Determination of Tooth Clearances at Trochoidal Pump

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The paper describes the development of a mathematical model of trochoidal gearing with clearances. Gearing of a trochoidal pump's gear set with an outer gear having one gear tooth more than an inner gear is analyzed. The inner gear tooth profile is described by peritrochoidal equidistance and the outer gear profile by a circular arc. Upon the basic principles of ideal profile generation, a mathematical model of gearing with clearances is developed. Using an analytical model, the calculation of the minimal clearance between gear teeth profiles is done. On the basis of the analytical calculation conducted on real pump's gear set, the influence of geometrical parameters of the profile on coupling with variation of the clearance is analyzed. The obtained results can be used for investigation of driving torque pulsation and for calculation of pump's volumetric losses.

Keywords: *trochoid, gearing, clearances.*

1. INTRODUCTION

Trochoidal pumps, widely known as a gerotor pumps, belong to the group of planetary rotated machines. Their kinematics is based on the principle of planetary mechanism with the internal gearing. Gerotor pumps are generally designed using a trochoidal inner gear and an outer gear formed by a circle with intersecting of circular arcs. Due to this principle, designers of engines, compressors, machine tools, tractors and other equipment, which require hydraulic systems, can build pump components integrally.

Trochoidal gearing, due to numerous advantages, are in the research focus of many scientists. Ansdale and Lockley have derived equations which define geometry of trochoidal profiles applied for Wankel engine design [1]. Colbourne presented an analytical model to calculate the force at each contact point by neglecting friction, and by that, the maximum contact stress in the gear teeth was obtained. The paper [2] also indicated that the main difficulty in calculating the contact stress is to determine the force that is transmitted through each contact point. Since there are many contact points, at any instance, the problem is statically indeterminate. Robinson and Lyon analyzed modification of epitrochoidal profiles of rotary pumps. They showed that equidistance of basic conjugated curves satisfies the fundamental law of gearing and that it can be applied for the definition of gearing profile [3]. Maiti gave detailed analysis of geometric, kinematic and functional characteristics of rotating machines with gerotor mechanisms [4,5]. He developed an analytical method for contact stresses calculation that can be applied for epitrochoidal hydraulic pumps and engines. The developed theoretical model is illustrated by numerical examples. Beard et al. [6] derived the relationships that show the influence of the trochoid ratio, the pin size ratio and the radius of the generating pin on the curvature of the epitrochoidal gerotor. Shung and Pennock [7] presented a unified and compact equation for describing the geometry, the geometric properties of the different types of trochoid and the geometric properties of a conjugate envelope. Blanche and Yang [8] developed an analytical model of the cycloid drive with machining tolerance. They investigated the effect of machining tolerance on the backlash and torque ripple. Litvin and Feng [9] investigated the envelope's relation to surface family by considering the envelopes formed by several branches for cycloidal pumps and conventional worm gear drives. Demenego et al. [10] developed computer program for tooth contact analysis (TCA) and discussed avoidance of tooth interference and rapid wearing through modification of the rotor profile geometry of a cycloidal pump whose one pair of teeth is in mesh at every instant. Paffoni [11] used vector analysis to precisely describe the geometry of a hydrostatic gear pump from which full general parametric equations are deduced. Paffoni et al. [12] describes the teeth clearance influence on the tooth number contact in a hydrostatic pump in which circular arc profiles are used. Gamez-Montero et al. [13] presented a simplified analytical model of a trochoidal-type machine when friction at the contact points is neglected. The study [13] presented the calculation of the maximum normal contact stress by finite-element model and then compared it with experimentally obtained results on the prototype model through photoelasticity measurement techniques. Hwang and Hsieh [14] proposed the method of how to correctly determine the region of feasible design without any undercutting on the outer and inner rotor profiles. The non-undercutting equations are derived by the theory of gearing and the region of feasible design is determined by considering the non-undercutting curves of the inner and the outer rotors.

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The objective of this paper is to develop a comprehensive model of trochoidal gearing with clearances based on described investigations, which can be applied to all teeth at every moment of meshing. Besides, by setting up certain kinematic relations, the model can be used not only for gearing pairs with orbital motion, but also for those with stationary axes. The paper deals with kinematic pair model with outer element being stationary, while the inner one does the planetary motion. It is adopted that all deviations from theoretical measures are reflected on modification of trochoidal profile of the inner gear. Minimum clearances between teeth were analyzed subsequently.

2. MATHEMATICAL MODEL OF THEORETICAL TROCHOIDAL PROFILES

A theoretical model for generation of gearing with peritrochoidal profiles based on double trochoid realization theorem is developed in this paper. Simultaneously the coupling between all gear teeth is obtained at trochoidal gear sets with theoretical gearing profiles and it is illustrated by the example of a gear pump (Fig. 1).

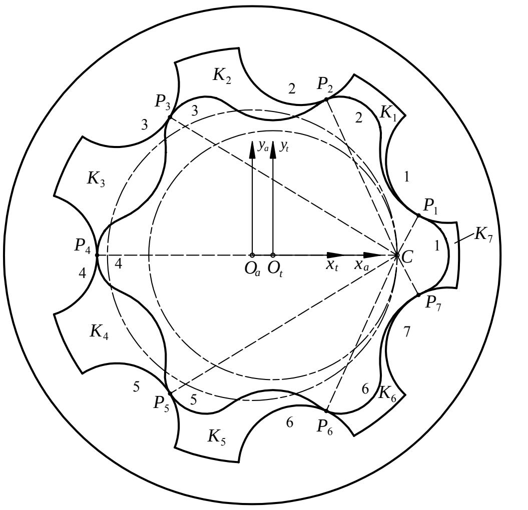

Figure 1. Numeration of teeth and chambers of trochoidal pump

It is necessary to numerate contact points and all gear teeth in order to determine which gear teeth are coupled during gear teeth modeling and coupling simulation. Figure 1 illustrates the numeration of teeth, contact points and operating chambers at initial moment and for gear set with z = 7 chambers. Thereafter, the outer gear teeth are marked with i = 1, 2, …, z, corresponding chambers with K_i, while for inner gear teeth, marks j = 1, 2, …, $z - 1$ are adopted. Since common normal of the coupled profiles go through point C that represents the instantaneous pitch point, it means that trochoidal profiles of all inner gear teeth simultaneously touch circular profiles of outer gear teeth at points P_1, P_2, ..., P_z.

At real designs, simultaneous coupling between all gear teeth at every moment is not possible for the following reasons:

- real profiles are manufactured with technological clearances necessary to prevent the occurrence of jams,
- errors due to assemblage and manufacture may cause profiles to interfere,
- abrasive particles transported by the pump may cause wear of the profile, which provokes increase of clearances between profiles, as a consequence.

Although gear teeth clearances are inevitable, they may lead to fluid losses and occurrence of additional dynamic forces, decrease stability and increase noise and vibration, particularly at high speeds.

Due to reasons mentioned above, this paper deals with modeling of coupling between real profiles. Therefore, it is assumed that all deviations from theoretical measures affect the modification of inner gear's trochoidal profile. If toothing profile of the inner gear would have larger dimensions than ideal, a mechanism could not be mounted, due to the simultaneous touch between all gear teeth. Hence, deviations may be modeled as constant decrease along axes normal to ideal trochoidal profile. Consequently, the applied coordinate systems will be the same as in the mathematical model of theoretical profiles and all equations will be developed for trochoid's coordinate system.

Since there is simultaneous coupling of all teeth at trochoidal gearing, general equations of profile points' coordinates applicable to all gear teeth must be determined. Generalization of geometrical relations between rotation angles of elements of trochoidal gear set is necessary for derivation of coordinates of any contact point P_i. We adopt the model of kinematical set where the outer element (the envelope) is observed as fixed, while the inner element (equidistant of a peritrochoid) performed planetary motion (Fig. 2).

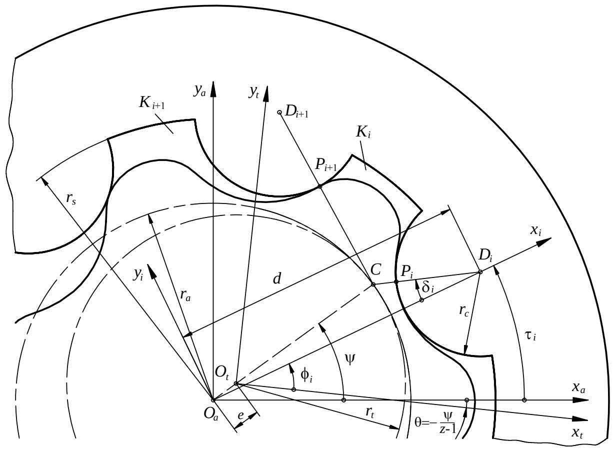

Figure 2. Gear set layout of trochoidal pump with basic geometrical variables

In addition, it is assumed that the driving shaft is connected to the inner element through eccentric gear, causing that all equations should be expressed as functions of the driving angle, labeled as ψ.

Before analyzing the kinematical relations during coupling of ideal trochoidal profiles, the coordinate systems and geometrical relations between rotation angles in different coordinate systems have to be introduced and applied throughout the modeling approach.

Figure 2 shows basic geometrical relations during the generation of peritrochoid adopted for definition of the basic profile of the considered gear pump. The center and the radius of moving (generating) circle are denoted with O_a and r_a, respectively, while O_t and r_t

denote the center and the radius of a fixed (basic) circle. Peritrochoid eccentricity, e, is a distance between the two centers of circles. The coordinate system, $O_a x_i y_i$, is connected to the center of a moving circle. The generating point, D_i, is the point that moves along by trochoid, located on the x_i axis at the distance d from O_a, and represents the radius of the trochoid. The line that connects the centers O_t and O_a and runs through the contact point of the two circles (the pitch point, C) determines the reference line. In order to present the trochoidal profile in an analytical form, the trochoid's coordinate system $O_t x_t y_t$ is introduced and its origin is set at the center of the fixed circle with an abscissa running through the initial contact point between the presented kinematical circles. The envelope's coordinate system, $O_a x_a y_a$, is connected to the center of the moving circle. All coordinate systems are right-handed. At the initial moment, the positive part of the x_t axis of inner gear is connected to the initial gear tooth's top land, while the positive part of the x_a axis of the outer circle coincides with the centerline of the coupled gear tooth. The angles are assumed to be positive when measured in counterclockwise direction.

Since the position of profile points is observed in relation to different coordinate systems, the application of coordinate systems transformation is necessary and the simplest form to describe their equations is the matrix form. Generally, transition from S_i coordinate system to the next, S_n, is determined by the equation of coordinate transformation in the form of:

$$\mathbf{r}_n = \boldsymbol{M}_{ni}\mathbf{r}_i \,, \tag{1}$$

where: $\mathbf{r}_i$ is the position vector of profile contact point in coordinate system S_i, $\mathbf{r}_n$ is the position vector of the same point in coordinate system S_n, $\boldsymbol{M}_{ni}$ is the coordinate transformation matrix from coordinate system S_i to coordinate system S_n.

The following matrices of the third order are defined for coordinate transformation:

$$\boldsymbol{M}_{ta} = \begin{bmatrix} \cos\frac{\psi}{z-1} & -\sin\frac{\psi}{z-1} & -e\cos\frac{z\psi}{z-1} \\ \sin\frac{\psi}{z-1} & \cos\frac{\psi}{z-1} & -e\sin\frac{z\psi}{z-1} \\ 0 & 0 & 1 \end{bmatrix},$$

$$\boldsymbol{M}_{at} = \begin{bmatrix} \cos\frac{\psi}{z-1} & \sin\frac{\psi}{z-1} & e\cos\psi \\ -\sin\frac{\psi}{z-1} & \cos\frac{\psi}{z-1} & e\sin\psi \\ 0 & 0 & 1 \end{bmatrix}. \tag{2}$$

The angle τ_i between the centerline of the outer gear's tooth (coordinate axis x_i) and coordinate axis x_a can be expressed in the form of:

$$\tau_i = \frac{\pi(2i-1)}{z}, \tag{3}$$

while, for the adjacent profile, this angle can be calculated by the following expression:

$$\tau_{i+1} = \frac{\pi(2i+1)}{z}. \tag{4}$$

Generalization of the profile equations of the outer gear can be done in the following manner. The gear tooth profile $i = 1$, is defined with the vector equation:

$$\mathbf{r}_{D_1}^{(a)} = \begin{bmatrix} x_{D_1}^{(a)} \\ y_{D_1}^{(a)} \\ 1 \end{bmatrix} = \begin{bmatrix} ez\lambda\cos\frac{\pi}{z} \\ ez\lambda\sin\frac{\pi}{z} \\ 1 \end{bmatrix}, \tag{5}$$

where λ is the coefficient of trochoid, $\lambda = d/ez$.

Since teeth are equally spaced, their mutual angular distance in relation to the corresponding axis of rotation is:

$$\tau_a = \frac{2\pi}{z}. \tag{6}$$

Coordinates of other gear teeth points can be obtained by rotation of the already known coordinates of points of gear tooth profile for an angle determined by the relation (Fig. 2):

$$\tau_{ai} = \frac{2\pi}{z} i. \tag{7}$$

Thereby, the position vector of the contact point at the profile of the i-th gear tooth of the outer gear is defined by matrix equation:

$$f\mathbf{r}_{P_i}^{(a)} = \boldsymbol{L}_{i1}\mathbf{r}_{P_1}^{(a)}, \tag{8}$$

where $\boldsymbol{L}_{i1}$ is the transformation matrix of coordinates of the contact points P_1 to P_i:

$$\boldsymbol{L}_{i1} = \begin{bmatrix} \cos\tau_{ai} & -\sin\tau_{ai} & 0 \\ \sin\tau_{ai} & \cos\tau_{ai} & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{9}$$

Now, vector equations for the point D_i of the center of circular gear tooth profile can be written in the envelope's coordinate system as:

$$\mathbf{r}_{D_i}^{(a)} = \begin{bmatrix} x_{D_i}^{(a)} \\ y_{D_i}^{(a)} \\ 1 \end{bmatrix} = \begin{bmatrix} ez\lambda\cos\tau_i \\ ez\lambda\sin\tau_i \\ 1 \end{bmatrix}. \tag{10}$$

In the trochoid's coordinate system, by the use of transformation matrix, the vector equation of the point D_i is:

$$\mathbf{r}_{D_i}^{(t)} = \boldsymbol{M}_{ta}\mathbf{r}_{D_i}^{(a)} = \begin{bmatrix} ez\lambda\cos\left[\tau_i + \frac{\psi}{z-1}\right] - e\cos\frac{z}{z-1}\psi \\ ez\lambda\sin\left[\tau_i + \frac{\psi}{z-1}\right] - e\sin\frac{z}{z-1}\psi \\ 1 \end{bmatrix}. \tag{11}$$

The position vector of the contact point P_i in the envelope's coordinate system can be written in the form of the following matrix relation:

$$\mathbf{r}_{P_i}^{(a)} = \begin{bmatrix} x_{P_i}^{(a)} \\ y_{P_i}^{(a)} \\ 1 \end{bmatrix} = \begin{bmatrix} e\left\{z\lambda\cos\tau_i - c\cos\left[\tau_i + \delta_i\right]\right\} \\ e\left\{z\lambda\sin\tau_i - c\sin\left[\tau_i + \delta_i\right]\right\} \\ 1 \end{bmatrix}, \quad (12)$$

where c is the equidistant radius coefficient, $c = r_c/e$. The angle δ_i is defined as the gear mesh angle and calculated by the following equation:

$$\delta_i = \arctan\frac{\sin\left[\frac{\pi(2i-1)}{z} - \psi\right]}{\lambda - \cos\left[\frac{\pi(2i-1)}{z} - \psi\right]}. \quad (13)$$

After the application of coordinate transformation, (12) has the following form in the trochoid's coordinate system:

$$\mathbf{r}_{P_i}^{(t)} = \boldsymbol{M}_{ta}\mathbf{r}_{P_i}^{(a)} =$$

$$= \begin{bmatrix} e\left\{\begin{matrix} z\lambda\cos\left[\tau_i + \frac{\psi}{z-1}\right] - \cos\frac{z}{z-1}\psi - \\ -c\cos\left[\tau_i + \frac{\psi}{z-1} + \delta_i\right] \end{matrix}\right\} \\ e\left\{\begin{matrix} z\lambda\sin\left[\tau_i + \frac{\psi}{z-1}\right] - \sin\frac{z}{z-1}\psi - \\ -c\sin\left[\tau_i + \frac{\psi}{z-1} + \delta_i\right] \end{matrix}\right\} \\ 1 \end{bmatrix}. \quad (14)$$

Derived equations enable modeling of gear set meshing during rotation and determination of necessary parameters for any teeth couple at an arbitrary moment.

3. MATHEMATICAL MODEL OF TROCHOIDAL PROFILES WITH CLEARANCES

Geometrical and kinematical model of trochoidal gear set with theoretical profiles described in the previous section are used as the basis for the analysis of real profile's meshing. Real profile of the inner gear will be generated as equidistant of a basic trochoid with equidistant radius greater than the theoretical one by a clearance size ε:

$$r_c^* = r_c + \varepsilon\,. \quad (15)$$

Thereby, equations of the trochoidal profile with tolerances have the following form in the coordinate system of trochoid:

$$\mathbf{r}_{P_i}^{*(t)} = \begin{bmatrix} e\left\{\begin{matrix} z\lambda\cos\left[\tau_i + \frac{\psi^*}{z-1}\right] - \cos\left(\frac{z\psi^*}{z-1}\right) - \\ -(c+\varepsilon^*)\cos\left[\tau_i + \delta_i^* + \frac{\psi^*}{z-1}\right] \end{matrix}\right\} \\ e\left\{\begin{matrix} z\lambda\sin\left[\tau_i + \frac{\psi^*}{z-1}\right] - \sin\left(\frac{z\psi^*}{z-1}\right) - \\ -(c+\varepsilon^*)\sin\left[\tau_i + \delta_i^* + \frac{\psi^*}{z-1}\right] \end{matrix}\right\} \\ 1 \end{bmatrix}, \quad (16)$$

where is $\varepsilon^* = \varepsilon/e$.

At gear sets with theoretical profiles and ideal geometrical measures, the transfer of motion is done with constant ratio. However, as operating elements of the pumps are manufactured with tolerances, the existing clearances in the mechanism make the input shaft to rotate for an angle before the inner gear begins to rotate. That is, the real position of the gear with trochoidal profile delays (demonstrates „lagging“) after its theoretical position during rotation.

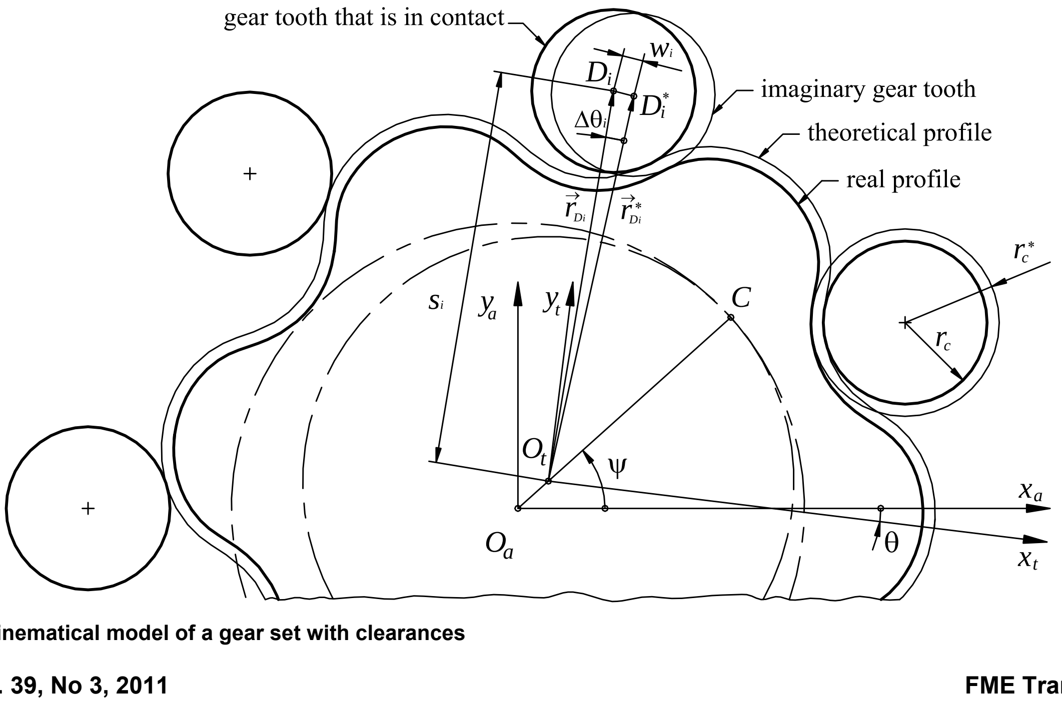

Figure 3. Kinematical model of a gear set with clearances

The angle between the theoretical and real position of the inner gear is called the lag angle [8], and is denoted as $\Delta\theta_i$ (Fig. 3).

Usually, it is not known in advance which gear tooth of the outer gear is in contact with the trochoidal gear. Therefore, in order to determine the lag angle, it is necessary to identify the gear tooth that, for a given angle of rotation of the input shaft, ψ, would get in contact and then to determine a corresponding lag angle. The change in value of the rotation angle induces the change of the gear tooth engaged and the lag angle.

In further analysis it is assumed that the engaged gear tooth is known and that it is necessary to determine the gear lag angle. Geometrical relations that apply to the real profile are given in Figure 3. Based on presented relations, necessary equations for determination of the lag angle can be derived. At the initial moment, the inner gear is in position concentric with the position of the theoretical profile. The vector of the centre of a circular gear tooth profile arriving to contact is marked as $\mathbf{r}_{D_i}^{(t)}$. In order to simplify the analysis, an imaginary gear tooth of a circular shape is introduced. This imaginary tooth rolls over the trochoidal gear surface until the intensity of the vector $\mathbf{r}_{D_i}^{*(t)}$ that defines the position of its center in trochoid's coordinate system, reaches the intensity of the vector $\mathbf{r}_{D_i}^{(t)}$. Then, the relative position of imaginary and trochoidal gear tooth corresponds to the position that real gear tooth with circular profile and trochoidal gear will take at the moment the contact is achieved. Since trochoidal gear rotates in the direction opposite to the direction of the rotation angle of input shaft, the lag angle will have the same direction as input angle.

To determine the position of the center of the imaginary gear tooth, we analyze the moment when the contact with the trochoidal gear is achieved. At that moment, gears are in contact and have a mutual normal that runs through the center of the circular profile. Furthermore, it is clear that the center of the circular profile is located at the normal that runs through the contact point of the real trochoidal profile, at the distance r_c from the contact point. Taking this fact into account, as well as the (14), the equation that defines the position of the center of imaginary gear tooth at the moment of contact can be defined as:

$$\mathbf{r}_{D_i}^{*(t)} = \begin{bmatrix} e\left\{ \begin{array}{c} z\lambda\cos\left[\tau_i + \dfrac{\psi_i^*}{z-1}\right] - \cos\left(\dfrac{z}{z-1}\psi_i^*\right) - \\ -\varepsilon^*\cos\left[\tau_i + \delta_i^* + \dfrac{\psi_i^*}{z-1}\right] \end{array} \right\} \\ e\left\{ \begin{array}{c} z\lambda\sin\left[\tau_i + \dfrac{\psi_i^*}{z-1}\right] - \sin\left(\dfrac{z}{z-1}\psi_i^*\right) - \\ -\varepsilon^*\sin\left[\tau_i + \delta_i^* + \dfrac{\psi_i^*}{z-1}\right] \end{array} \right\} \\ 1 \end{bmatrix}. \quad (17)$$

As it was mentioned before, in order to make a contact, it is necessary that vectors, defined by (11) and (17), have the same intensity s_i, which is described by:

$$\left|\mathbf{r}_{D_i}^{(t)}\right|^2 = \left|\mathbf{r}_{D_i}^{*(t)}\right|^2 = a_i^2 . \quad (18)$$

The expression for determination of the position vector intensity of the real gear tooth center that comes in contact is obtained from (11) in the following form:

$$r_{D_i}^2 = e^2\left\{1 + z^2\lambda^2 - 2z\lambda\cos\left[\tau_i - \psi\right]\right\} . \quad (19)$$

Similarly, based on (17), an expression for determination of the position vector's intensity of the imaginary gear tooth center that comes in contact is obtained in the following form:

$$(r_{D_i}^*)^2 = e^2\left\{1 + z^2\lambda^2 + (\varepsilon^*)^2 - 2z\lambda\cos\left[\tau_i - \psi_i^*\right] - \right.$$
$$\left. -2z\lambda\varepsilon^*\cos\delta_i^* + 2\varepsilon^*\cos\left[\tau_i + \delta_i^* - \psi_i^*\right]\right\} . \quad (20)$$

Substituting the (19) and (20) in (18), the final equation for determination of the imaginary angle ψ_i^* that represents the angle of rotation during generation of the contact point of the real trochoidal profile, is formed as:

$$\cos\left[\tau_i - \psi\right] + \frac{(\varepsilon^*)^2}{2z\lambda} - \cos\left[\tau_i - \psi_i^*\right] - \varepsilon^*\cos\delta_i^* +$$
$$+ \frac{\varepsilon^*}{z\lambda}\cos\left[\tau_i + \delta_i^* - \psi_i^*\right] = 0 . \quad (21)$$

Equation (21) is transcendent trigonometric equation and angle ψ_i^* cannot be explicitly expressed, but the solution is gained by iterative procedure. Then, the solution of (21) is used to determine the lag angle.

The lag angle is determined as a difference of the angles Θ_i and Θ_i^* between the coordinate axis x_t and vectors $\mathbf{r}_{D_i}^{(t)}$ and $\mathbf{r}_{D_i}^{*(t)}$, respectively (Fig. 4):

$$\Delta\theta_i = \Theta_i - \Theta_i^* , \quad (22)$$

where

$$\Theta_i = \arctan\frac{y_{D_i}^{(t)}}{x_{D_i}^{(t)}} \quad (23)$$

and

$$\Theta_i^* = \arctan\frac{y_{D_i}^{*(t)}}{x_{D_i}^{*(t)}} . \quad (24)$$

Consequently, potential values of lag angles can be determined by variation of the ordinal number of the gear tooth with the circular profile, $i = 1, 2, ... , z$. A real value of the lag angle will be the one that defines the minimum distance w_i between the circular gear tooth profile and the real trochoidal profile in theoretical position, for a given angle of rotation, ψ. That distance can be determined according to the following relation:

$$w_i = 2a_i \sin\frac{\Delta\theta_i}{2}\ ,\ i = 1, 2, \dots, z. \qquad (25)$$

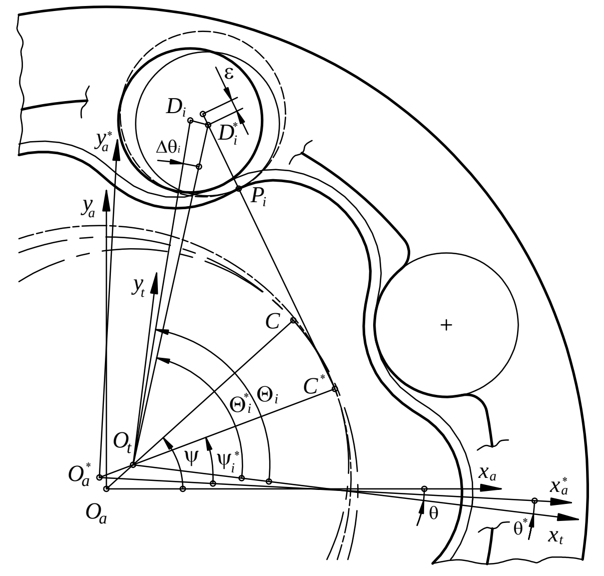

Figure 4. Geometrical relations for determination of the lag angle

Comparing the calculated values of the distance and identifying the minimum one, a gear tooth of a circular profile that is in contact with the trochoidal gear and the corresponding lag angle will be known, for every value of the rotation angle of the input shaft as:

$$w^{\Delta} = \min\left(w_1, w_2, \dots, w_z\right) \ \rightarrow\ \Delta\theta . \qquad (26)$$

Now, we can determine the minimal value of the clearance between the gear tooth profiles. We analyze the moment when the contact between real profiles is achieved, as shown in Figure 5.

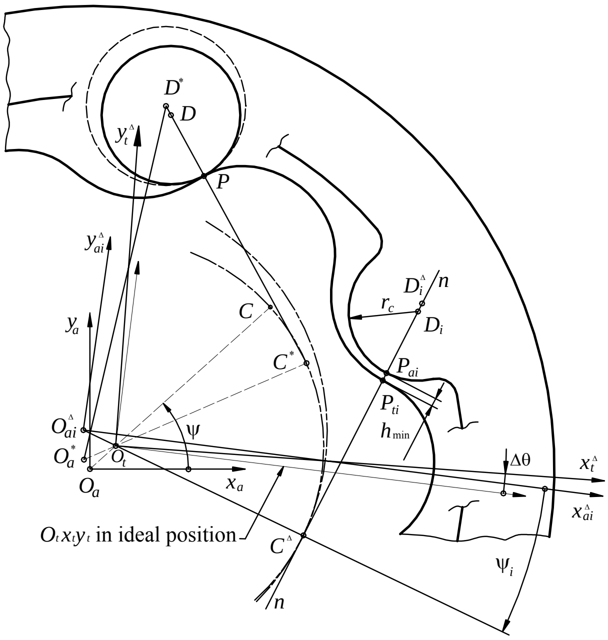

Figure 5. Geometrical relations for determination of the minimal clearance

Identification of the gear teeth that are in contact is conducted for the given values of the reference angle ψ, and the corresponding angle $\Delta\theta$ is determined. Between the other teeth pairs there exist the clearances, defined by a mutual distance between points P_t and P_a at the gear teeth profiles of the inner and outer gear, respectively. This distance of the points will be minimal at the position where the profile tangents are parallel to each other, that is, when these points lay on the common normal of the profiles. Since the outer gear teeth profile is circular, its normal always runs through the center of the circular arc (point D_i). Considering this, it is necessary to define the axis that is perpendicular to trochoidal profile and runs through the point D_i. Firstly, the general form of the equation of normal to the trochoid curve is defined. Then, the value of the angle that defines the position of the point P_t of the trochoidal profile, through which the common normal runs, is identified by variation of angle ψ_i for the observed gear tooth i of the outer gear.

In the case when the curve is expressed in a parametric form, the equation of the normal at point $P_{ti}\,(x_t, y_t)$ of that curve is defined in the following manner:

$$(y_n - y_t)\frac{\mathrm{d}y_t}{\mathrm{d}\psi} = -(x_n - x_t)\frac{\mathrm{d}x_t}{\mathrm{d}\psi}\ , \qquad (27)$$

where: x_n and y_n are the coordinates of the point of where the normal cuts modified trochoid, x_t and y_t are the coordinates of the point-form modified trochoid, defined by (16).

It is necessary to determine the coordinates of the circular profile center in trochoid's coordinate system, after its rotation by the angle $\Delta\theta$.

Coordinate transformation in the form of matrix equation is applied:

$$\mathbf{r}_{D_i}^{(t^{\Delta})} = \left[\boldsymbol{M}_{tt^{\Delta}}\right]\mathbf{r}_{D_i}^{(t)} =$$

$$= \begin{bmatrix} e\left\{z\lambda\cos\left[\tau_i + \dfrac{\psi}{z-1} - \Delta\theta\right] - \cos\left(\dfrac{z}{z-1}\psi - \Delta\theta\right)\right\} \\ e\left\{z\lambda\sin\left[\tau_i + \dfrac{\psi}{z-1} - \Delta\theta\right] - \sin\left(\dfrac{z}{z-1}\psi - \Delta\theta\right)\right\} \\ \\ 1 \end{bmatrix}. \qquad (28)$$

where the coordinate transformation matrix is:

$$\left[\boldsymbol{M}_{tt^{\Delta}}\right] = \begin{bmatrix} \cos\Delta\theta & \sin\Delta\theta & 0 \\ -\sin\Delta\theta & \cos\Delta\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \qquad (29)$$

Starting from (27) and (16), considering (13), the final equation for determination of angles ψ_i that define the position of points P_{ti} can be written as:

$$y_n \left\{ \begin{array}{c} z\cos\left[\pi(2i-1) + \dfrac{z\psi_i}{z-1}\right] + \lambda z\cos\left(\tau_i + \dfrac{\psi_i}{z-1}\right) - \\ -(c+\varepsilon^*)\delta_i'\cos\left(\tau_i + \dfrac{\psi_i}{z-1} + \delta_i\right) \end{array} \right\} -$$

$$-x_n \left\{ \begin{array}{c} z\sin\left[\pi(2i-1) + \dfrac{z\psi_i}{z-1}\right] + \lambda z\sin\left(\tau_i + \dfrac{\psi_i}{z-1}\right) - \\ -(c+\varepsilon^*)\delta_i'\sin\left(\tau_i + \dfrac{\psi_i}{z-1} + \delta_i\right) \end{array} \right\} +$$

$$+e\begin{bmatrix} \lambda z\,(z-1)\sin(\tau_i-\psi_i)+ \\ +\lambda z\,(c+\varepsilon^*)(1-\delta_i')\sin\delta_i+ \\ +(c+\varepsilon^*)(\delta_i'-z)\sin(\tau_i-\psi_i+\delta_i) \end{bmatrix}=0 \quad (30)$$

where $x_n = x_{D_i}^{(t^\Delta)}$, $y_n = y_{D_i}^{(t^\Delta)}$, $\delta_i{}' = \frac{\mathrm{d}\delta_i}{\mathrm{d}\psi}$.

This equation is transcendent and it can be solved by iterative procedure. Obtained solution is used for determination of normal distance between points D_i and P_{ti}, which is defined by the well-known relation from analytical geometry:

$$\overline{D_i P_{ti}} = \left[\,(y_{D_i}-y_{P_{ti}})^2+(x_{D_i}-x_{P_{ti}})^2\,\right]^{\frac{1}{2}}. \quad (31)$$

Finally, minimal clearance between gear teeth profiles can be calculated as follows:

$$(h_{\min})_i = \overline{D_i P_{ti}} - r_c\,,\; i = 1, 2, \ldots, z. \quad (32)$$

By using the analog procedure, the values of the observed parameters can be determined for every value of the angle of rotation, ψ.

The corresponding programs for calculation of the lag angle and the value of the minimal clearance between the gear teeth profiles are formed based on presented mathematical model [15]. The programs are tested for given values of geometrical parameters and the value of technological clearance. Based on calculated values, the diagrams of the variation of clearance height during rotation are drawn. Graphical interpretation of the obtained results is given in the following section.

4. TEST RESULTS OF THE PROGRAM FOR CLEARANCE CALCULATION

In this section of the paper, the influence of the rotation angle, ψ, trochoid coefficient, λ, and technological clearance, ε, to the clearance height, $h_{\min}$, between the non-contacting teeth profiles of the gear pump with trochoidal gearing is given. For that purpose, the program for identification of gear teeth in contact and calculation of the minimal clearance height between the gear teeth profiles is tested for two different gear sets with the geometrical parameters given in Table 1.

Table 1. Parameters of the trochoidal pump's gear sets

Parameters for both gear sets: z = 6, e = 3.56 mm, ε = 0.07 mm, r_s = 26.94 mm		
	Gear set I	Gear set II
λ	1.375	1.575
c	2.75	3.95

Variation of the clearance height is analyzed for a period of one revolution. In the case when the criterion $h_{\min} \le 0.001$ mm is satisfied the program assigns the height of $h_{\min} = 0$ to the computational clearance height.

Graphical presentation of the results analysis is given in Figure 6. Periodical variation of the clearance height is noticeable on the graphical presentation and the variation period corresponds to one revolution of the driving shaft. In addition, it can be noticed that chart flows of clearance height variation are equal for all chambers, and only shifted in phase.

The value of phase angle is equal to the value of the angle that defines one phase of fluid distribution in the

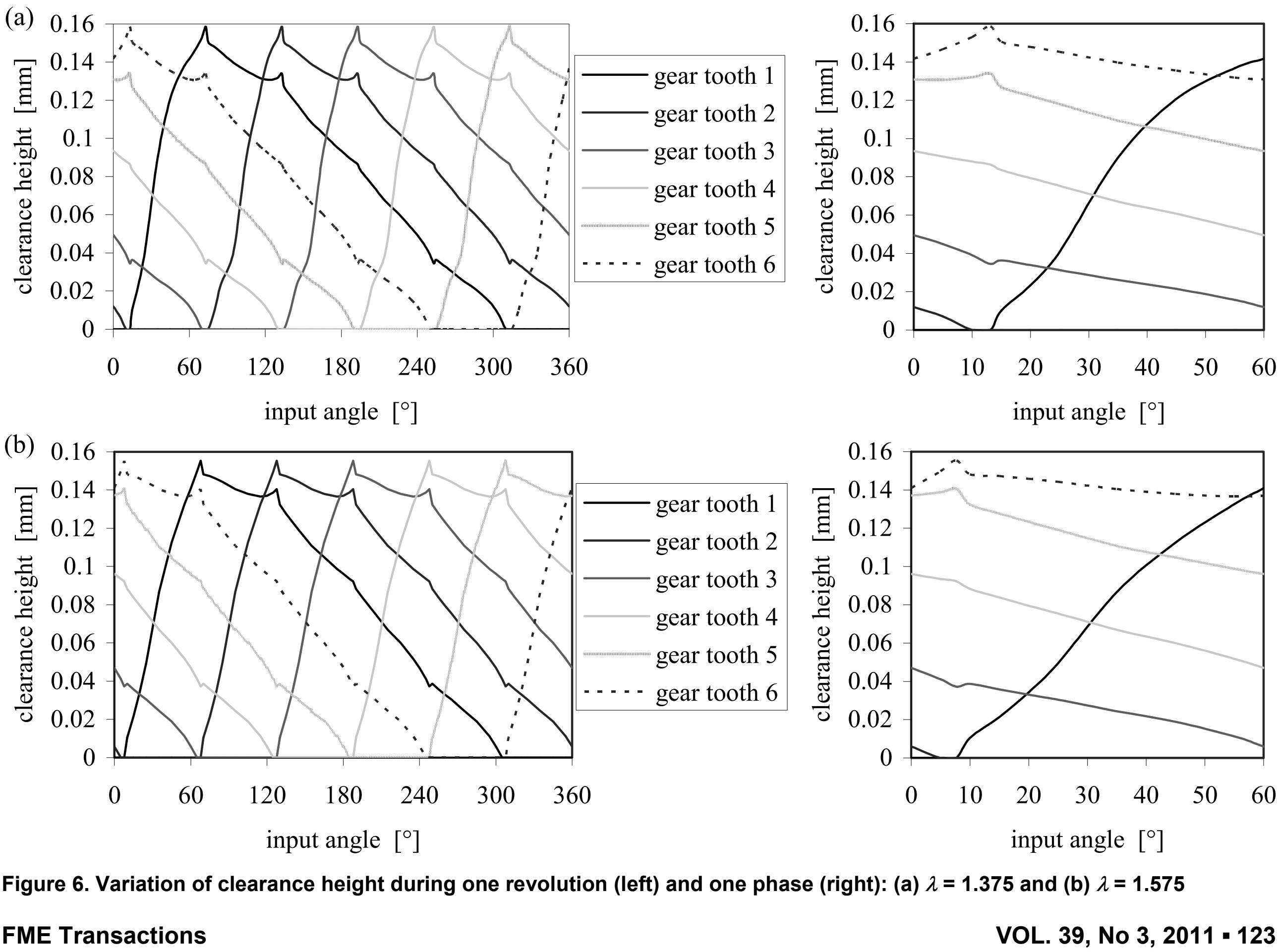

Figure 6. Variation of clearance height during one revolution (left) and one phase (right): (a) λ = 1.375 and (b) λ = 1.575

chamber and in concrete example, it is equal to $\psi_0 = 2\pi/z = 60°$. Thereby, variations of clearance and other characteristics can be observed only for a period from 0 to ψ_0. Figure 6 presents chart flows of clearance variation in all chambers during that period. From the given graphical presentation, the clearance value can be read and some conclusions can be drawn. The same gear couple stays in contact during one phase – the couple whose ordinal number is greater than ordinal number of fluid distribution phase by one. Consequently, in a short angular interval, there is simultaneous meshing of two adjacent gear teeth. Diagrams show peaks, the first and the most expressive of them taking place at the moment when the contact between adjacent couple of gear teeth ends, while the second of them takes place after a period that corresponds to one phase angle. Further decrease in clearance has nearly linear character with noticeable deviation at the moment when a new couple of gear teeth comes into contact.

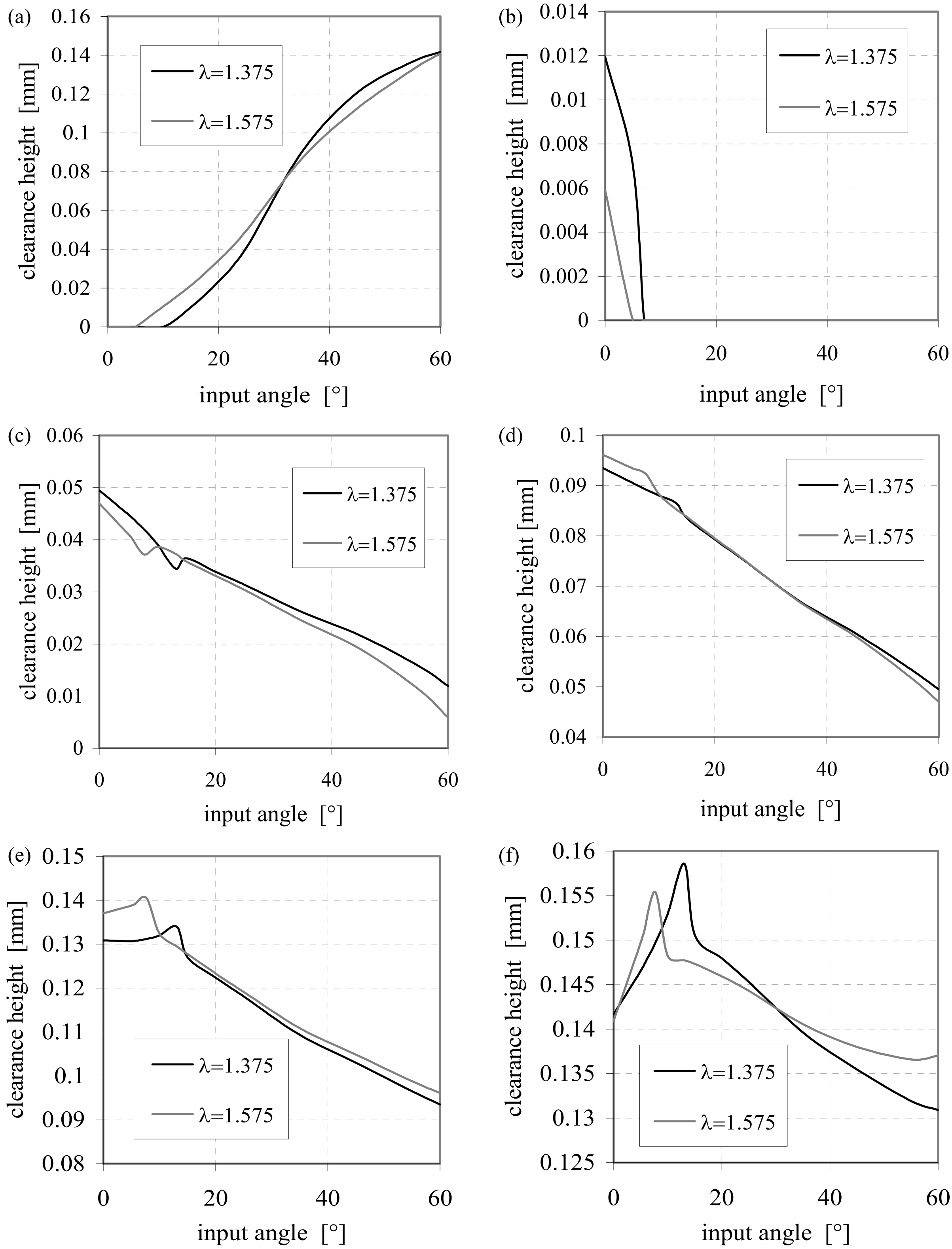

Figure 7. Comparative diagrams of minimal clearance height variation: (a) gear tooth 1, (b) gear tooth 2, (c) gear tooth 3, (d) gear tooth 4, (e) gear tooth 5 and (f) gear tooth 6

Comparative diagrams of the influence of parameter λ on the clearance height are provided in the Figure 7 as well. Diagrams in Figures 7a, 7d and 7f show that the clearance height at the first and the fourth teeth couple increases with the increase of parameter λ in the first half of the phase period, and decreases at the sixth gear teeth couple. At the second half of the phase period, the situation is inverse. Diagrams in Figures 7b, 7c and 7e show that decrease of parameter λ induces greater clearances at the second and the third gear teeth couple and smaller clearances at the fifth couple. Diagrams in Figure 7a show that the length of the interval of double couplings are bigger for smaller values of parameter λ.

The values obtained through the presented analysis can be used for calculation of volumetric losses of the pump and for the analysis of their influence on pressure variation in chambers.

5. CONCLUSION

The following conclusions summarize the research results:

- Generalization of gear tooth profile equations enables modeling of gear set meshing during rotation and determination of necessary parameters for any tooth couple at any moment;
- Mathematical model of toothing profiles with clearance enables identification of teeth that are in contact and determination of instantaneous minimal clearance height between gear teeth profiles;
- Computer program for modeling and simulation of meshing of gear sets with real profiles is written based on mathematical model. Thus, mutual position of gear set elements in different mesh phases can be followed and analyzed;
- The program was tested on real example and graphical interpretation of obtained results shows that trochoid coefficient has an influence on double coupling interval;
- Developed mathematical model can be used in the analysis of the influence of technological clearance height on functional characteristics of the pump.

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NOMENCLATURE

$O_a x_1 y_1$	coordinate system attached to the generating point

$O_t x_t y_t$	coordinate system attached to the internal gear
$O_a x_a y_a$	coordinate system attached to the external gear
$O_f x_f y_f$	fixed coordinate system
O_t, O_a	center of the internal gear and the external gear, respectively
C	pitch point
D	generating point
z	teeth number of the external gear
$z - 1$	teeth number of the internal gear
e	center distance between the internal and external gear (eccentricity)
r_t	radius of pitch circle of the internal gear $r_t = e\,(z - 1)$
r_a	radius of pitch circle of the external gear $r_a = ez$
r_c	radius of equidistance
c	equidistant coefficient
d	distance joining the generating point D and the center of the external gear
P_t, P_a	contact point on the profile of the internal gear and the contact point on the profile of the external gear, respectively
K	chamber
a	intensity of the position vector of the generating point D
w	distance between the circular gear tooth profile and the real trochoidal profile
h	clearance height

Greek symbols

ϕ	generating rotation angle
λ	trochoid coefficient
δ	leaning angle
ψ	referent rotation angle
τ_i	angle between coordinate axis x_i and coordinate axis x_a
τ_a	angular pitch
ε	technological clearance
θ	angle from the axis x_a to the axis x_t
$\Delta\theta$	lag angle
Θ	position angle of point D
Θ_i^*	position angle of point D^*

Superscripts

t	trochoide
a	envelope
f	fixed
*	with tolerances
Δ	in contact

ОДРЕЂИВАЊЕ ВЕЛИЧИНЕ ЗАЗОРА ИЗМЕЂУ ЗУБАЦА ТРОХОИДНЕ ПУМПЕ

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У овом раду је описан развој математичког модела трохоидног озубљења са зазорима. Анализирано је озубљење зупчастог пара трохоидне пумпе, код којег спољашњи зупчаник има један зубац више од унутрашњег. Профил унутрашњег зупчаника је описан еквидистантом перитрохоиде, а спољашњег кружним луком. Полазећи од основних принципа генерисања идеалног профила развијен је математички модел озубљења са зазорима. На основу добијених аналитичких израза прорачунате су минималне висине зазора између профила зубаца. На конкретном примеру анализиран је утицај геометријских параметара профила на процес спрезања и ток промене висине зазора. Добијени резултати могу да се користе за испитивање пулзација погонског момента, као и за прорачун запреминских губитака пумпе.