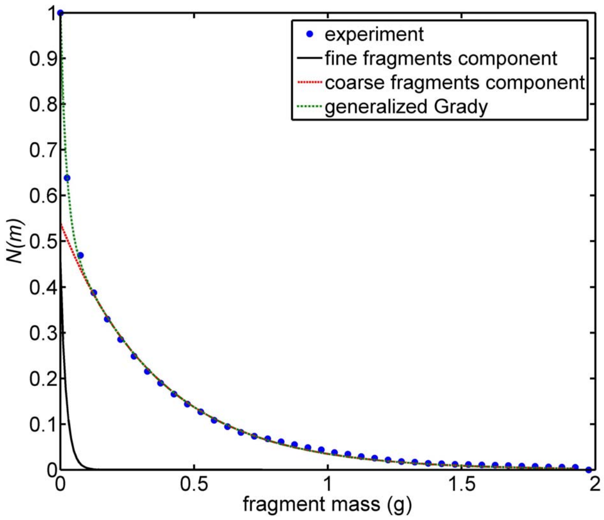

Figure 3. Fine and coarse fragments components of the generalized Grady distribution fit of the experimental data (projectile 1 [19])

4. CONCLUSION

The analysis of the most relevant theoretical fragment mass distribution models has been undertaken. Using the comprehensive statistical approach based on comparison with the representative experimental database of 30 projectiles, it has been concluded that the generalized Grady distribution provides the best description of the mass distribution of fragments generated by detonation of the HE projectiles. The main characteristic of this distribution are physically justified. The suggested distribution model can be applied in HE projectile efficiency modeling.

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Table 3. Relative error of the median $\tilde{m}_M$ for the three analyzed distributions (generalized Mott, generalized Grady and Weibull) based on 30 experimental fragmentation results. Ranks of calculated results are indicated in brackets

	Relative error of median		
	Gen. Mott	Gen. Grady	Weibull
1	0.2459 (3)	0.0554 (1)	0.0876 (2)
2	0.3895 (3)	0.0985 (1)	0.1870 (2)
3	0.2768 (3)	0.0149 (1)	0.0698 (2)
4	0.4140 (3)	0.1160 (1)	0.2213 (2)
5	0.3918 (3)	-0.0006 (1)	0.0762 (2)
6	0.1080 (3)	0.0932 (2)	0.0735 (1)
7	0.0508 (1)	0.3209 (3)	0.0612 (2)
8	0.1751 (3)	-0.1112 (2)	0.0003 (1)
9	0.3604 (3)	-0.0698 (1)	0.0841 (2)
10	0.3769 (3)	-0.0856 (2)	0.0829 (1)
11	0.2425 (3)	0.0515 (1)	0.0831 (2)
12	0.2691 (3)	0.0689 (1)	0.1193 (2)
13	0.0833 (3)	0.0607 (2)	-0.0012 (1)
14	0.1210 (3)	0.0763 (2)	0.0105 (1)
15	0.8198 (3)	0.7621 (1)	0.7933 (2)
16	0.5482 (3)	0.3365 (1)	0.4569 (2)
17	0.1668 (2)	0.1699 (3)	0.1360 (1)
18	0.2058 (3)	-0.0878 (1)	-0.1178 (2)
19	0.2361 (3)	0.1656 (2)	0.1203 (1)
20	0.3259 (3)	0.1357 (1)	0.1666 (2)
21	1.2373 (3)	0.1409 (1)	0.3726 (2)
22	0.6374 (2)	0.6458 (3)	0.6212 (1)
23	0.0592 (1)	-0.5295 (3)	-0.4608 (2)
24	-0.0417 (1)	-0.2846 (3)	-0.2746 (2)
25	0.3071 (3)	-0.1612 (2)	-0.0497 (1)
26	2.8439 (3)	-0.0271 (1)	0.3419 (2)
27	0.1269 (3)	-0.0902 (1)	-0.1147 (2)
28	0.3896 (3)	0.2499 (1)	0.2702 (2)
29	4.5932 (3)	-0.9182 (2)	-0.7485 (1)
30	0.8376 (3)	-0.6784 (2)	-0.3911 (1)

Note: The same as for Table 2.

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РАСПОДЕЛА МАСЕ ФРАГМЕНАТА БОЈНИХ ГЛАВА СА ПРИРОДНОМ ФРАГМЕНТАЦИЈОМ

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У раду се разматрају статистички аспекти фрагментације разорних бојних глава. Моделирање расподеле масе фрагмената је од великог значаја при одређивању ефикасности разорних пројектила. Дат је преглед седам релевантних теоријских модела расподеле масе парчади: Мотов (Mott) модел, генерализовани Мотов модел, Грејдијев (Grady) модел, генерализовани Грејдијев модел, логнормална расподела, Вајбулова (Weibull) и Хелдова (Held) расподела. Поређење ових модела са репрезентативном базом података за 30 разорних пројектила показало је веома добро подударање теоријских и експерименталних резултата. Анализа коефицијената детерминације указала је да генерализована Мотова, генерализована Грејдијева и Вајбулова расподела најбоље описују резултате експеримената. Даље поређење ових модела засновано на анализи медијане фаворизује генерализовану Грејдијеву расподелу чија се бимодалност може физички оправдати. Предложени закон расподеле масе фрагмената може се применити у сложеном симулационом моделу ефикасности разорних пројектила.