

# Moving Loads in Structural Dynamics of Cranes: Bridging the Gap Between Theoretical and Practical Researches

*The moving load problem is one of the fundamental problems in structural dynamics. A lot of work has been reported during more than a century ago dealing with the dynamic response of structures at first in the field of transportation such as railway bridges, later on highway bridges and finally various constructions such as are cranes, under the influence of moving loads. Research discussing the moving load effects on structural dynamics of cranes attracted the interest of several authors in the past years. However, regardless of the theoretical accomplishments in, after checking the adopted crane's parameters it is obvious that the obtained results in many papers are not relevant for practical considerations for already existing cranes. On the other hand, one of the challenges for engineers and scientists is the conversion of theoretical ideas and researches into practical and efficient approaches that can be used by designers of mechanical equipment. However, sometimes it is needed to move from research findings of existing machines to future systems, particularly having in mind that cranes performances have increased over the years and this intension is not yet finished. For instance, high-performance mega quayside container cranes (QCCs) and gantry cranes have already tripled in outreach, capacity and trolley velocity compared to the machines built more than 50 years ago. This reason makes the investigations on mathematical models of a crane moving trolley necessary. In recent years, considerable efforts have been made to better understand the dynamic behavior and vibration of large QCCs and gantry cranes under a moving trolley. The other direction in research is how to adopt the most appropriate moving load model. The goal of this paper is to discuss a wide spectrum of the most significant published papers in this field with respect to the future perspective of cranes structures development and to point out how to bridge the gap between "pure theoretical researches" (their usefulness) and practical requirements that are imposed by the structural engineers who are involved in the design of high-performance cranes.*

**Keywords:** moving load, structural dynamics, cranes, modelling.

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## 1. INTRODUCTION

The moving load problem is one of the fundamental problems in structural dynamics. In contrast to other dynamic loads these loads vary not only in magnitude but also in position. Structures subjected to moving bodies have been analyzed ever since the first railway bridges were built in the early 19th century. Interest in analysis of moving load problems originated in civil engineering (from observation that when an elastic structure is subjected to moving loads, its dynamic displacements and stresses can be significantly higher than those due to equivalent static loads) for the design of rail-road bridges and highway structures. The importance of this problem is manifested in numerous

applications in the field of transportation. Bridges, guideways, cranes, cableways, rails, roadways, runways and pipelines are examples of structural elements to be designed to support moving masses. Applications of moving load problem have been presented in mechanical engineering studies for the past 30 years. Extensive references to the literature on the subject can be found in the monograph by Fryba [1] with many analytical solution methods for simple cases. The basic approaches in trolley modeling are: the "moving force" model; the "moving mass" model and the trolley "suspension model".

The simplest dynamic trolley models are the "moving force" models. The consequences of neglecting the structure-vehicle interaction in these models may sometimes be minor. In most moving force models the magnitudes of the contact forces are constant in time. A constant force magnitude implies that the inertia forces of the trolley are much smaller than the dead weight of the structure. Thus the structure is affected dynamically through the moving character of

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the trolley only. All common features of all “moving force” models are that the forces are known in advance. Therefore structure-trolley interaction cannot be considered. On the other hand the “moving force” models are very simple to use and yield reasonable structural results in some cases. The review paper concerning with the fundamental load problem of a uniform simply supported Euler-Bernoulli beam subjected to a constant vertical force moving at a constant speed is written by [2] and gives a basic understanding of the moving load phenomenon. An excellent overview of moving load dynamic problem is given in [3].

## 2. MOVING LOAD PROBLEM IN STRUCTURAL DYNAMICS OF CRANES

The last decades have seen a mounting interest in research on modelling and control of cranes. The adopted models can be distinguished by different complexity of modelling and by the nature of neglected parameters. The most common modelling approaches are the lumped-mass and distributed mass approach. The comprehensive literature review of crane modelling and control, starting from 1961, was given in [4], where the conducted study has covered 150 journal papers, conference papers, and reports. The application of moving load problem in cranes dynamics has obtained special attention in the engineering researches in the last few years, but unfortunately little literature on the subject is available. Some of the most interesting papers will be discussed. The paper [5] is according to the authors’ best knowledge the first attempt to increase the understanding of the dynamics of cranes due to the moving load. Simply supported uniform Euler-Bernoulli beam carrying a crane carriage and payload is modeled, Figure 1 [5].

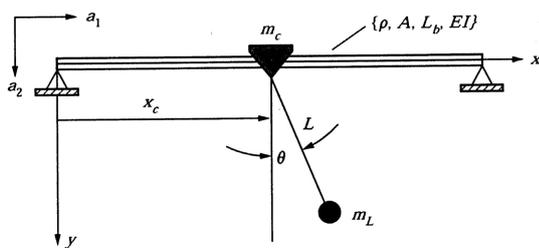


Figure 1. Oguamanam's model [5]

The crane carriage is modeled as a particle as is the payload which is assumed to be suspended from the carriage on a massless rigid rod and is restricted to motion in the plane defined by the beam axis and the gravity vector. The natural frequencies of vibration of the beam-crane system for a stationary crane are investigated and the explicit frequency equation is derived for that set of cases. Numerical examples are presented which cover a range of carriage speeds, carriage masses, pendulum lengths and payload masses. It is observed that the location and the value of the maximum beam deflection for a given set of carriage and payload masses is dependent upon the carriage speed. At very fast carriage speeds, the maximum beam deflection occurs close to the end of the beam where the carriage stops as a result of inertial effects and at very

slow speeds occurs near the middle of the beam because the system reduces to a quasi static situation.

Oguamanam's model was extended in [6]. An overhead crane, modeled as a point mass carriage traversing a simply supported Euler-Bernoulli beam that is allowed to travel in a direction perpendicular to its span, is considered. The point mass payload is attached to the carriage via a massless beam and is allowed both in-plane and out-of-plane motion, Figure 2 [6]. The Rayleigh-Ritz solution technique is used to obtain the equations of motion of the system. The influences of traverse and travel motions, pendulum length and payload mass on the pendulum motion are investigated.

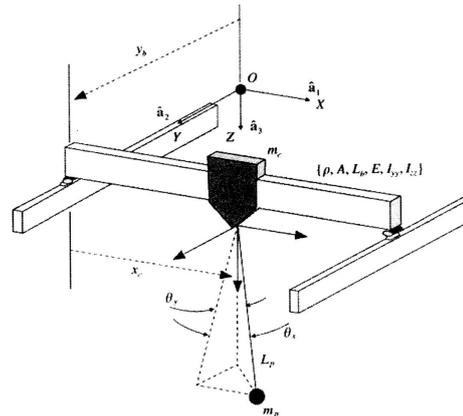


Figure 2. 3D Oguamanam's model [6]

Wu analyzed in paper [7] dynamic responses of the three-dimensional framework of a tyred overhead crane under the action of a moving trolley hoisting a swinging object which were calculated using the finite element method and the direct integration method. Instead of the conventional moving force problem where only the vertical inertia effect of the moving trolley was considered, the three-dimensional inertial effects due to the masses of both the moving trolley and the swinging object have been considered in this paper, Figure 3 [7]. To this end, an equivalent moving mass matrix has been presented and which is dependent on both the instantaneous swinging angle of the hoisted object and the instantaneous position of the moving trolley so that the contribution of the moving mass on the overall mass matrix of the entire structure itself is easily tackled. Finally, the title problem was solved by calculating the forced vibration responses of the three-dimensional framework with time-dependent overall mass and damping matrices and subjected to an equivalent moving force.

In the paper that comes after, [8], Wu presents a technique to replace the moving load by an equivalent moving finite element so that both the transverse and the longitudinal inertial effects due to the moving mass may easily be taken into account simultaneously. The mass, damping and stiffness matrices of the moving finite element are determined by the transverse (y) inertial force, Coriolis force and centrifugal force of the moving mass, respectively.

From the numerical examples illustrated, it has been found that, in addition to the conventional transverse (y) responses, the inertial effects of the moving load also affect the longitudinal (x) responses of the portal-frame structure significantly, Figure 4 [8].

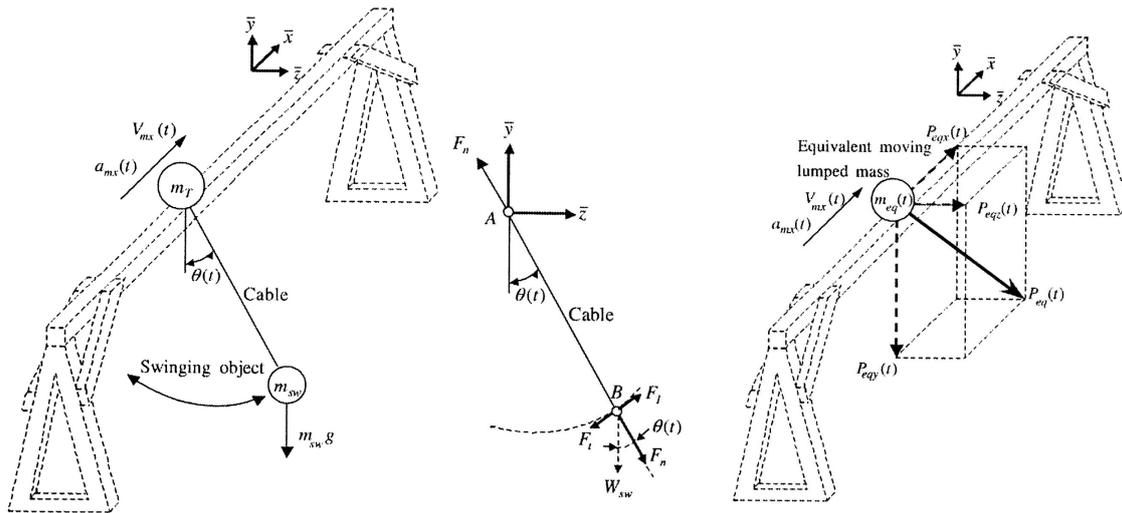


Figure 3. Wu's model [7]

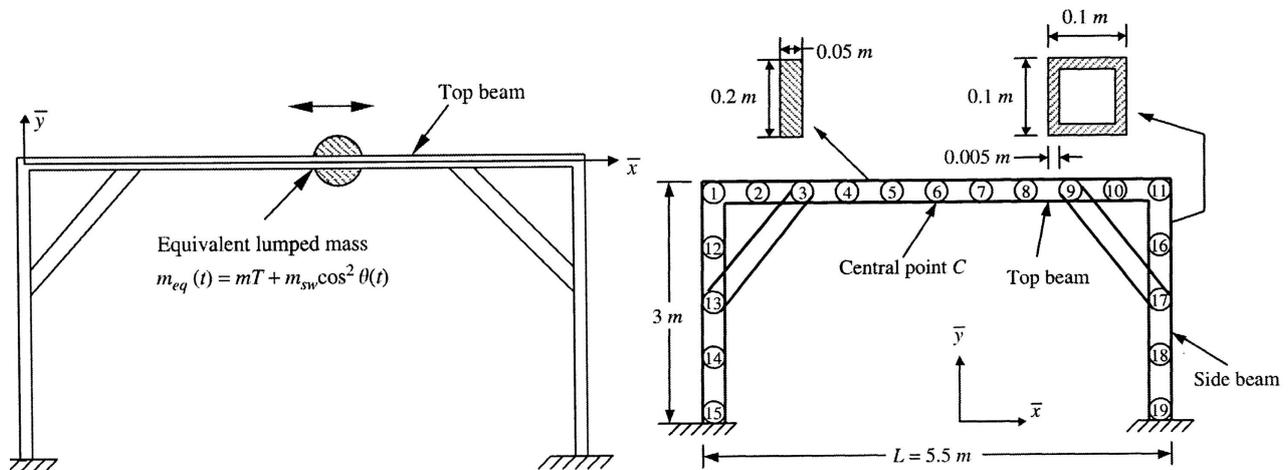


Figure 4. Wu's mathematical model for a crane hoisting a swinging mass [8]

Younesian has presented in [9], Figure 5, nonlinear vibration of a three-dimensional moving gantry crane carrying a trolley hoisting a swinging object. A finite element method is used to solve nonlinear coupled governing equations of the structure. A combinational technique (Newmark-Runge-Kutta) is employed for direct integration procedure. To develop a comprehensive parametric study and sensitivity analysis of the coupled nonlinear system, sequence of numerical simulations are carried out. Parametric study is directed to find out how different parameters like speed and acceleration of the trolley and gantry crane as well as the mass of the moving trolley and swinging object may affect the linear and nonlinear responses of the structure. It is found that the nonlinearity arises from large amplitude of three-dimensional motion of the swinging object.

The dynamics of an overhead crane system with the suspended payload on the trolley moving at a specified constant speed is considered in paper [10]. The beam is discretized by 10 elements, while the trolley is modeled as particle along with payload suspended with rope system modeled as spring, Figure 6 [10]. The overall mass, damping and stiffness matrix is calculated at each time interval, along with finite element formulation of equivalent force vector. Equations of motion of MDOF

system are given for oscillator moving on beam structure. Dynamic responses in the vertical direction for all DOFS are obtained by solving the governing equations with direct integration method. For validation purposes, the technique is first applied to a simple beam subjected to a force moving along the beam with constant velocity. The influence of moving velocity and spring stiffness are investigated.

The moving load problem is also considered for jib cranes. Dynamics of a two-dimensional jib crane structure subjected to a moving trolley with hoist and payload is investigated, Fig. 7 [11]. Dynamic responses of the structure, both in the vertical (Y) and horizontal direction (X), are calculated using the finite element method and the direct integration method. Instead of the conventional moving force problem, the paper [11] deals with the two-dimensional inertial effects due to the masses of trolley, hoist and payload. For this purpose, the moving mass matrix has been used to give contribution to the overall mass matrix of the entire system. The title problem was solved by calculating the forced vibration responses of the jib crane structure with time-dependent overall mass while subjected to an equivalent moving force. Factors as magnitude, speed and acceleration of the moving trolley were studied as well.

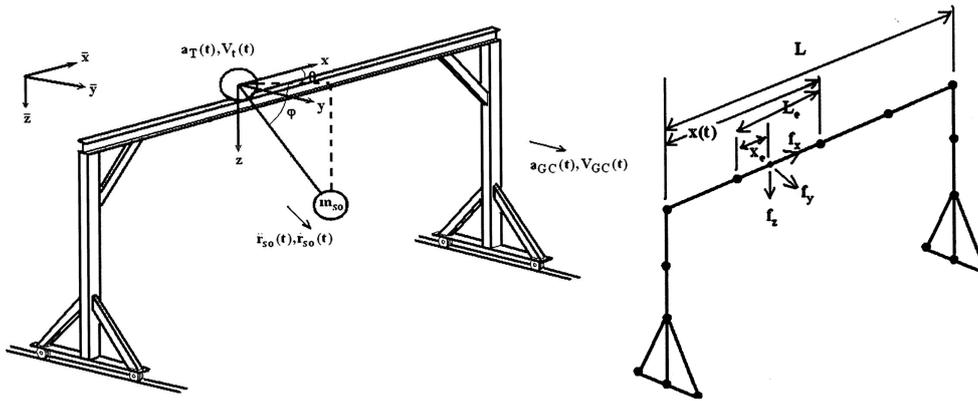


Figure 5. 3D sketch of Younesian model [9]

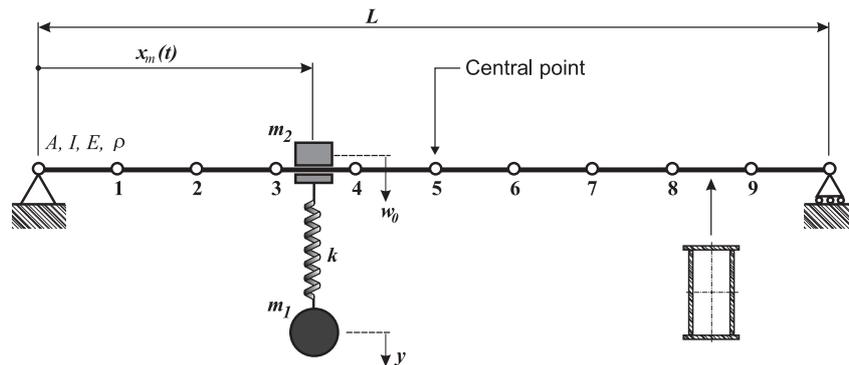


Figure 6. Dynamic model of the system [10]

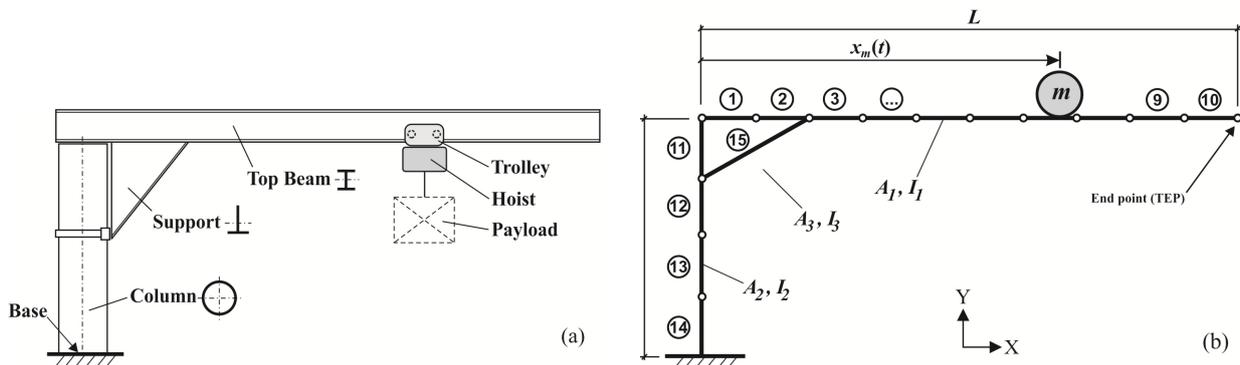


Figure 7. Gasic's model [11]

In recent years, considerable efforts have been made in order to better understand the dynamic behavior and vibration of large QCC under a moving trolley, particularly because of a construction of faster and heavier trolleys and the design of slender support structures without strict deflections limits. This fact resulted in several papers discussing the application of moving load problem in analysis of structural behavior of QCCs [12, 13, 14]. This is due to the fact that modern high-performance mega QCCs have already tripled in outreach and load capacity compared to the first QCC built in 1959.

This is not easily accomplished given the cantilevered nature of QCC. A cantilever (waterside boom) identified as the more important structural part is structurally inefficient because almost all of the structural strength and weight is needed to support its own weight. In practice it is very complicated and expensive to do an experimental research on a real size mega QCC. This reason makes the investigations on mathematical models of moving load necessary,

especially during the design stage and particularly having in mind the large dimensions of the boom and trolley mass [15]. Mathematical model of moving load action on QCC is presented in Figure 8 [12, 15].

The dynamics of a two-dimensional gantry crane structure subjected to a moving trolley with hoist and payload is examined in [16].

Dynamic responses of the structure, both in vertical (Y) and horizontal direction (X), are postulated using the combined finite element and analytical method and solved with the direct integration method. Instead of the conventional moving force problem, the two-dimensional inertial effects due to the overall mass of trolley, hoist and payload have been considered in this paper, Figure 9 [16]. The title problem was solved by calculating the forced vibration responses of the jib crane structure with time-dependent overall mass and subjected to an equivalent moving force. Factors as speed and acceleration of the moving trolley were studied.

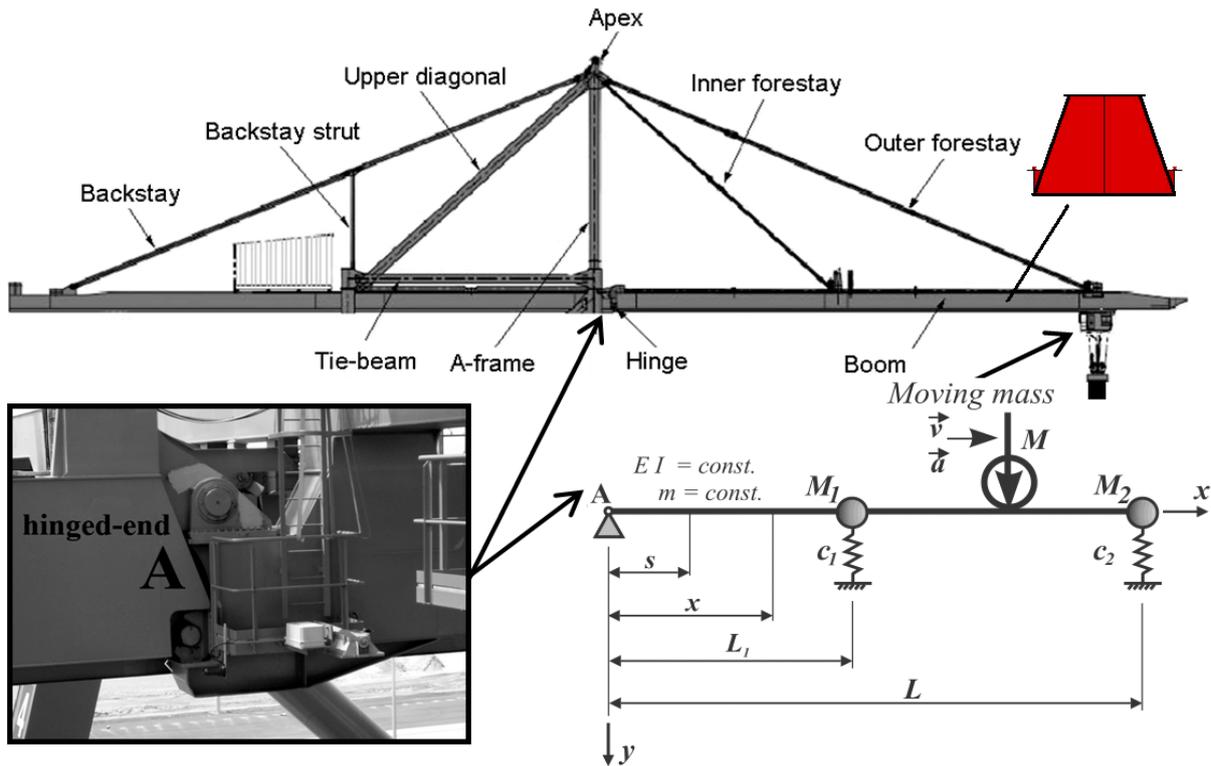


Figure 8. Zrnic's model [13,15]

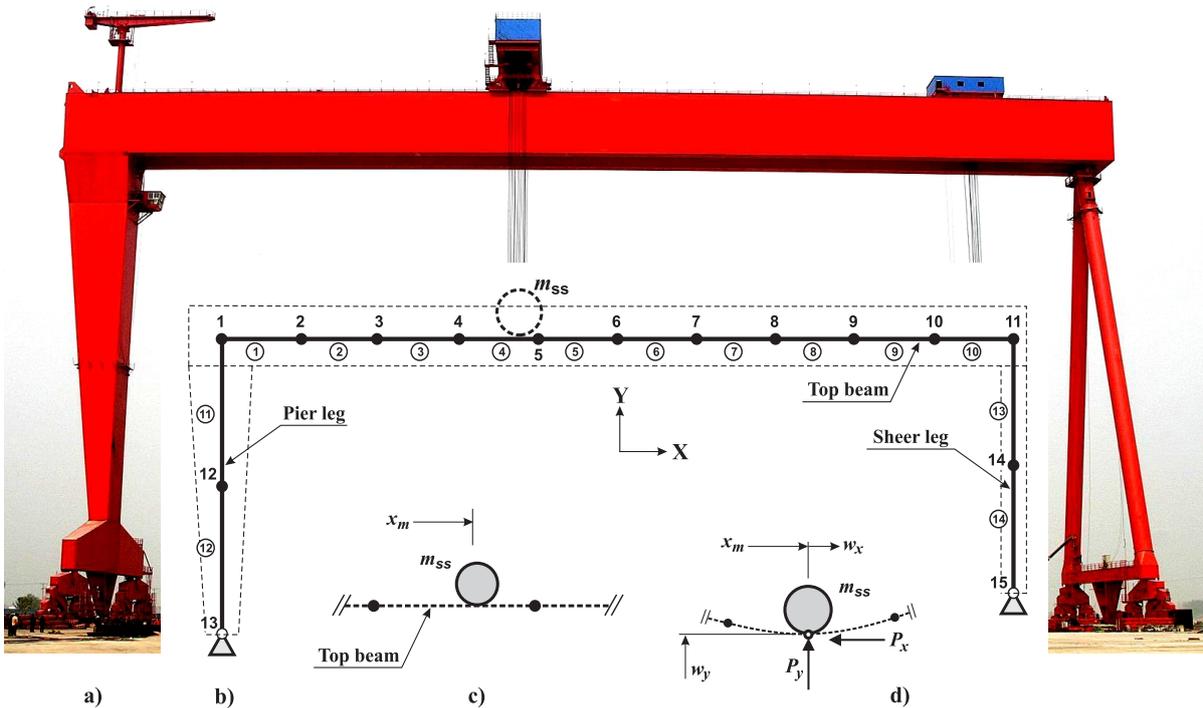


Figure 9. a) Real model of the gantry crane, b) FE model of the framework, c) Moving mass, d) Dynamic interaction [16]

### 3. DISCUSSION

Results from paper [5] are obtained for beam length of 10 m with too small (unrealistic) square tube cross section. The ratio of length/section dimension is extremely big which can raise doubts in the usage of Euler-Bernoulli theory for bending in this case. Maximal static deflection due to own beam self weight is 37 cm which also describe the practical faults of the

research. Extended research [6] also uses unrealistic (too small) value for inertial moment for cross section of beam which is 6 m long. Hence, only swinging angles of payload are presented with values ( $<1,71^\circ$ ) which can't be of much importance. Periods for acceleration/deceleration of 15 s are very big to raise interest in crane dynamics. However, both papers show the complex mathematical approach for moving load problem.

Basic assumption in paper [7] is the reduction of the moving pendulum problem to moving mass problem with equivalent moving mass as  $m_{eq}(t)=m_T+m_{sw}\cos^2\theta(t)$ . This approximation neglects the centrifugal forces due to payload swinging and assumes that swinging of the payload is known in advance. Also, structural deflections of 3 mm are very small to gain practical usage. Moreover, paper [8] studies the crane dynamics with trolley speeds up to 100 m/s. These (unachievable) speeds can not stand for conducting proper discussion. Nevertheless, results for speeds of trolley in range of 3-5 m/s are not suitably presented which certainly could be more interesting for discussion of real range of trolley performances in close future.

Results in paper [9] are based on top beam discretization with only 4 elements which can generate bottom level of satisfactory results and can be used only for *small* gantry cranes which aren't affected to moving load problems because of short runway.

Paper [10] presents the middle span deflection of bridge crane as main parameter in crane design. It is noted that when the trolley speed is relatively low (2 m/s) the dynamic response in moving mass problem resembles the static case. For the trolley speed of 5 m/s one can find dynamic amplification factor (DAF) of 1.085 which comes from inertial effect of moving mass, performed on bridge span of 40 m and sectional properties which satisfies basic static design check. Extended analysis with moving oscillator model give mid span deflection of 11,5 cm which is increase of 12 % from the static case, even for the critical spring stiffness.

For the push-to-limits performances of the jib crane structure, results from [11] show that moving load effects are noticeable mainly in acceleration/deceleration periods. The DAF factor goes to 1.11 for the value of the flexural moment for the jib crane basement compared to static case and DAF is 1.08 for the maximal horizontal deflection.

Results from paper [12] reveals that values of acceleration of moving mass in vertical direction is 0.0165 g which belongs to the class of clearly appreciable accelerations in the frequency range between 1 - 10 Hz and it is not as high to disturb the crane operator.

Trolley speed pattern in paper [13] assumes that speed is 6 m/s and acceleration is 1.2 m/s<sup>2</sup>, performed on QCC with span of 65.8 m. The values for DAFs are obtained in relation to the maximal static results. With moving mass model DAF for the boom deflection is 1.142 while for the bending moment is 1.114. It is important here to note that DAFs from moving mass model and moving force model are very close which was obtained in [15] with maximal value of boom deflection of 0.407 m.

Paper [16] gives results that show the trolley speed of 5 m/s don't have significant influence on vertical displacements of the structure, but only for horizontal displacements. The decrease of amplitudes for the central displacement can be gained with structural damping of 0.06, but is only descriptive because it is very hard to achieve this value of damping in crane structures.

#### 4. CONCLUSION

After discussing the practical relevance of the obtained results in many analyzed papers [5,6,7], regardless of the theoretical accomplishments in, it is obvious that the obtained results are not relevant for practical considerations for already existing industrial cranes. On the other hand in some papers discussing large high-performance container and gantry cranes [10,11,15] are considered both actual crane performances and the ones predicted to be reached in the future, although the perspective in design of such machines is uncertain.

So, the intention in research dealing with moving load problem in structural dynamics of cranes should go towards mega cranes with increased performances. Obviously, dynamic behaviour of a mega structure as a movable flexible structure is different than of a smaller crane. Vibrations are a serious problem in crane systems that are required to perform precise motion in the presence of structural flexibility. Not only the vibration of the crane is unacceptable operationally, it may be unacceptable structurally because of additional fatigue damage. Of course, it is very difficult and expensive in practice to do an experimental research on a real size mega crane or even on a scale-model. For this reason the investigations on mathematical models are necessary, especially during the design stage. Simpler models of mega cranes enable easier mathematical analysis and give better insight in the design and the possibilities of different control algorithms. On the other hand, more complex models are necessary to approximate the reality closer, e.g. the flexibility of the crane structure will certainly affect the behaviour of the controller. However, it is impossible to include all effects of the real life in a mathematical model of large crane [13].

Finally, the last direction in research can be towards how to adopt the most appropriate moving load model. It can be found in paper [15] that the basic structural response "moving force" model is appropriate for use in engineering problems because it gives a very slight difference comparing to the "moving mass" model. This fact applies to the extreme up-to-date parameters of QCC. Although it very difficult to predict the future development of large cranes it seems unlikely to reach in the next decades such an increase of performances that will favor "moving mass" model to be suitable for engineers in design process. However, "moving mass" model will be viable for scientific approach and structures of cranes can be taken as a good example in modeling.

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**ПРОБЛЕМАТИКА ПОКРЕТНОГ ОПТЕРЕЂЕЊА  
ПРИ АНАЛИЗИ ДИНАМИЧКОГ ПОНАШАЊА  
ДИЗАЛИЦА: ПРЕМОШЋАВАЊЕ ЈАЗА ИЗМЕЂУ  
ТЕОРИЈСКИХ И ПРАКТИЧНИХ ПРИСТУПА**

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Проблематика покретног оптерећења је једно од основних поглавља динамике структура. Током последњих сто година спроведена су многа истраживања која се баве анализом динамичких одзива структура, најпре код железничких и друмских мостова. У последње време предмет ових истраживања постају и структуре дизалица. у последњих неколико година истраживања из области утицаја покретног оптерећења на структуре дизалица су привукла пажњу неколицине истраживача. Па ипак, независно од теоријских достигнућа у овој области, након провере усвојених параметара дизалица, очигледно је да добијени резултати у многим радовима нису релевантни за практичну примену на постојећим дизалицама. Са друге стране, изазов за инжењере и научнике је управо конверзија теоријских идеја у практичан приступ, који се може искористити за пројектовање дизалица. Међутим, понекад је потребно претпоставити параметре који превазилазе тренутни ниво техничких решења, посебно имајући у виду да се перформансе дизалица константно побољшавају, што ће и даље бити настављено. На пример, данашње обалске контејнерске дизалице и порталне дизалице су троструко повећале распоне, носивости и брзине колица у поређењу са истим направљеним пре 50 година. Због тога је неопходно разматрање динамичких модела дизаличних колица. У последње време велика пажња је посвећена бољем разумевању динамичког понашања и вибрација код структура обалских контејнерских дизалица и порталних дизалица услед дејства кретања колица. Један од основних проблема у истраживањима јесте усвајање одговарајућег модела покретног оптерећења. Циљ овог рада је да да освети на широк спектар најважнијих објављених радова из ове области у односу на будући развој структура дизалица и да нагласи како се може премостити јаз између "чистог" теоријског приступа (њихове корисности) и практичних потреба конструктора који пројектују дизалице високих перформанси.