Modeling the Motion and Mass Quantity of Fruit by Rotating Sizing Machines

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This paper analyzes theoretically the motion and mass quantity of eight fruit types, roundish in shape, along rotating sizing machines. Mathematical model of the motion of fruit on the rotating disc is solved in spherical coordinate system which is tied to the disc. The exact solution is simplified introducing assumpions that are do not significantly affect the accuracy of the solution. Starting from differential equation of fruit motion on a rotating disk of the sizing machine, all forces acting on fruit were determined as well as velocities and time necessary for fruit to reach the disk rim if found in some upper position on the cone. Mass quantity analysis comprised sized fruit mass and weight capacity. New empirical coefficients were introduced: extent ratio, feed ratio and distribution ratio. The results obtained for the adopted values $k_e = 0.7$, $k_f = 1$, $k_d = 0.5$ coincide approximately with those reported to date for fruit mass quantity rate on sizing machines. It was found that mass quantity rates vary considerably, depending on fruit diameter and mass. Fruit numbers mass quantity ranges from 8949 crops/h for apple to 40,157 crops/h for deep frozen raspberry. Mass quantity varies from 229.1 kg/h for cherry to 2054.7 kg/h for apple.

Keywords: *fruit sizing, fruit motion, mass quantity.*

1. INTRODUCTION

The fresh fruit postharvest sector is dynamic, due largely to increasing consumer demand for quality produce. After harvest, fruit crops differ in many properties. Fruit sorting performed after harvest is a set of technological operations, whose aim is to sort crops for placement on the market, preservation or consumption as fresh fruit, or industrial processing. In order to put fruit crops on the market, they must be of uniform quality and size.

The fact that numerous factors influence the possibility of standardizing crop quality, according to prescribed criteria, indicates that subsequent crop processing is a complex and important process [1]. The term ‘crop processing’ primarily refers to crop sorting and grouping by quality and size characteristics to as high extent as possible. Without disregarding other operations preceding or following the two mentioned, it should be stressed that mechanized operations should be applied in fruit sorting by quality, but to a higher extent when this is done by fruit size or mass [2]. The factor that makes sorting by geometric or weight properties more difficult is the diversity of crop shapes: round, elongated or flat. To illustrate the complexity, scale and level of costs for above mentioned processing and packing of crops, the following data are presented: the processing of 1.0 t of fruit crops takes 20-40 working hours, which is equivalent to 50-60% of harvest costs and nearly one-third of the total fruit production [3].

2. MATERIALS AND METHODS

According to the working principle, all sizing machines can be translational or rotational, depending on fruit trajectory on them. The majority of today’s commercial and industrial sizing machines are based on translational, translational-vibratory system of fruit sorting by the size [4]. The capacity of those machines ranges between 3 and 10t/h and is determined by product type and quality. Rotating sizing machines, on the other hand, are characterized by considerably lower capacity. Their mass quantity is certainly lower than 3t/h, which in some cases can be advantageous unless higher capacity is required, but this makes the machine more complex and reduces its productivity and cost-effectiveness in many ways. Reduction in capacity can be compensated for by engaging a number of parallel rotating sizing machines and coupling their equivalent classes with a conveyor system. The disadvantage of such a system would be to increase the length of fruit trajectory during sizing, whereby the possibility of fruit damage is greater. Also, rotating sizing machines are superior to those translational for their smaller dimensions.

The main parts of fruit sizing machine comprise a rotating disk, a sizing board, a feeding tray, receiving trays and a power drive. The model is shown in Figure 1. The top surface of the rotating disk is formed into a conical shape, which allows the fruit to roll down to the sizing gaps by gravity. During operation, crops are continuously poured onto the feeding tray and then rolled down onto the rotating disk in clusters of 6-10 pieces at a time [5]. Each fruit is then brought into

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contact with the sizing board and the rim of the rotating disk through gravitational and centrifugal forces. The fruit move along the sizing board and drop down to the receiving tray whenever the diameter of the fruit is less than the constant metering gap. Small fruit thus will be sized before big fruit.

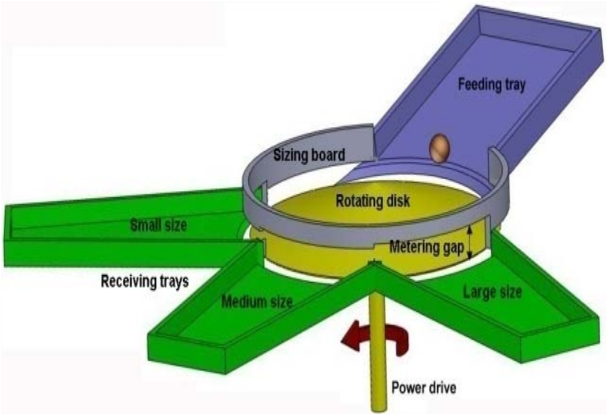

Figure 1. Shematic diagram of fruit sizing machine

3. RESULTS AND DISCUSSION

3.1. Motion of fruit

Fruit motion on a rotating disk of the sizing machine (Fig. 2) can be viewed as relative motion of the particle M, of mass m, along the disk rotating at constant angular velocity ω about vertical axis z. The moving coordinate system, fixed to a rotating disk, is represented by spherical coordinates: r, θ and φ, as shown by their axes in Fig. 2.

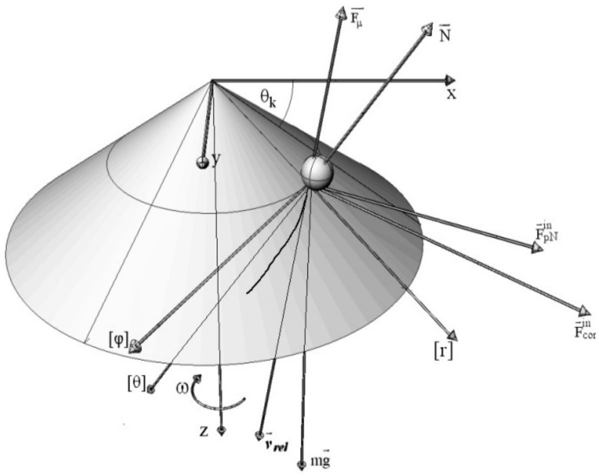

Figure 2. Forces acting on fruit and rotating disk

Differential equation of fruit motion relative to a rotating cone is given by the relation:

$$m\vec{a}_{rel} = m\vec{g} + \vec{N} + \vec{F}_{\mu} + \vec{F}_{p}^{in} + \vec{F}_{cor}^{in}. \tag{1}$$

The friction force is collinear with the vector of relative velocity $\vec{v}_{rel}$ and is opposite in direction relative to the direction of relative motion, i.e.

$$\vec{F}_{\mu} = -\mu N \frac{\vec{v}_{rel}}{v_{rel}}. \tag{2}$$

and magnitude that depends on the sliding friction coefficient μ and intensity of normal reaction N.

The force of inertia $\vec{F}_{p}^{in}$, presented in (1), is reduced to normal component due to constant angular velocity of the cone rotation ω, of the magnitude:

$$F_{pN}^{in} = m\omega^2 r \sin\theta_k, \tag{3}$$

and of direction normal to the axis of rotation, where spherical coordinate $r = \overrightarrow{OM}$, while the angle θ_k is designated in Fig. 2.

Coriolis force of inertia, presented in (1), is given by the relation:

$$\vec{F}_{cor}^{in} = -2\vec{\omega} \times \vec{v}_{rel}. \tag{4}$$

Relative velocity $\vec{v}_{rel}$, presented in (2) and (4), is determined by its projections v_r, v_θ and v_φ on coordinate axes of the spherical coordinate system:

$$v_r = \dot{r},\ v_\varphi = r\dot{\varphi}\cos\theta,\ v_\theta = r\dot{\theta} \tag{5}$$

Taking into account that angle $\theta = \theta_k = const$ on the cone surface, the projection v_θ given by a corresponding expression in (5), is annulled, i.e. $v_\theta = 0$ so that relative velocity $\vec{v}_{rel}$ becomes:

$$\vec{v}_{rel} = v_r\vec{e}_1 + v_\varphi\vec{e}_2 = \dot{r}\vec{e}_1 + r\dot{\varphi}\cos\theta_k\vec{e}_2 \tag{6}$$

where $\vec{e}_1$ and $\vec{e}_2$ are corresponding vectors of coordinate axes $[r]$ and $[\varphi]$. Due to the orthogonality of coordinate axes, the intensity of relative velocity can be written in the form:

$$v_{rel} = \sqrt{v_r^2 + v_\varphi^2} = \sqrt{\dot{r}^2 + \left(r\dot{\varphi}\cos\theta_k\right)^2} \tag{7}$$

The force of friction $\vec{F}_{\mu}$, determined by (2), now obtains the form:

$$\vec{F}_{\mu} = -\mu N \frac{1}{v_{rel}}\left(\dot{r}\vec{e}_1 + r\dot{\varphi}\cos\theta_k\vec{e}_2\right) \tag{8}$$

the intensity v_{rel} being determined by (7).

The projections of acceleration $\vec{a}_{rel}$ on coordinate system axes are:

$$a_r = \ddot{r} - r\dot{\varphi}^2\cos^2\theta - r\dot{\theta}^2$$
$$a_\varphi = \frac{1}{r\cos\theta}\frac{\mathrm{d}}{\mathrm{d}t}\left(r^2\dot{\varphi}\cos\theta\right) \tag{9}$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} + r\dot{\varphi}^2\sin\theta\cos\theta.$$

If particle M, during its motion, does not leave the cone surface, then $\theta = \theta_k = const$ therefore the relation (9) obtains a simpler form:

$$a_r = \ddot{r} - r\dot{\varphi}^2\cos^2\theta_k,$$

$$a_\varphi = \frac{1}{r\cos\theta_k}\frac{\mathrm{d}}{\mathrm{d}t}\left(r^2\dot{\varphi}\cos\theta_k\right), \qquad (10)$$

$$a_\theta = r\dot{\varphi}^2 \sin\theta_k \cos\theta_k .$$

Differential equation of relative motion (10) is equivalent to the system of scalar differential equations:

$$ma_r = mg\sin\theta_k - \mu N\frac{\dot{r}}{v_{rel}} + m\omega^2 r\cos^2\theta_k + 2m\omega r\dot{\varphi}\cos^2\theta_k,$$

$$ma_\varphi = -\mu N\frac{r\dot{\varphi}\cos\theta_k}{v_{rel}} - 2m\omega\dot{r}\cos\theta_k, \qquad (11)$$

$$ma_\theta = mg\cos\theta_k - m\omega^2 r\cos\theta_k\sin\theta_k - -N - 2m\omega r\dot{\varphi}\sin\theta_k\cos\theta_k .$$

whose exact solution, with corresponding initial conditions, due to extreme complexity, should be sought in the application of some numerical method. In order to simplify the analysis of influence of some geometric and constructive parameters, it is convenient to obtain an approximative solution of the system (11). Assuming that the friction force can be neglected, and by leaving out the second term on the right side of the second equation of the system (11), as a small quantity, it is obtained:

$$ma_\varphi = 0, \qquad (12)$$

wherefrom

$$r^2\dot{\varphi}\cos^2\theta_k = const = r_0^2\dot{\varphi}_0^2\cos^2\theta_k . \qquad (13)$$

If it is assumed that the initial relative velocity was collinear with the cone generating line, i.e.

$$v_{\varphi 0} = v_\varphi(0) = r_0\dot{\varphi}\cos\theta_k = 0 \qquad (14)$$

there follows that $\dot{\varphi}_0 = 0$, therefore from the relation (13) it comes out that $\varphi = 0$ too, i.e. particle M will be moving along the cone generating line [6]. Taking into account this fact, it can be deduced that $a_\theta = 0$, so that (11) obtains the form:

$$m\ddot{r} = mg\sin\theta_k + m\omega^2 r\cos^2\theta_k \qquad (15)$$

having in mind that the second equation of the system (11) is identically satisfied. From the second equation of the system (15), the intensity of normal reaction is determined:

$$N = mg\cos\theta_k - m\omega^2 r\sin\theta_k\cos\theta_k . \qquad (16)$$

The final equation of fruit motion relative to the moving cone becomes:

$$r(t) = \left(r_0 + \frac{g\sin\theta_k}{\omega'^2}\right)\cos h(\omega' t) + + \frac{\dot{r}_0}{\omega'}\sin h(\omega' t) - \frac{g}{\omega'^2}\sin\theta_k \qquad (17)$$

where the quantity $\omega' = \omega\cos\theta_k$ while r_0 and $\dot{r}_0$ are a coordinate of the fruit M and projection of its relative velocity at initial moment $t_0 = 0$ respectively. If l is the length of the cone generating line, time t_k, needed for fruit to reach the cone (disk) rim from initial position determined by the coordinate r_0, is obtained from the condition $r(t_k) = l$, wherefrom it follows:

$$t_k = \frac{1}{\omega'}\ln\frac{a + \sqrt{a^2 + b^2 - 1}}{1 + b} \qquad (18)$$

the quantities a and b being determined by expressions:

$$a = \frac{l + \frac{g}{\omega'^2}\cdot\sin\theta_k}{r_0 + \frac{g}{\omega'^2}\cdot\sin\theta_k}, b = \frac{\dot{r}_0}{\omega'(r_0 + \frac{g}{\omega'^2}\cdot\sin\theta_k)} . \qquad (19)$$

The relative speed of the crop can be determined by the relation:

$$\dot{r}(t_k) = \left(r_0 + \frac{g\sin\theta_k}{\omega'^2}\right)\sin h(\omega' t) + + \frac{\dot{r}_0}{\omega'}\cos h(\omega' t) - \frac{g}{\omega'^2}\sin(\omega' t) \qquad (20)$$

where t_k time is determined by (18).

Speed of the fetus, given by the relation (20) is the functional dependence of several variables and is shown graphically in Figure 3, the initial rate constant value of $\dot{r}_0$.

The cone angle of the disk stimulates disk discharge through metering gap between the rotating disk and sizing board [7]. Also, it decreases the chance of congestion by increasing fruit relative velocity.

When crops are moving along a metering gap, being sized in a single batch, which is most often the case, fruit relative velocity does not exist, therefore absolute velocity is equal to the transmission speed of fruit. Functional dependence of this speed on the number of rotations and diameter of metering gap is graphically presented in Fig. 3.

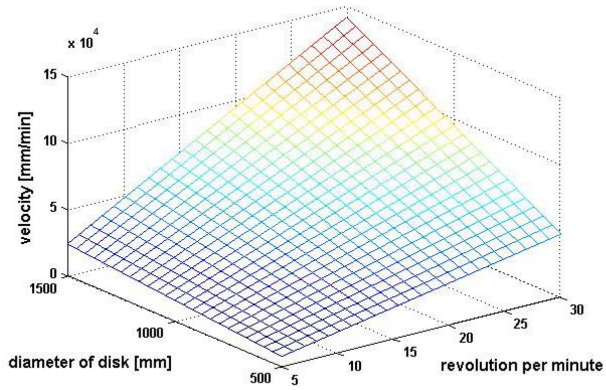

Figure 3. Velocity as function of diameter of disk and revolution per minute

3.2. Mass quantity of fruit

Just when the crop is found against the metering gap on the sizing board, whose dimensions exceed the dimensions of the crop itself, then the board stops it no longer in relative motion, so the crop receives the component of relative velocity too, moving down the rotating disk and falling onto the receiving tray for a certain sizing. Linearization of the expression in which speed participates, leads to an expression for the mass quantity of fruit with influential factors.

Total numbers mass quantity of all sized fruit batches at the sizing machine exit is:

$$Q_N = 60 k_e k_f k_d \frac{(D-d)\pi}{d} \cdot n \,. \qquad (21)$$

Total mass quantity of all sized fruit batches at the sizing machine exit is:

$$Q_m = 60 k_e k_f k_d \frac{(D-d)\pi}{d} \cdot m \cdot n \,. \qquad (22)$$

Due to the simplification of the model, it is assumed that the relative velocity linearly depends on the fruit of a conical disk angular velocity. The disk extent ratio, $k_e < 1$, is the design parameter that depends on the width of feeding tray for fruit conveying and dosing, i.e. the space it occupies above the rotating disk. The larger the cone angle of the rotating disk, the larger the feed ratio, $k_f \geq 1$, and crops are conveyed to the disk in larger amounts than needed, so as to be distributed to the disk rim, only in one batch along the sizing board. When moving along the rotating disk, the crops are not distributed along the entire arc against the sizing board, so their speed is reduced due to stopping, sliding, rebounding and rubbing. This requires the correction of speed using the distribution ratio $k_f \geq 1$ that indirectly indicates how much fruit conveying and dosing should be slowed down. The average diameter of selected fruit is generally equal to the height of the middle of metering gap.

Calculations of theoretical mass quantity expressed by the relations (20) and (21) were applied for various types of fruit and the results obtained are shown in Tab. 1 and by a diagram in Figs 4 and 5. The values adopted were: disk diameter $D = 600mm$ and number of disk revolutions $n = 21\min^{-1}$ sized fruit damage being the least [5]. The adopted coefficients figuring in these expressions were: $k_e = 0.7$, $k_f = 1$, $k_d = 0.5$.

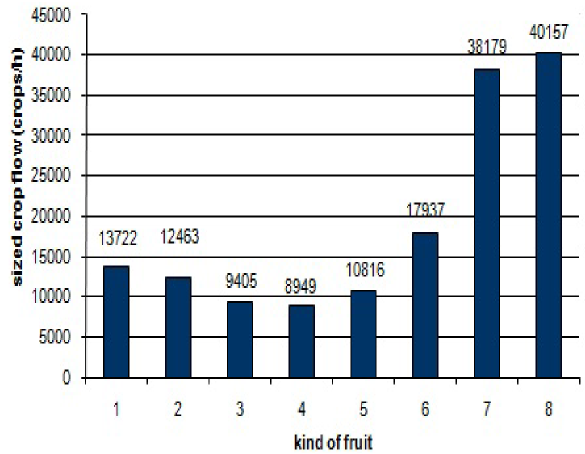

Figure 4. Diagram of sized crop numbers

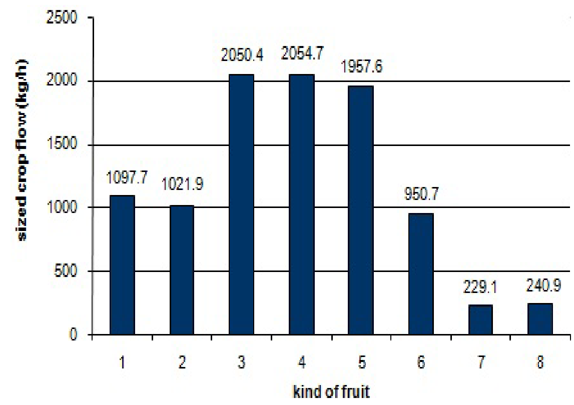

Figure 5. Diagram of sized crop numbers and mass quantity

The majority of fruit types vary considerably in physical properties determined by fruit variety. Therefore, Table 1 shows fruit varieties that can be claimed to be exactly the characteristic representatives of the variety. Functional dependence of mass quantity on fruit average diameter and its average mass is shown by the graph in Fig. 6. It is noticeable from the graph that the capacity and efficiency of the rotating disk sizing machine grow as fruit mass increases and its diameter decreases.

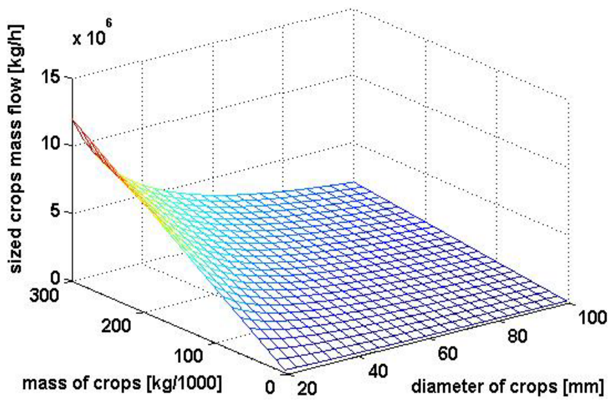

Figure 6. Sized crop mass quantity as a function of disk diameter and revolutions per minute

4. CONCLUSION

Mathematical model of rotating sizing machine is described in details in this paper. Instead of exact numerical solution of very complex system of differential equations an approximative solution is obtained. In that way, influences of some geometric and construction parameters such as extent ratio k_e, feed ratio k_f and distribution ratio k_d, become more transparent. When fruit sizing is performed it is always possible to damage crops by hitting or bruising, while deep frozen fruit sizing can cause cracking of crops. In both cases it is necessary to carry out the optimization of disk speed and capacity of such sizing machines to make the sizing process as efficient as possible with as small fruit damage as possible. As the motion of fruit crops across sizing machines depicted in this paper is complex, this machine is almost exclusively suitable for fruit crops with ball-like fruit, while sizing of other types of fruit would require the increase of disk cone angle. In the case of aggregating several individual rotating sizing machines such advantage of theirs is eliminated compared to translational sizing machines of the same rank by capacity.

Further studies of rotating sizing machines should be adapted to each type of fruit, due to their specific shape and dimensions. Also, studies should be oriented to optimizing the parameters of sizing machines, such as: metering gap diameter, disk rotating speed, disk cone angle, coefficients involved in calculations of theoretical capacity, given and described in this paper, as well as the speed of dosing.

Under conditions of fruit crops production, processing and trade, the sizing machines described can be used on wholesale fruit markets and in small-capacity factories for fruit processing and packing.

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NOMENCLATURE

d (mm)	- average diametar of calibrated crops,
D (mm)	- diametar of metering gap,
F (N)	- force,
g (m/s^2)	- constant of gravitation,
k_e	- extent ratio,
k_f	- feed ratio,
k_d	- distribution ratio,
m (kg)	- average mass per one crop,
n (min^{-1})	- number of disk revolutions,
N (N)	- cone normal reaction
Q (kg/h)	- mass quantity rate of calibrated crops,
t (s)	- time,
v (mm/min)	- velocity,

***Greek symbols**:*

μ	- coefficient of sliding friction,
ω (rad/min)	- angular velocity
θ_k (º)	- cone angle of rotating disk,

Indexes:

0	- initial value,
cor	- Coriolis force (acceleration),
in	- inertia force,
m	- mass,
N	- numbers,
p	- transmission motion,
rel	- relative motion.

МОДЕЛИРАЊЕ КРЕТАЊА И ПРОТОКА ВОЋА НА РОТАЦИОНИМ КАЛИБРАТОРИМА

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У овом раду теоретски је анализирано кретање и проток осам врста воћа дуж ротационог калибратора. Полази се од диференцијалне једначине кретања воћа на ротирајућем диску калибратора. Развијен је општи модел који може бити примењен за одређивање протока за све типове ротационих калибратора. Анализа протока обухватила је масу и проток калибрираног воћа. Уведени су нови искуствени коефицијенти: коефицијент процентуалног искоришћења обима диска, коефицијент пуњења и коефицијент расподеле плодова. Посебно је истражен утицај релативне брзине воћа на капацитет калибратора. За усвојене вредности наведених коефицијената $k_e = 0.7$, $k_f = 1$, $k_d = 0.5$ добијени су резултати који се приближно поклапају са до сада познатим протоцима воћа на калибраторима. Установљено је да протоци веома варирају у зависности од дијаметра и масе воћа. Количински проток варира од 8949 plod/h за јабуку до 40157 plod/h за дубоко замрзнуту малину. Масени проток варира од 229,1 kg/h за вишњу до 2054,7 kg/h за јабуку.

Table 1. Properties of eight types of fruit and sized crop mass and numbers mass quantity [5,8,9,10,11,12,13]

No	Kind of fruit and variety	Average crop diametar (mm)	Average crop mass (kg/1000)	By reference	Sized crops mass quantity (numbers/h)	Sized crops mass quantity (kg/h)
1.	mangosteen	55	80	*Jarimopas et al., 2007*	13,722	1097.7
2.	apple Redspar	80.4	229.6	*Kheiralipour et al., 2008*	8949	2054.7
3.	apricot Rajabali	43	53	*Mirzaee et al., 2009*	17,937	950.7
4.	cherry Chabestar	21	6	*Naderiboldaji et al., 2008*	38,179	229.1
5.	tangerine Clementine	60	82	*Sahraroo et al., 2008*	12,463	1021.9
6.	orange Tompson	77	218	*Sharifi et al., 2007*	9405	2050.4
7.	peach Filina	68.1	181	*Zhivondov, 2010*	10,816	1957.6
8.	deep frozen raspberry	20	6	big deep freezing fruit processing company from Western Serbia	40,157	240.9