Interaction of the Gaussian Pulse EM Wave with Suddenly Created Cold Magnetized Plasma

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The linear transformation of the source Gaussian pulse electromagnetic wave, with propagating in a free space along an external static magnetic field, due to the sudden creation of cold linear plasma, is studied. This transformation has been analyzed by using the first order perturbation theory in radio approximation. Spatial distribution of the new created static magnetic field mode is presented in corresponding diagram.

Keywords: suddenly created plasma, perturbation technique, Fourier transform, Laplace transform, static magnetic mode generation.

1. INTRODUCTION

Rapidly created plasmas appear practically in all pulse gas discharges, laser created plasmas, lightning, and plasmas created by nuclear explosions. If the rise time of plasma is much smaller than the decay time, it is possible to approximate the time variation of plasma parameters with the Heaviside step function. Linear transformation of the plane electromagnetic wave (EMW) in suddenly created plasma was investigated in [1]. It was shown that for $t < 0$, the source EMW propagating along the $z$-direction in free space with wave number $k_0$ and angular frequency $\omega_0$ transforms in the suddenly created plasma having an infinite extent into two new modes propagating in opposite directions with identical upshifted frequencies $\omega_p = \sqrt{\omega_0^2 + \omega_p^2}$, with electron plasma angular frequency $\omega_p = \sqrt{N_0 e^2 / \varepsilon_0 m} \approx 18\pi N_0^{1/2}$, where $N_0$ is electron plasma density. The basic results of the transformation of EMW in such time varying linear media have been summarized by Kalluri [2]. Some of these results have been verified by Particle In Cell simulation [3] and by experimentally created microwave plasma previously illuminated by stationary microwave source [4]. By the use of second-order perturbation technique, the transformation of the source EMW in the nonlinear media was done in [5-10]. The process of third harmonic generation (THG) [11,12] is caused by the coupling of the transverse first harmonic and longitudinal second harmonic modes through Lorentz force and convective term in the equation of electron field, due to the sudden creation of cold linear plasma, is studied. This process of third harmonic generation (THG) [11,12] is caused by the coupling of the transverse first harmonic and longitudinal second harmonic modes through Lorentz force and convective term in the equation of electron field, due to the sudden creation of cold linear plasma, is studied. This process of third harmonic generation (THG) [11,12] is caused by the coupling of the transverse first harmonic and longitudinal second harmonic modes through Lorentz force and convective term in the equation of electron field, due to the sudden creation of cold linear plasma, is studied. This transformation has been analyzed by using the first order perturbation theory in radio approximation. Spatial distribution of the new created static magnetic field mode is presented in corresponding diagram.

2. PROBLEM FORMULATION AND SOLUTION

Electric and magnetic fields of the source EMW propagating in free space for $t < 0$ are given by:

$$\mathbf{e}_0(z,t) = x \cdot E_0 \exp \left[ -\frac{\alpha_0 t - k_0 z}{2\alpha_0 T} \right] \cos(\alpha_0 t - k_0 z), \tag{1}$$

$$\mathbf{h}_0(z,t) = y \cdot H_0 \exp \left[ -\frac{\alpha_0 t - k_0 z}{2\alpha_0 T} \right] \cos(\alpha_0 t - k_0 z), \tag{2}$$

where $x$ and $y$ are unit vectors in positive direction of $x$ and $y$-axis, with $H_0 = \sqrt{\varepsilon_0 / \mu_0 E_0}$, where $\varepsilon_0$ and $\mu_0$ are electric permittivity and magnetic permeability of the free space, respectively.

The EM and electron velocity fields $\mathbf{e}(z,t)$, $\mathbf{h}(z,t)$ and $\mathbf{u}(z,t)$ in magnetoplasma medium have to satisfy the following equations:

$$\nabla \times \mathbf{e}_1(z,t) = -\mu_0 \frac{\partial \mathbf{h}_1(z,t)}{\partial t}, \tag{3}$$

$$\nabla \times \mathbf{h}_1(z,t) = -N_0 g \mathbf{u}_1(z,t) + \varepsilon_0 \frac{\partial \mathbf{e}_1(z,t)}{\partial t}, \tag{4}$$

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\[
\frac{\partial \mathbf{u}_t(z,t)}{\partial t} = - \frac{q}{m} \mathbf{e}_t(z,t). \tag{5}
\]

In order to solve the system of partial differential equations (3) to (5) we have applied Laplace transform in time:

\[
L(f(z,t)) = \int_0^\infty f(z,t) \exp(-st)dt = F(z,s). \tag{6}
\]

and, as the plasma is unbound, Fourier transform in space:

\[
F(f(z,s)) = \int_{-\infty}^{+\infty} f(z,s) \exp(-jkz)dz = F(k,s). \tag{7}
\]

In a domain of complex frequency \( s = j\omega \) (collisionless plasma), \( j = -1 \), and wave number \( k \) the EM and velocity fields are defined by the following system of linear algebraic equations:

\[
j k E_1(k,s) - \mu_0 H_1(k,s) = \mu_0 H_1(k,t = 0), \tag{8}
\]

\[
j k H_1(k,s) + \varepsilon_0 \omega_0 E_1(k,s) - N_0 q U_1(k,s) = -\varepsilon_0 \omega_0 E_1(k,t = 0), \tag{9}
\]

\[
\frac{q}{m} E_1(k,s) + s U_1(k,s) = 0, \tag{10}
\]

where

\[
E_1(k,t = 0) = \frac{E_0 \sqrt{\pi}}{K_0} \left\{ \exp \left[ -\left( \frac{k - k_0}{K_0} \right)^2 \right] + \exp \left[ -\left( \frac{k + k_0}{K_0} \right)^2 \right] \right\},
\]

\[
H_1(k,t = 0) = \sqrt{\frac{\varepsilon_0}{\mu_0}} \omega_0 E_1(k,t = 0),
\]

\[
K_0 = \frac{k_0}{\varepsilon_0 \omega_0 T}. \tag{11}
\]

Solving the algebraic equations (8) to (10) one obtains the transformed EM and electron velocity fields in the form:

\[
E_1(k,s) = \frac{s - jkc}{s^2 + k^2c^2 + \omega_0^2} E_1(k,t = 0), \tag{12}
\]

\[
H_1(k,s) = \left( \frac{jkc}{\mu_0 s^2 + k^2c^2 + \omega_0^2} + \frac{1}{s} \frac{\omega_0}{\mu_0} \right) E_1(k,t = 0), \tag{13}
\]

\[
U_1(k,s) = \frac{\varepsilon_0}{N_0 q} s E_1(k,s). \tag{14}
\]

After performing an inverse Laplace and Fourier transform to (12) and (13), the following EM components are obtained:

\[
\mathbf{e}_t(z,t) = x \left[ E_1' \cos(\omega_0 t - k_0 z - \theta_1) + E_1'' \cos(\omega_0 t + k_0 z - \theta_1) \right], \tag{15}
\]

\[
\mathbf{h}_t(z,t) = y \left[ H_1' \cos(k_0 z - \theta_2) + H_1'' \cos(\omega_0 t - k_0 z - \theta_2) \right]. \tag{16}
\]

Amplitudes and phase shift of the electric field components have the following form:

\[
E_{1r}^r = \sqrt{(A_{1r}^r)^2 + (A_{1r}^\perp)^2},
\]

\[
tg \theta_1 = \frac{A_{1r}^\perp}{A_{1r}^r},
\]

\[
A_{1r}^r = E_0 \left( \frac{1}{2} \left( \frac{\omega_0}{\omega_1} + \frac{1}{4T^2} \frac{3\omega_0 \omega_1^2}{\omega_1^2} - \frac{\omega_0^2}{\omega_1^2} + \frac{\omega_0}{\omega_1} \left( \frac{\omega_0^2}{\omega_1^2} + \frac{z}{c} \right)^2 \right) \right),
\]

\[
A_{1r}^\perp = E_0 \left[ \frac{2\omega_0^2 \omega_0}{\omega_1^2} \left( \frac{\omega_0}{\omega_1} T + \frac{z}{c} \right) - \frac{\omega_0^2}{\omega_1^2} \left( \omega_0 \pm \omega_0 \theta_1 \right) \right]. \tag{17}
\]

The new created EM wave in plasma has the angular frequency \( \omega_1 = \sqrt{\omega_0^2 + \omega_1^2} \). Amplitudes and phase shifts of the magnetic field components have the following form:

\[
H_{10} = B_1 + B_2',
\]

\[
tg \theta_2 = \frac{B_{1r}^\perp}{B_{1r}^r},
\]

\[
B_1 = H_0 \left[ \frac{\omega_0^2}{\omega_1^2} + \frac{\omega_0^2}{2T^2 \omega_1^2} - \frac{1}{2} \left( \frac{\omega_0}{\omega_1} \right)^2 \left( \frac{z}{c} \right)^2 \right],
\]

\[
B_2 = H_0 \left[ \frac{\omega_0^2}{\omega_1^2} - \frac{\omega_0^2}{2T^2 \omega_1^2} \frac{z}{c} \right],
\]

\[
tg \theta_3 = \frac{C_{1r}^\perp}{C_{1r}^r},
\]

\[
C_{1r}^r = H_0 \left( \omega_1 \pm \omega_0 \right) \left[ \pm \frac{\omega_0}{\omega_1} + \frac{1}{2T^2 \omega_1^2} \left( \omega_1 \pm \omega_0 \right) \left( \frac{z}{c} \right)^2 \right],
\]

\[
C_{1r}^\perp = H_0 \left( \omega_1 \pm \omega_0 \right) \left[ \frac{2}{2T^2 \omega_1^2} \frac{\omega_0}{\omega_1} \left( \omega_1 \pm \omega_0 \right) \frac{z}{c} \right], \tag{18}
\]

In the above equations the superscript \( t \) and upper
sign refer to transmitted and superscript $r$ and lower sign to the reflected wave.

Spatial distribution of the new created static magnetic mode—the first term in (16), see Figure 1, has made with specific values of following parameters: $\omega_0 \sim 10^6$ Hz, the source wave is in a radio frequency range, $N_0 \sim 10^{22}$ m$^{-3}$, rapidly created plasma is generated by lightning and $T \sim 10^8/\omega_p$, plasma time duration is about 100 µs.

Figure 1. Spatial distribution of the new created static magnetic field mode, normalized on the source wave magnetic field amplitude

3. CONCLUSION

For the first time in the theory of the interaction of a EMW with time-varying media, the initial value problem of interaction of the Gaussian pulse EM source wave with suddenly created cold magnetoplasma medium, in the particular case of longitudinal propagation, is solved in the closed form. The source wave, due to sudden generation of the cold plasma, splits into two traveling EM waves (one transmitted and one reflected with the same upshifted angular frequency) and one spatial-varying static magnetic mode with the peak value the same as the amplitude of the magnetic field of the source EM wave and spatial period $k_0 z = 3.2$.

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ИНТЕРАКЦИЈА ГАУСНО-ИМПУЛСНОГ ЕЛЕКТРОМАГНЕТСКОГ ТАЛАСА СА НАГЛО СТВОРЕНОМ ХЛАДНОМ МАГНЕТИЗОВАНОМ ПЛАЗМОМ

Зоран М. Трифковић

По први пут у теорији интеракције електромагнетских таласа са временско променљивим просторима пропагације анализирана је трансформација изворног таласа у облику Гаусовог импулса. Као временски променљив простор узета је нагло створена магнетизована

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плазма. Анализа је спроведена коришћењем пертубационе теорије првог реда у радио апроксимацији. Решења новонасталих поља у плазми су добијена у затвореној форми и приказана је просторна расподела статичког магнетског поља добијеног као последица ове интеракције.