A Constant Wall Temperature Microbearing Gas Flow

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A non-isothermal two-dimensional compressible gas flow in a slider microbearing with constant and equal wall temperature is investigated in this paper analytically. The slip flow is defined by the Navier-Stokes and energy continuum equations along with the velocity slip and the temperature jump first order boundary conditions. Knudsen number is in the range of 10^{-3} to 10^{-1}, which corresponds to the slip flow. The gas flow is subsonic and the ratio $\kappa M^2/Re$ is taken to be a small parameter. Moreover, it is assumed that the microbearing cross-section varies slowly, which implies that all physical quantities vary slowly in x-direction. The model solution is treated by developing a perturbation scheme. The first approximation corresponds to the continuum flow conditions, while the second one involves the influence of inertia as well as rarefaction effect. The analytical solutions of the pressure, velocity and temperature for moderately high Reynolds numbers are obtained.

Keywords: *microbearing, non-isothermal, slip flow, inertia, analytical solution, low Mach number, high Reynolds number.*

1. INTRODUCTION

Gas lubrication is a component of most micro-electro-mechanical systems (MEMS) such as microbearings, micropumps, microvalves or magnetic disk storages [1]. The hard disc industry demands nanometer distances between slider with read/write head and rotating recording disk. The gas slider bearing flow is traditionally modelled by the Reynolds lubrication equation which is derived from the Navier-Stokes and continuity equations under the no slip continuum boundary conditions. The thickness of the lubricating film in microdevices is of the order of the mean free path of gas molecules and the continuum theory is not applicable. A wide range of Knudsen numbers is possible in microdevice flows, but the slip flow regime with $10^{-3} < Kn < 0.1$ is the most frequent. Therefore, solutions for such flow conditions in microbearings are very useful.

Analytical and numerical investigations of the slip gas flow in microbearings were performed. Burgdorfer [2] made the Reynolds equation correction by including the Maxwells first order slip conditions at the wall. Mitsuya [3] set up 1.5-order slip model for ultra thin gas lubrication. Hsia and Domoto [4] developed the second order model by incorporating their second order boundary condition in the Reynolds lubrication equation. They also carried out experiments with different gases in microbearings, and compared the obtained load carrying capacity with analytical results. Sun et al. [5] incorporated expression for the effective viscosity in the Navier-Stokes equation and obtained modified Reynolds equation. Bahukudumbi and Beskok [6] developed semi analytical model for gas lubricated microbearings. They remarked that the viscosity coefficient depends on the Knudsen number. Since the proposed relation is not general, the rarefaction correction parameter is introduced in this relation. Values of the rarefaction correction parameter are defined for certain Knudsen number and surface accommodation parameter values by comparing obtained flow rate results with numerical solutions of the Boltzmann equation under the same conditions [7,8]. Finally, the derived function of the viscosity coefficient is introduced in the model, and the new modified Reynolds equation is obtained. Liu and Ng [9] analysed the posture effects of a slider air bearing and the influence of the lower plate velocity on the pressure distribution and velocity field with a direct simulation Monte Carlo method.

The model developed in this paper for non-isothermal microbearing gas flow with constant and equal wall temperature is based on already verified results for a isothermal pressure driven gas flow in a microchannel with slowly varying cross-section [10] and isothermal gas flow in the microbearing [11,12]. The low Mach number gas flow is considered, which enables a definition of the small parameter $\varepsilon = \kappa M^2/Re$. Moreover, it is assumed that the channel cross-section varies slowly, which also implies that all physical quantities vary slowly in the flow direction. All these assumptions together with the defined relations between the Reynolds, Mach and Knudsen number and the small parameter ε, enable a precise estimation of each term in the dimensionless governing equations, as well as in the boundary conditions. In the solving procedure, the pressure, velocity and temperature are expressed as the perturbation series of the Knudsen number. The system of nonlinear second order differential equations is obtained, and it is solved numerically.

2. PROBLEM DESCRIPTION

Two-dimensional and compressible gas flow in microbearing with constant wall temperature (as presented in Figure 1) is considered. Although the

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temperature of the walls is the same and constant, and the distance between the walls is of the micron scale, the gas flow is not treated as isothermal. In that way, apart from the continuity, momentum, equation of state, and slip boundary condition at the wall, the energy equation and temperature jump boundary condition has to be involved too. These equations are transformed into a dimensionless form by the introduction of the following scales: exit microbearing height $\tilde{h}_e$ for all lengths, wall velocity $\tilde{u}_w$ for all velocity components, walls temperature $\tilde{T}_w$ for temperature and the pressure and density are scaled with the corresponding values $\tilde{p}_e$ and $\tilde{\rho}_e$ at the channel outlet cross-section. Then the assumption of the low Mach number flow conditions enables a definition of the small parameter

$$\varepsilon = \kappa M_e^2 / Re_e \quad (1)$$

where $\kappa = c_p/c_v$ is the ratio of specific heats, M_e is the referent Mach number value defined as

$$M_e = \tilde{u}_w \Big/ \sqrt{\kappa\, \tilde{p}_e / \tilde{\rho}_e} \quad (2)$$

and Re_e is the referent Reynolds number

$$Re_e = \tilde{\rho}_e \tilde{u}_w \tilde{h}_e / \tilde{\mu} \, . \quad (3)$$

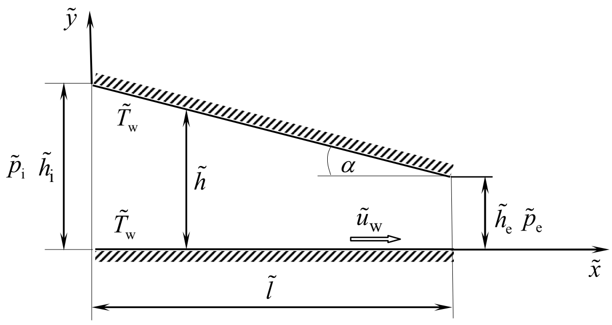

Figure 1. Slider microbearing geometry

Dynamic viscosity $\tilde{\mu}$ is assumed to be constant. Now, the expression for the small parameter ε follows: $\varepsilon = \tilde{\mu}\tilde{u}_w \big/ \left(\tilde{p}_e \tilde{h}_e\right)$.

The assumption of the slowly varying channel cross-section $\alpha \approx \varepsilon << 1$, where α is the channel wall inclination (Fig. 1) implies that all flow parameters change vary slowly in the x-axes direction, which is explicitly expressed by the introduction of the slow coordinate $\xi = \varepsilon x$. Also, the crosswise velocity component v is much smaller than the streamwise component u, which leads to the following relation: $v(x,y) = \varepsilon V(\xi,y)$, $V = O(1)$.

The continuity equation, the Navier-Stokes equations for the stream-wise and cross-wise directions and the equation of state in dimensionless form are:

$$\partial(pu)/\partial\xi + \partial(pV)/\partial y = 0 \quad (4)$$

$$\kappa M_e^2 p\left(u\frac{\partial u}{\partial \xi} + V\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial \xi} + \frac{\partial^2 u}{\partial y^2} + O\left(\varepsilon^2\right) \quad (5)$$

$$\frac{\partial p}{\partial y} = O\left(\varepsilon^2\right) \quad (6)$$

$$\kappa M_e^2 Pr\rho\left(u\frac{\partial T}{\partial \xi} + V\frac{\partial T}{\partial y}\right) = M_e^2 Pr(\kappa - 1)u\frac{\partial p}{\partial \xi} +$$

$$+\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right) + M_e^2 Pr(\kappa - 1)\left(\frac{\partial u}{\partial y}\right)^2 + O\left(\varepsilon^2\right) \quad (7)$$

$$p = \rho T \, . \quad (8)$$

Further, all dimensionless parameters are denoted without a bar, i.e. pressure as p, stream-wise velocity component as u, temperature as T, etc. The thermal conductivity $\tilde{k}$ is treated as constant. Therefore, the Prandtl number $Pr = c_p \tilde{\mu} / \tilde{k}$ is constant, where c_p is the specific heat at the constant pressure. In accordance with the slip flow theory, the gas velocity and the temperature at the wall in the dimensionless form are respectively:

$$y = 0: \; u\text{-}u_w = \frac{\left(2-\sigma_{v1}\right)}{\sigma_{v1}} \frac{Kn_e\sqrt{T}}{p}\frac{\partial u}{\partial y}, \; V = 0 \quad (9)$$

$$y = h(\xi): \; u = -\frac{\left(2-\sigma_{v2}\right)}{\sigma_{v2}} \frac{Kn_e\sqrt{T}}{p}\frac{\partial u}{\partial y}, \; V = u\frac{\mathrm{d}h}{\mathrm{d}\xi} \quad (10)$$

$$y = 0, \; T = T_w + \frac{\left(2-\sigma_T\right)}{\sigma_T}\frac{2\kappa}{(\kappa+1)}\frac{Kn_e}{Pr}\frac{\sqrt{T}}{p}\frac{\partial T}{\partial y} \quad (11)$$

$$y = h(\xi), \; T = T_w - \frac{\left(2-\sigma_T\right)}{\sigma_T}\frac{2\kappa}{(\kappa+1)}\frac{Kn_e}{Pr}\frac{\sqrt{T}}{p}\frac{\partial T}{\partial y} \, . \quad (12)$$

These are well-known Maxwell-Smoluchowski first-order slip boundary conditions, where σ_v and σ_T are momentum and thermal accommodation coefficients and Kn_e is the reference Knudsen number defined as $Kn_e = \tilde{\lambda}_e / \tilde{h}$. Since the molecular mean-free path is defined as $\tilde{\lambda} = \tilde{\mu}\sqrt{\pi R\tilde{T}/2} \big/ \tilde{p}$ [6], the relation between local $Kn = \tilde{\lambda}/\tilde{h}$ and the reference Knudsen number Kn_e is $Kn = Kn_e\sqrt{T}/p$. Furthermore, the relation between Kn_e, M_e and Re_e is:

$$Kn_e = \frac{M_e}{Re_e}\sqrt{\frac{\pi\kappa}{2}} \, . \quad (13)$$

The presumption of extremely subsonic flow in the slip regime enables the relation between the Mach and Knudsen numbers and the small parameter ε: $\kappa M_e^2 = \beta\varepsilon^m$, $\beta = O(1)$ and $Kn_e = \eta\varepsilon^n$, $\eta = O(1)$. Due to the relation between the Reynolds, Mach and Knudsen number (13) and the definition of the small parameter ε, the exact expression for the Reynolds number and relations among introduced parameters m and n, as well as β and η follows: $Re_e = \beta\varepsilon^{m-1}$, $2n + m = 2$ and $\eta = \sqrt{\pi/2\beta}$. Supposition of the low Mach and Knudsen number flow, limited m and n values to positive domain, which together with the relation 2n + m = 2 gives that this parameters must be in the following ranges: 0 < m < 2 and 0 < n < 1. In this frame two characteristic problems could be analysed: *Re* < 1 if 1 < m < 2 $\Rightarrow$ 0 < n < 1/2 and *Re* > 1 if 0 < m < 1 $\Rightarrow$ 1/2 < n < 1. In derivation of the Reynolds equation for lubrication theory, the inertia term is neglected which corresponds to the low Reynolds number case which was already obtained [11]. In this paper solution for *Re* > 1 is presented. Values for parameters m and n are chosen to enable attendance of the inertia effect together with the rarefaction: m = n = 2/3. The relations for the dimensionless numbers are: $Re_e = \beta\varepsilon^{-1/3}$, $\kappa M_e^2 = \beta\varepsilon^{2/3}$, $Kn_e = \eta\varepsilon^{2/3}$.

All dependant variables from (4) to (8), i.e. pressure, temperature and velocity components, are presented in the form of perturbation series

$$f = f_0 + \varepsilon^{2/3} f_1 \tag{14}$$

where f_0 is the solution for the flow with no-slip boundary conditions, and f_1 comprise the corrections for the inertia effect and the slip on the wall. The systems of equations for two approximations together with corresponding boundary conditions are obtained by substitution perturbation series for pressure and velocities in (4) to (8) and (9) to (12). In order to catch up the slip effect already in the second approximation, the power for small parameter in the second term on the r.h.s. of (14) is the same as for the Knudsen number ($\varepsilon^{2/3}$). As inertia in (5) is of the order $\kappa M_e^2 = \beta\varepsilon^{2/3}$, for the perturbation series in (14), the inertia effect is included also in the second approximation. The velocity, temperature and pressure perturbation expressions, in the form of equation (14), are introduced in the continuity equation (4), the momentum conservation equation (5) energy equation (7), equation of state (8) and the boundary conditions (9) to (12). From these equations, the terms of the order $O(1)$ and $O(\varepsilon^{2/3})$ are extracted, and the following sets of equations are obtained

- for $O(1)$

$$\frac{\partial(\rho_0 u_0)}{\partial\xi} + \frac{\partial(\rho_0 V_0)}{\partial y} = 0 \tag{15a}$$

$$\frac{\partial p_0}{\partial\xi} = \frac{\partial^2 u_0}{\partial y^2} \tag{15b}$$

$$\frac{\partial^2 T_0}{\partial y^2} = 0 \tag{15c}$$

$$p_0 = \rho_0 T_0 \tag{15d}$$

$$y = 0:\ u_0 = 1,\ V_0 = 0,\ T_0 = 1 \tag{15e}$$

$$y = h(\xi):\ u_0 = 0,\ V_0 = 0,\ T_0 = 1 \tag{15f}$$

- for $O(\varepsilon^{2/3})$

$$\frac{\partial(\rho_0 u_1 + \rho_1 u_0)}{\partial\xi} + \frac{\partial(\rho_0 V_1 + \rho_1 V_0)}{\partial y} = 0 \tag{16a}$$

$$\beta\rho_0\left(u_0\frac{\partial u_0}{\partial\xi} + V_0\frac{\partial u_0}{\partial y}\right) = -\frac{\partial p_1}{\partial\xi} + \frac{\partial^2 u_1}{\partial y^2} \tag{16b}$$

$$\beta Pr\rho_0\left(u_0\frac{\partial T_0}{\partial\xi} + V_0\frac{\partial T_0}{\partial y}\right) = \beta Pr\frac{(\kappa-1)}{\kappa}u_0\frac{\partial p_0}{\partial\xi} + \frac{\partial^2 T_1}{\partial y^2} + \beta Pr\frac{(\kappa-1)}{\kappa}\left(\frac{\partial u_0}{\partial y}\right)^2 \tag{16c}$$

$$p_1 = \rho_1 T_1 \tag{16d}$$

$$y = 0:\ u_1 = \frac{2-\sigma_{v1}}{\sigma_{v1}}\frac{\eta\sqrt{T_0}}{p_0}\frac{\partial u_0}{\partial y},\ V_1 = 0,\quad T_1 = \frac{2-\sigma_T}{\sigma_T}\frac{2\kappa}{(\kappa+1)}\frac{\eta}{Pr}\frac{\sqrt{T_0}}{p_0}\frac{\partial T_0}{\partial y} \tag{16e}$$

$$y = h(\xi):\ u_1 = -\frac{(2-\sigma_{v2})}{\sigma_{v2}}\frac{\eta\sqrt{T_0}}{p_0}\frac{\partial u_0}{\partial y},\ V_1 = u_1\frac{\mathrm{d}h}{\mathrm{d}\xi},\quad T_1 = -\frac{(2-\sigma_T)}{\sigma_T}\frac{2\kappa}{(\kappa+1)}\frac{\eta}{Pr}\frac{\sqrt{T_0}}{p_0}\frac{\partial T_0}{\partial y}. \tag{16f}$$

The solution procedure for each system of these equations is the same. The approximations of the temperature T_0, T_1 are derived from the corresponding energy equations (15c) and (16c), then stream-wise velocity component u_0, u_1 from the corresponding momentum equations (15b) and (16b). The pressure approximations p_0, p_1 are derived from the corresponding continuity equations (15a) and (16a). Temperature, velocity and pressure equations for the first two approximations are obtained in the following form

- the first approximation

$$T_0 = 1 \tag{17}$$

$$u_0 = 1 - \frac{y}{h}\left(1 + p_0'\frac{h^2}{2\xi_l}\right) + p_0'\frac{y^2}{2\xi_l} \tag{18}$$

$$\left[h^3(p_0 p_0')\right]' - 6\xi_l(p_0 h)' = 0 \tag{19}$$

- the second approximation

$$T_1 = -Pr\beta\frac{\kappa-1}{\kappa}\left[\frac{(p_0')^2}{8\xi_l^2}y^4 - \frac{p_0'}{2\xi_l}\left(\frac{hp_0'}{2\xi_l} + \frac{1}{h}\right)y^3 + \left(\frac{2p'}{\xi_l} + \left(\frac{hp_0'}{2\xi_l}\right)^2 + \frac{1}{h^2}\right)\frac{y^2}{2} - \left(\frac{hp_0'}{2\xi_l} + \frac{1}{2h}\right)y\right] \tag{20}$$

$$u_1 = A\left(y^6 - h^5 y\right) + B\left(y^5 - h^4 y\right) + C\left(y^4 - h^3 y\right) + D\left(y^3 - h^2 y\right) + E\left(y^2 - hy\right) - \frac{2-\sigma_{v2}}{\sigma_{v2}}\frac{\eta}{hp_0}\left(\frac{p_0'h}{2\xi_l} - \frac{1}{h}\right)y + \frac{2-\sigma_{v1}}{\sigma_{v1}}\frac{\eta}{p_0}F \tag{21}$$

where:

$$A=\frac{\beta p_0}{60\xi_l^3}\left(\frac{p_0' p_0''}{2}-\frac{(p_0 p')'}{3p_0}\right) \quad (22)$$

$$B=\frac{1}{20}\left[\left(F\frac{p_0'}{2\xi_l}\right)'-F\frac{(p_0 p_0')'}{6p_0\xi_l}+\right.$$
$$\left.+\left(\frac{(p_0 p_0' h)'}{2\xi_l}+\left(\frac{p_0}{h}\right)'\right)\frac{p_0'}{2p_0\xi_l}\right] \quad (23)$$

$$C=\frac{1}{12}\left[FF'+\frac{p_0''}{2\xi_l}+\right.$$
$$\left.+\left(\frac{(p_0 p_0' h)'}{2\xi_l}+\left(\frac{p_0}{h}\right)'\right)\frac{F}{2p_0}-\frac{(p_0')^2}{\xi_l p_0}\right] \quad (24)$$

$$D=\frac{1}{6}\left(F'-F\frac{p_0'}{p_0}\right) \quad (25)$$

$$E=\frac{p_1'}{2\xi_l} \quad (26)$$

$$F=-\left(\frac{p_0' h}{2\xi_l}+\frac{1}{h}\right) \quad (27)$$

$$\left[-p_0\left(\frac{Ah^7}{42}+\frac{Bh^6}{30}+\frac{Ch^5}{20}+\frac{Dh^4}{12}+\frac{Eh^3}{6}\right)\right]'+$$
$$+\left[-\frac{2-\sigma_{v2}}{\sigma_{v2}}\frac{\eta h}{2}\left(\frac{p_0' h}{2\xi_l}-\frac{1}{h}\right)+\frac{2-\sigma_{v1}}{\sigma_{v1}}\eta F h\right]'-$$
$$-Pr\beta\frac{\kappa-1}{\kappa}\left\{p_0\left[\left(\frac{p_0'}{\xi_l}\right)^3\frac{h^7}{112}+\left(\frac{p_0'}{\xi_l}\right)^2\frac{Fh^6}{16}+\right.\right.$$
$$+\frac{3h^5}{20}\frac{p_0'}{\xi_l}\left(\frac{p_0'}{2\xi_l}+F^2\right)+\frac{h^4 F}{8}\left(\frac{3p_0'}{\xi_l}+F^2\right)+\frac{h^2 F}{2}+$$
$$\left.\left.+\frac{h^3}{6}\left(\frac{p_0'}{\xi_l}+3F^2\right)\right]\right\}'+\left[p_1\left(\frac{p_0' h^3}{6\xi_l}+\frac{Fh^2}{2}+h\right)\right]'=0 \quad (28)$$

where the stream-wise coordinate is $X=\xi/\xi_l$. The prime denotes d/dX, while h is the channel cross-section in dependence on X defined as: $h(X)=h_i-X(h_i-1)$, where h_i is dimensionless parameter defined as ratio of the inlet and outlet microbearing height $h_i=\tilde{h}_i/\tilde{h}_e$.

The channel length expressed by the slow coordinate is: $\xi_l=\varepsilon\tilde{l}/\tilde{h}_e=\tilde{\mu}\tilde{u}_W\tilde{l}/\left(\tilde{p}_e\tilde{h}_e^2\right)$. The bearing number definition is $\Lambda=6\tilde{\mu}\tilde{u}_W\tilde{l}/\left(\tilde{p}_e\tilde{h}_e^2\right)$ and it is evident relation with parameter ξ_l

$$\xi_l=\Lambda/6 \quad (29)$$

This means that pressure distribution and velocity field which are obtained from (17) to (21) and (28) is defined by bearing number Λ, referent Knudsen number Kn_e, channel geometry h and parameter η. The system of the two second order differential equations (19) and (28) that enables the prediction of pressure along the microbearing, demands four boundary conditions at the channel inlet – p_0, p_1, p_0', p_1'. But, the first derivate of pressure is not known. This problem is overcome by using the known pressure at channel outlet instead of the first pressure derivate at the inlet, which imposed the application of the shooting method for the solving of system of equations. The boundary conditions for pressure, prescribed at the inlet and outlet are $X=0$, $p=p_0=1$, $p_1=0$ and $X=1$, $p=p_0=1$, $p_1=0$.

3. RESULTS AND DISCUSSION

The defined perturbation analysis shows that the inertia is already involved in the second order momentum equation for the moderately high Reynolds number flows (16b). Moreover, except for the sole conduction term in the first approximation, the convection, dissipation and rate at which work is done in compressing the element of fluid terms appear in the second approximation of the energy equation, too (16c). Hence, the acquired gas temperature field is non-isothermal.

In addition, it has been proven here that the temperature solution does not comprise the temperature jump effect at the wall even in the second approximation. However, the velocity slip boundary condition is present in the second approximation of the problem solution. The obtained results for the pressure, velocity and temperature field for the moderately high Reynolds number flow conditions depend on the Reynolds, Knudsen and Prandtl numbers.

All results shown in Figures 2, 3 and 4 are obtained for $\sigma_v=1$, $\sigma_T=1$, $\kappa=1.4$ and Pr = 0.667. Besides, the results are obtained for the ratio of inlet to outlet heights $\tilde{h}_i/\tilde{h}_e=2$, two bearing number values, $\Lambda=1$ and $\Lambda=10$, two Knudsen number values $Kn_e=0.1$ and $Kn_e=0.05$. Besides the bearing number Λ, the inlet to outlet ratio $\tilde{h}_i/\tilde{h}_e$ and the Knudsen number, which are usually defined flow conditions in the microbearing according with the Reynolds lubrication theory which negligible the inertia influence, in this model the parameter η is also need for the solving of the system of differential equations (17) to (21) and (28). This is the consequence of the incorporation of the inertia effect in the model. All results presented in this paper are obtained for $\eta=1$.

The inertia, slip and temperature influence on the pressure distribution along the microbearing are presented in Figure 2. It is evident that the inertia leads to the pressure increase in the microbearing, while the non-isothermal influence for the same and constant wall temperature flow conditions leads to the pressure decrease. Hence, for bearing number $\Lambda=1$, calculation which comprises non-isothermal effect together with inertia lids to pressure lower then pressure obtained by omitting non-isothermal and inertia influence. Effect of the temperature field on the pressure distribution in the microbearing is less pronounced for higher values of the bearing number (Λ = 10). On Figure 2 results obtained for continuum flow condition (Kn = 0) are also presented and it is obvious that slip effect at the wall leads to the lower pressure in the microbearing. Figure 2 shows excellent agreement of presented model with no inertia and non-isothermal effect (dashed line) with Fukui and Kaneko numerical solution of the Boltzmann equation [7,8], obtained also with inertia and non-isothermal influence omit.

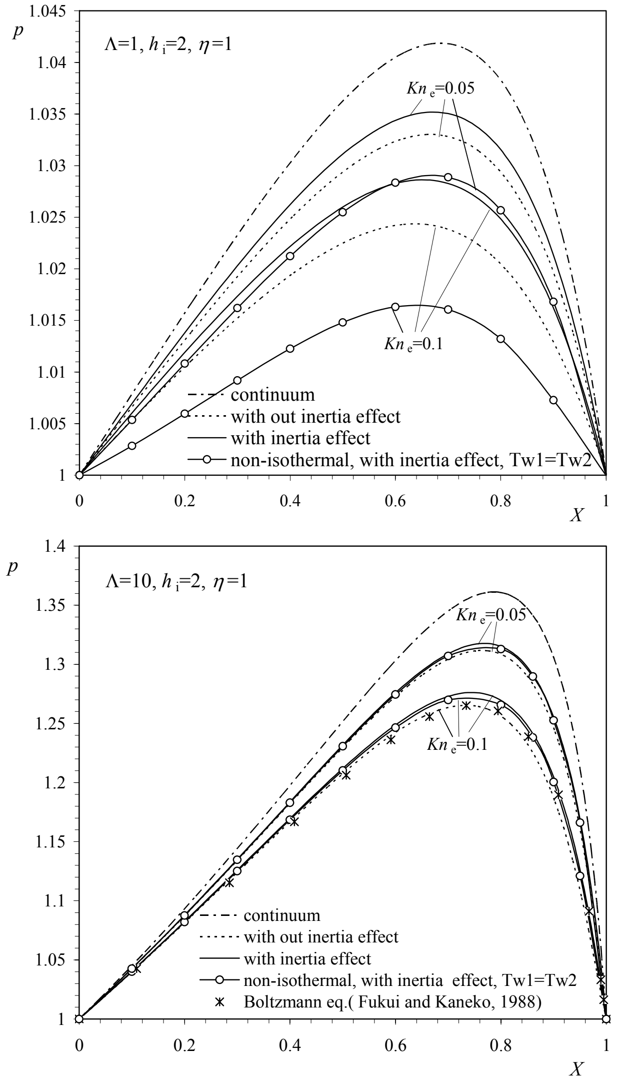

Figure 2. Pressure distribution in microbearing for Kn_e = 0.1, h_i = 2, η = 1 and two bearing numbers: Λ = 1 and Λ = 10

The temperature profiles in the microbearing gas flow at various cross-sections are depicted in Figure 3. These are obtained from the (17) and (20). The first approximation corresponds to the continuum and isothermal flow conditions, while in the second one the non-isothermal effect appears as the influence of the conduction, the dissipation and the rate at which work is done in compressing the element of fluid terms. The convection term wanes in the second approximation of the temperature, since its first approximation is T_0 = 1 = const. Also, this caused no temperature jump effect in (16e) and (16f) and the fluid temperature at the wall is $\tilde{T}_w$, while the temperature values in the remaining flow field are different than the wall temperature. The lowest temperatures are at channel exit.

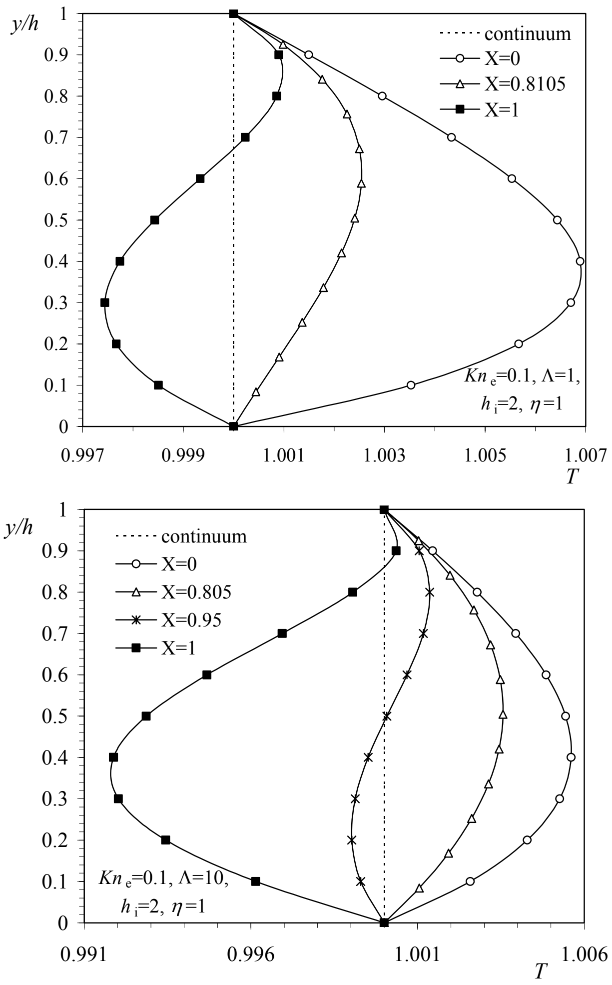

Figure 3. Temperature profiles in microbearing with constant and equal wall temperature at various cross-sections, for Kn_e = 0.1, h_i = 2, η = 1, and different bearing numbers: Λ = 1 and Λ = 10

In Figure 4 velocity profiles for the flow condition defined with Kn_e = 0.1, $\tilde{h}_i / \tilde{h}_e = 2$, Λ = 1, η = 1 are presented. The full lines present velocity profiles with inertia influence, obtained by omitting non-isothermal effect, while the dashed lines present velocity profile with no inertia and non-isothermal effect. Full lines with circles show velocity profiles in microbearing obtained by including non-isothermal effect along with inertia. The difference between velocity profiles calculated from the four different models (continuum, slip flow conditions, slip flow conditions with inertia effect and slip flow conditions with inertia and non-isothermal effect) is evident. The non-isothermal and inertia effect have no influence on the slip at walls. Slip velocity at the upper wall increase along the microbearing, while at the lower decrease.

4. CONCLUSION

The analytical solutions for the non-isothermal subsonic slip gas flow in the microbearing with constant wall temperature have been obtained. The results for the pressure, velocity and temperature fields have been presented for the moderately high Reynolds number flow conditions. The small parameter has been defined by (1) and the Mach, Knudsen and Reynolds numbers have been expressed with it. Moreover, the exact relation between these numbers has been used for a precise estimation of each term's contribution in the continuum, momentum and energy equations, as well as in the boundary conditions. All physical quantities have been assumed with perturbation series. The first two approximations have been taken into account. The first one corresponds to the continuum flow conditions, while the second represents the contribution of the rarefaction effect. In addition, for the moderately high Reynolds numbers, the second approximation includes the inertia effect, as well as a non-isothermal character of the flow. Hence, although the temperature of the walls is the same and constant, and the distance between walls is of micron scale, the obtained gas temperature profile is non-uniform. It has been shown that, for the prescribed flow conditions, the temperature solution does not comprise the temperature jump effect at the wall even in the second approximation. However, the velocity slip boundary condition is present in the second approximation of the problem solution.

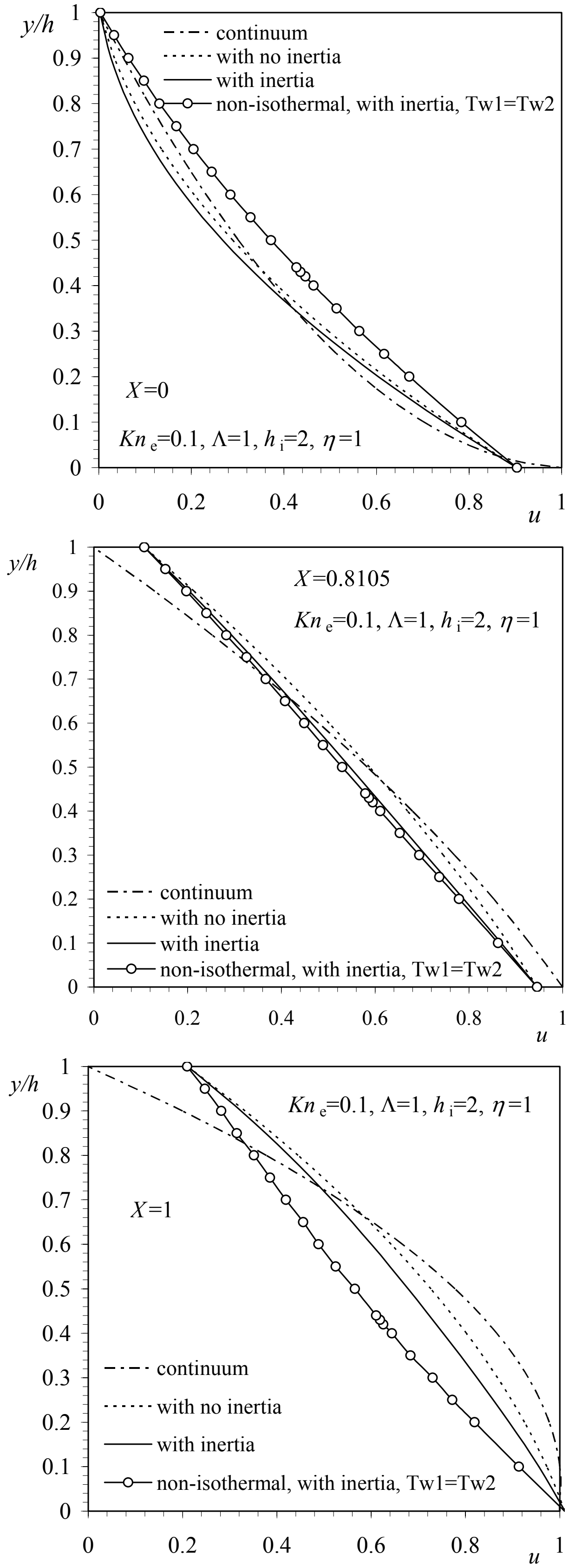

Figure 4. Velocity profiles in microbearing for $Kn_e = 0.1$, $h_i = 2$, $\eta = 1$, $\Lambda = 1$ at three cross-sections: $X = 0$, $X = 0.8105$ and $X = 1$

The presented method incorporates the energy equation, which leads to the prediction of the temperature field and its influence on the pressure and velocity distribution. Besides, this analytical model enables the inclusion of inertia effect on the pressure, velocity and temperature fields. It is concluded that inertia and non-isothermal effect have the opposite influence on the pressure filed.

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СТРУЈАЊЕ ГАСА У МИКРОЛЕЖАЈИМА СА ЗИДОВИМА КОНСТАНТНИХ ТЕМПЕРАТУРА

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У раду је анализирано неизотермско дводимензијско стишљиво струјање гаса у микролежају константних и једнаких температура зида. Вредност Кнудсеновог броја је између 10^{-3} и 10^{-1}, што одговара режиму струјања са клизањем. Овај режим струјања дефинише се једначинама континуума: Навије-Стоксовом и једначином енергије и граничним условом клизања и температурског скока на зиду. Струјање гаса је дозвучно, па се мали параметар дефинише као $\varepsilon = \kappa M^2/Re$. Осим тога претпостављено је се попречни пресек микроканала мења споро, што доводи до споре промене свих величина у правцу струјања. Решење је добијено пертурбационом методом. Прва апроксимација представља решење за случај струјања гаса без клизања, док се у другој апроксимацији јавља утицај клизања и инерције. Добијена су аналитичка решења за расподелу притиска, брзине и температуре у микролежају при умерено великим вредностима Рејнолдсовог броја.