Dragan Lj. Milanović

Assistant Professor University of Belgrade Faculty of Mechanical Engineering

Dragan D. Milanović

Associate Professor University of Belgrade Faculty of Mechanical Engineering

Mirjana Misita

Assistant Professor University of Belgrade Faculty of Mechanical Engineering

The Evaluation of Risky Investment Projects

The subject of this investigation are risky investment projects that some project evaluation methods (NPV, AE, IRR etc) are insufficient to be applied to, because the risk of wrong decision-making is considerable. That is the reason why in such cases some other methods taking into account project risk must be implemented. In practice, the NPV and IRR methods are most often used for project evaluation. However, these methods cannot provide reliable evaluation of project cost-effectiveness for the case of risky projects. Therefore, in this paper another combination of methods is proposed that would give a more reliable picture of the project. Using a concrete example, NPV analysis, sensitivity analysis and Monte Carlo simulation were done. In this way, certain results were arrived at providing a more reliable evaluation of an engineering investment project, which represents the contribution of this paper.

Keywords: risky project, NPV method, sensitivity analysis, Monte Carlo simulation.

1. INTRODUCTION

Until the 1960s the dominant methods for evaluating engineering investment projects were the so-called static methods (payback period, accounting rate of return, breakeven point) that did not involve time value of money, i.e. the fact that money available earlier is more worth than money available later.

The beginning of transition towards recognizing the dynamic methods (net present value, internal rate of return, profitability index, discount return period) was marked by the World Bank's promotion of this group of methods for the needs of evaluating projects, strategies, enterprises as a whole. The promotion of analysis becomes evident through introducing and applying the Discount Account, which formalized the appreciation of the time value of money concept in the evaluation procedure itself.

The experience of thirty years or so in applying dynamic methods pointed to certain system weaknesses. The key weakness is that these methods are all based on fixed cash-flow projections, meaning that their implementation assumes or claims that future is certain.

To eliminate these weaknesses, "the third generation" of methods for financial evaluation of projects appeared in the late 1990s headed by the method of real options, Monte Carlo simulation, decision tree, and optimization methods.

The above methods emerged as a response to inadequacy of traditional methods for evaluating the projects in terms of uncertainty. The foundation for their implementation should be the creation of diverse scenarios and simulations of future effects and recognition of the fact that managerial flexibility has a value and such value must be included in the value of the project as a whole. The main advantage of new methods for evaluating the projects is a significant reduction of space for making mistakes in the projection of essential inputs and making a final investment decision [1].

2. LITERATURE REVIEW

In principle, whenever it is possible to adequately evaluate the issue of risky investment with analytical methods, in general, it is better to do so. However, there are many investment situations that cannot be solved using analytical methods. In that case Net Present Value (NPV) distribution must be developed (or some other measure for investment value – Annual Equivalent value (AE), Internal Rate of Return (IRR) etc) by simulation methods, such as Monte Carlo simulation [2].

The term Monte Carlo refers to a broad spectrum of mathematical models and algorithms, whose most prominent feature is the use of random numbers in solving various problems. These are most commonly mathematical problems whose solutions cannot be analytically found, or there are no efficient numerical algorithms for them. In addition, they are often used to test the results obtained by analytical or some other methods. Due to a volume of mathematical operations and recursion, the Monte Carlo method has become widely used only with rapid development of computers. In general, to term something a Monte Carlo experiment, it is enough to use random numbers to examine the likely results of the experiment.

The name Monte Carlo was introduced by Stanislaw Ulam, John von Neumann and Nicholas Metropolis in 1946 even though the idea had existed long before. Enrico Fermi employed similar methods in 1930 to determine the properties of the newly discovered neutron. In the 1940s and 1950s the USA Forces used these methods intensively in the development of the atomic and later the hydrogen bomb despite a very limited power of computers in those days.

Received: February 2010, Accepted: April 2010 Correspondence to: Dr Mirjana Misita Faculty of Mechanical Engineering, Kraljice Marije 16, 11120 Belgrade 35, Serbia E-mail: mmisita@mas.bg.ac.rs

Today, the Monte Carlo method is applied in various areas of science: from computational mathematics, physical chemistry and statistical physics to the development of semi-conductors, computer graphics and finances [3].

Other scientists also consider Monte Carlo method applicable in different fields. For example, Woller [4] says that you can find Monte Carlo methods used in everything from economics to nuclear physics to regulating the flow of traffic. Of course, the way they are applied varies widely from field to field. But, strictly speaking, to call something a Monte Carlo experiment, all you need to do is to use random numbers to examine some problem.

Young [5] considers that for practical cash-flow sets the only way to estimate the entire worth distribution is by Monte Carlo simulation, which samples the possible combinations of parameters in proportion to their probability of occurring. The weakness of Monte Carlo simulation is that it requires a computer and that it does not provide elegant, compact answer such as a formula, but rather gives tables and histograms.

In to date practice of applying Monte Carlo simulation for evaluation of engineering investment projects the most commonly used input parameters have been revenues, operating costs, investment costs, salvage value etc. Lončar [1], for example, says that the goal of Monte Carlo simulation is to evaluate the distribution of dependent variable probabilities (Net Present Value of the project) based on distribution of probabilities of a larger number of independent input variables (demand, prices, costs, investments and the like).

A logical sequence of operations for risky engineering investment project simulation is as follows [2]:

- Identification of all parameters affecting the result of applied method for the evaluation of investment (e.g. net present value – NPV method);
- Classification of all parameters into two groups: parameters whose values are known with certainty and parameters whose values cannot be determined at the time of decision-making – random variables;
- Identification of relationship between variables by using NPV or some other equation. These equations create a model that is attempted to be analyzed;
- Determining distributions for all random variables. Distributions can be based on statistical data, if they are available, or on subjective judgment (e.g. for some random variables the distributions of probabilities can be based on the past objective indicators if decision-makers feel the same trend will continue in the future, but if not, subjective probabilities must be employed);
- Execution of the Monte Carlo testing and calculations of distribution parameters;
- Interpretation of simulation results.

3. DESCRIPTION OF THE INVESTIGATED PROBLEM

Using a concrete example of the case of postal charges, it will be first determined if this is a risky project (in the

sense whether fixed cash-flow projections yield net present value approximating zero), and if this is the case, a model for project evaluation will be applied, containing NPV method, sensitivity analysis, Monte Carlo simulation, and adequate decision will be made.

The example of the case of postal charges is as follows: if more precise scales for weighting the consignments were installed, this would reduce errors in calculating postal charges. Annual savings were estimated at 420 \in , while investment costs of scales procurement and its salvage value after a 5-year exploitation period were estimated at 1500 \in and 150 \in respectively. Current discount rate is 12 % and tax rate is 10 %. Annual depreciation charges amounted to

$$\frac{1500 - 150}{5} = 270 \ \ (1)$$

For the above case, the NPV equation can be written in the following form:

$$v_{(k)} = -i + (r+d) \cdot \frac{(1+k)^n - 1}{k \cdot (1+k)^n} + \ell \cdot \frac{1}{(1+k)^n} .$$
 (2)

Incorporating the given values into (2), $v_{(12 \%)} =$ 45.05 € is obtained. Relying on the NPV method only, a decision can be made on accepting this project, because NPV is larger than zero. However, as this value is approximating zero, sensitivity method will be applied in further analysis, which will show NPV sensitivity to variations of these three parameters becoming now three variables. Sensitivity analysis reveals how much NPV will change, depending on the given variables change. It starts from "the base case" obtained by employing the most probable values for each variable. A certain variable is changed then by a certain percentage above and below the most probable value, keeping other variables constant. Thereafter, NPV is calculated for each of these values. Varying the variables by ± 5 %, \pm 10 % and \pm 15 % yielded the following results:

Table 1. Summative results for varying of annual savings, investment and salvage value

Deviation [%]	Annual savings NPV [€]	Investment NPV [€]	Salvage value NPV [€]
- 15 %	- 159.34	270.05	32.29
- 10 %	- 91.21	195.05	36.54
- 5 %	- 23.08	120.05	40.80
0	45.05	45.05	45.05
5 %	112.79	- 29.95	49.31
10 %	181.32	- 104.95	53.57
15 %	249.45	- 179.95	57.82

As the above table (Tab. 1) does not show clearly the NPV sensitivity level in varying of these three variables, elasticity coefficients of NPV will be calculated for these variables [6].

Elasticity coefficient of NPV in relation to investment:

$$e_i = \frac{i}{v} \tag{3}$$

$$e_i = \frac{1,500}{45.05} = 33.30 .$$
 (4)

Elasticity coefficient of NPV in relation to annual savings:

$$e_r = \frac{r}{v} \cdot \frac{(1+k)^n - 1}{k \cdot (1+k)^n}$$
(5)

$$e_r = \frac{378}{45.05} \cdot \frac{(1+0.12)^5 - 1}{0.12 \cdot (1+0.12)^5} = 30.25.$$
 (6)

Elasticity coefficient of NPV in relation to salvage value:

$$\mathbf{e}_{\ell} = \frac{\ell}{\nu} \cdot \frac{1}{\left(1+k\right)^n} \tag{7}$$

$$e_{\ell} = \frac{150}{45.05} \cdot \frac{1}{(1+0.12)^5} = 1.89$$
. (8)

Net present value is sensitive the most to variations in investment values (highest coefficient value) and slightly less sensitive to variations in annual savings values, while it is least sensitive to variations in salvage value. Sensitivity analysis indicates that justifiable acceptance of this project is questionable when investment and annual savings values already change by 5 % upward and downward, respectively, because NPV is then smaller than zero.

4. MONTE CARLO SIMULATION

To solve the problem of such risky investment, Monte Carlo simulation method will be employed with inputs for a certain probabilities distribution. The above presented model of the development of Monte Carlo simulation will be applied (Items 1 to 6):

- 1. Inputs:
 - annual savings (*r*),
 - investment (*i*),
 - salvage value (ℓ).
- 2. All three inputs are random variables:
 - annual savings (X),
 - investment (Y),
 - salvage value (W).

Table 2. Simulation distribution of NPV frequency

3. Equation:

$$v_{(k)} = -i + (r+d) \cdot \frac{(1+k)^n - 1}{k \cdot (1+k)^n} + \ell \cdot \frac{1}{(1+k)^n} \Longrightarrow$$

$$v_{(12\%)} = -Y + 3.6048X + 0.5674W + 97.3296 . (9)$$

4. Probabilities distributions of random variables were obtained on the basis of statistical experiment and they are:
X – normal distribution: μ = 420 €, σ = 40 €

(standardized normal distribution Z (0; 1) will be used hereafter), Y – triangular distribution: minimum value L =

Y – triangular distribution: minimum value L = 1350 €, maximum value H = 1725 €, most likely value (mode) $M_o = 1500 \text{ €}$ [2], W – discrete distribution:

$$W = \begin{cases} 130 & 140 & 150 & 160 & 170 \\ 0.1 & 0.2 & 0.4 & 0.2 & 0.1 \end{cases}.$$
 (10)

- 5. Execution of Monte Carlo testing: 100 random tests were carried out by applying tables of random numbers between 0 and 1 and tables of random normal numbers (z).
- 6. The results are as follows: One hundred recursions yielded 100 net present values suitable to classify into 10 classes to make the procedure easier. The highest obtained NPV amounts to 415.95 €, while the lowest to 347.61 €. Class width is (415.95 + 347.61):10 = 76.356.

It can be assumed that with a large number of recursions relative frequencies are the representative of proportions that would be obtained if all possible combinations were tested. There remains to calculate arithmetic mean \overline{v} (which represents the expected current value of the basic set), empirical variance s^2 and standard deviation s (s^2 is an empirical adequate to the variance σ^2 – the characteristic of primary wholeness dispersion, whose part is a sample, while s is standard deviation differing a little from σ for high values of N).

$$\overline{\nu} = \frac{1}{100} \sum_{i=1}^{10} NSV_i \cdot f_i = 14.32 \quad \in \tag{11}$$

$$s^{2} = \frac{1}{100} \sum_{i=1}^{10} (NSV_{i})^{2} \cdot f_{i} - (\overline{NSV})^{2} = 29,984.59 (12)$$

Classes	Class midpoint	Frequency	Cumulative frequency	Relative frequency	Cumulative relative frequency
(-347.61) - (-271.25)	- 309.43	4	4	0.04	0.04
(-271.24) - (-194.88)	- 233.08	11	15	0.11	0.15
(- 194.87) - (- 118.51)	- 156.73	12	27	0.12	0.27
(-118.50) - (-42.16)	- 80.38	11	38	0.11	0.38
(-42.15) - 34.21	- 4.03	13	51	0.13	0.51
34.22 - 110.56	72.34	16	67	0.16	0.67
110.57 – 186.91	148.71	15	82	0.15	0.82
186.92 - 263.26	225.08	12	94	0.12	0.94
263.27 - 339.61	301.45	4	98	0.04	0.98
339.62 - 415.95	377.80	2	100	0.02	1.00
		100		1.00	

$$s = \sqrt{s^2} = \sqrt{29,984.59} = 173.16$$
 €. (13)

Believing that 100 tests was a sufficient number for the present project, relative frequencies from the above table (Tab. 2) can be interpreted as probabilities. Net present values in the range from $-347.61 \notin$ to $415.95 \notin$ indicate the probability of approximately 50 % of the project loss. In NPV distribution the expected value is $14.32 \notin$ and standard deviation $173.16 \notin$.

5. CONCLUSION

The subject of this investigation are risky investment projects that some methods for evaluation of projects (NPV, AE, IRR etc) are insufficient to be applied to, because the risk of wrong decision-making is considerable. A concrete example of a risky project demonstrated that NPV based on fixed cash-flow projection is not a sure indicator of project profitability. Sensitivity analysis showed high NPV sensitivity to variations in two key inputs – annual savings and investment, while elasticity coefficients indicated the sequence of inputs according to their impact on NPV.

The application of Monte Carlo simulation showed even lower value of expected NPV than originally designed and a high degree of dispersion around mean value, which indicates a highly risky project. Accordingly, this concrete project although subjected to three analyses (NPV method, Sensitivity analysis, Monte Carlo simulation) remains in the domain of high risk, because relatively small changes in input values result in negative net present value i.e. loss in possible realization of this project. Therefore, the proposed model for evaluation of risky projects assesses this project unprofitable, and as such it should be rejected.

REFERENCES

- [1] Lončar, D.: *Monte Carlo Simulation*, Faculty of Economics, University of Belgrade, Belgrade, 2009, (in Serbian).
- [2] Park, C.S.: *Contemporary Engineering Economics*, Addison-Wesley, Reading, 1993.
- [3] Posavec, M.: Monte Carlo method, On-Line Article, 2009, (in Croatian), http://www.rep.hr/ vijesti/strucni-clanci/strucni-clanak-monte-carlometoda/211/

- [4] Woller, J.: The basics of Monte Carlo simulations, On-Line Article, 1996, http://www.chem.unl.edu/ zeng/joy/mclab/mcintro.html
- [5] Young, D.: *Modern Engineering Economy*, John Wiley & Sons, New York, 1992.
- [6] Dubonjić, R. and Milanović, D.Lj.: *Engineering Economy*, ICIM, Kruševac, 2005, (in Serbian).

NOMENCLATURE

- v net present value
- *i* investment
- *r* annual after-tax savings
- *d* after-tax depreciation charges
- ℓ salvage value
- *k* discount rate
- n number of years of project exploitation
- \overline{v} arithmetic mean
- s standard deviation
- s^2 empirical variance

ОЦЕЊИВАЊЕ РИЗИЧНИХ ИНВЕСТИЦИОНИХ ПРОЈЕКАТА

Драган Љ. Милановић, Драган Д. Милановић, Мирјана Мисита

Предмет овог истраживања cy ризични инвестициони пројекти, за које није довољно применити само неки од метода оцене пројеката (НПВ, АЕ, ИРР итд), јер је ризик доношења погрешне одлуке знатан. Због тога се у оваквим случајевима морају применити неки други методи који узимају у обзир ризик пројекта. У пракси се најчешће за оцену пројеката користе методе НПВ и ИРР. Међутим, за оцену ризичних пројеката ове методе не могу да дају поуздану оцену о исплативости пројекта. Због тога се у овом раду предлаже једна друга комбинација метода, која би дала поузданију слику о пројекту. На конкретном примеру урађена је НПВ анализа, сензитивна анализа и Монте Карло симулација. На тај начин се дошло до одређених резултата који омогућују поузданије оцењивање инжењерског инвестиционог пројекта, што представља допринос овог рада.