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Buckling Behaviour of Dented Aluminium Alloy Cylindrical Shell Subjected to Uniform Axial Compression

Thin-walled cylindrical shells are commonly used in numerous branches of industry. Since they are subjected to axial load, the most common cause of their failure is buckling. This paper provides numerical analysis and experimental verification of the buckling of the thin-walled aluminium alloy cylindrical shell with special regard to the influence of dent, positioned in the middle of the shell. Numerical simulation was performed using ANSYS 16.2, and experimental verification was performed by means of hydraulic press Armavir, PSU-50, which was used to subject the specimen to the increasing axial load until the occurrence of buckling. Comparing the results it was concluded that there is significant decrease of the buckling resistance if compared the values of the specimen with no dent, and the specimen with 2 mm deep dent. On the contrary, resistance of the 2 mm and 4 mm dented specimen is quite similar. Position and shape of the deformations occurred due to buckling are matching if experimental and numerical results are compared.

Keywords: *Thin cylindrical shell, Buckling strength, Geometrical imperfections, Dents, Numerical simulations*

1. INTRODUCTION

Thin-walled cylindrical shells are widely used in aerospace and automotive industries and civil engineering [1]. In order to obtain the best mechanical properties, they are often made of the light-weight alloys such are Aluminium-based alloys [2], [3]. Their function is to transmit the axial loading, and thus to be under pressure. When the pressure reaches the critical value, it comes to the occurrence of the buckling of the walls. Furthermore, due to the small thickness of walls, it is very possible the occurrence imperfections, which additionally reduce the level of critical buckling stress. The imperfections present in thin cylinders are classified as geometrical imperfections, material imperfections and other imperfections. The imperfections such as circularity, cylindricity, local indentations, dents, crack, swellings, non-uniform thicknesses, etc., belong to geometrical imperfections whereas imperfections such as in-homogeneity, vacancies, impurities, etc., are classified as material imperfections. The residual stresses and strains induced during manufacturing, etc. are grouped as other imperfections [1].

Buckling load is the load at which the current equilibrium state of a structural element or structure suddenly changes from stable to unstable, and is, simultaneously, the load at which the equilibrium state suddenly changes from that previously stable configuration to another stable configuration with or without an accompanying large response (deformation or deflection). Thus, the buckling load is the largest load for which stability of equilibrium of a structural element or structure exists in its original (or previous) equilibrium configuration [5].

The importance of this subject is confirmed by numerous papers which refer to the buckling of cylindrical shell. Therefore, Prabu, Raviprakash and Venkatraman investigated the effect of a dent on strength of the short cylindrical shell [1], while the buckling of cylindrical shells with through cracks has been studied by Estekanchi and Vafai [6]. Simulation and FEM analysis of thin steel cylindrical shells of various lengths and diameters with elliptical cutouts have been studied by Shariati and Rokhi [7], [8]. The effect of random geometric imperfections on the critical load of isotropic, thin-walled, cylindrical shells under axial compression with rectangular cutouts is presented by Schenk and Schueller [9] and the effect of material and thickness imperfections on the buckling load of isotropic shells is investigated by Papadopoulos and Papadrakakis [10]. Using a double scale analysis including interaction modes, Jamal, Midani, Damil and Potier-Ferry also analysed influence of local imperfections [11]. Elastic cylindrical shells subjected to buckling considering the effect of localized axisymmetric imperfections were studied by Khamlichi, Bezzazi and Limam. Stull, Earls and Aquino have shown that leveraging nonlinear finite element analysis with a divide and conquer type stochastic search algorithm can identify the presence of localized denting imperfections in cylindrical shell structures [13]. Pircher

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and Wheeler presented method of measuring imperfections in circular cylindrical members, simple to implement in a laboratory environment while providing accurate measurements with numerical methods to process the measurements into three-dimensional imperfection maps [14].

Aim of this paper is to investigate how geometrical imperfections such as dents, positioned in the middle of aluminium-alloy cylindrical shell, have influence to critical buckling stress.

2. TYPES OF BUCKLING ANALYSIS

There are two approaches in analysing cylindrical tin-walled shells for buckling. These are Eigen (or linear) buckling analysis and Non-linear buckling analysis.

Eigen buckling analysis predicts the theoretical buckling strength of an ideal linear elastic structure. This analysis is used to predict the bifurcation point using linearized model of elastic structure. It is a technique used to determine buckling loads – critical loads at which a structure becomes unstable and buckled mode shapes - the characteristic shape associated with the structure's buckled response. The other name for this Eigen buckling analysis is “bifurcation analysis”. In Eigen buckling analysis, imperfections and non-linearities cannot be included. The basic form of the Eigen buckling analysis is given by:

$$[K]\{\phi_i\} = \lambda_i [S]\{\phi_i\} \tag{1}$$

where $[K]$ is the structural stiffness matrix, $\{\phi_i\}$ the Eigen vector, λ_i the Eigen value and $[S]$ the stress stiffness matrix.

Another linear approach is given by linear strain-displacement relations below:

$$\begin{aligned} \varepsilon_x &= \frac{\partial U}{\partial x} & k_x &= -\frac{\partial^2 V}{\partial x^2} \\ \varepsilon_y &= \frac{\partial V}{\partial y} + \frac{W}{R} & k_y &= -\frac{\partial^2 V}{\partial y^2} \\ \gamma_{xy} &= \frac{\partial V}{\partial x} + \frac{\partial U}{\partial y} & k_{xy} &= -\frac{\partial^2 V}{\partial x \partial y} \end{aligned} \tag{2}$$

where x and y are the axial and circumferential coordinates in the shell middle surface; U and V are the shell displacements along axial and circumferential directions, and W is the radial displacement, positive outward; ε_x, ε_y and γ_{xy} are strain components; k_x, k_y and k_{xy} are middle surface curvatures of the shell; R is the radius of the cylindrical shell [15].

Non-linear buckling analysis is a more accurate approach and hence, this FE analysis has capability of analysing the actual structures with imperfections. This approach is highly recommended for a design or evaluation of actual structures. This technique employs a non-linear structural analysis with gradually increasing loads to seek the load level at which the structure become unstable and this phenomenon is also explained in singular and thereby further load step is not possible. In order to overcome this problem, arc tangent iteration scheme is adopted [1].

Pseudo code for the nonlinear buckling analysis:

1. Geometrically nonlinear (GNL) analysis by arc-length method
2. Monitor and detect stability point during GNL analysis
3. Re-set all state variables to configuration at load step just before stability point – a precritical point
4. Perform eigenbuckling analysis on deformed configuration at load step before stability point

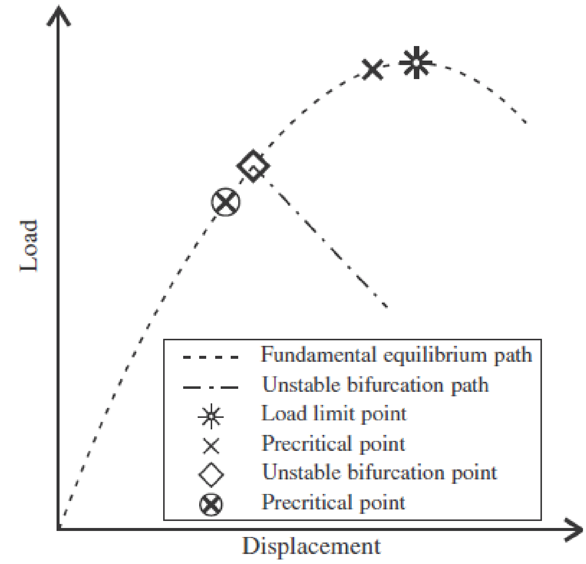


Figure 1. The proposed procedure for nonlinear buckling analysis

Total Lagrangian approach, i.e. displacements, refers to the initial configuration for the description of geometric nonlinearity. An incremental formulation is more suitable for nonlinear problems and it is assumed that the equilibrium at load step n is known and it is desired at load step n+1. Furthermore, it is assumed that the current load is independent on deformation. The incremental equilibrium equation in the Total Lagrangian formulation is written as

$$\mathbf{K_T}\left(\mathbf{D}^n, \gamma^n\right)\delta D = \mathbf{R}^{n+1} - \mathbf{F}^n, \text{ where}$$

$$\mathbf{K_T}\left(\mathbf{D}^n, \gamma^n\right) = \mathbf{K}_0 + \mathbf{K_L}\left(\mathbf{D}^n, \gamma^n\right) + \mathbf{K}_\sigma\left(\mathbf{D}^n, \gamma^n\right) \tag{3}$$

$$\text{i.e. } \mathbf{K_T^n} = \mathbf{K}_0 + \mathbf{K_L} + \mathbf{K_\sigma^n}$$

Here $\delta\mathbf{D}$ is the incremental global displacement vector, $\mathbf{F}^n$ global internal force vector, and $\mathbf{R}^{n+1}$ global applied load vector. The global tangent stiffness $\mathbf{K_T^n}$ consists of the global initial stiffness $\mathbf{K}_0$, the global stress stiffness $\mathbf{K_\sigma^n}$, and the global displacement stiffness $\mathbf{K_L^n}$. The applied load vector $\mathbf{R}_n$ is controlled by the stage control parameter (load factor) γ^n according to an applied reference load vector $\mathbf{R}$

$$\mathrm{R}^n = \gamma^n \mathrm{R} \tag{4}$$

During the nonlinear path tracing analysis at some converged load step it is possible to estimate an upcoming critical point, i.e. bifurcation or limit point, by utilizing tangent information. At a critical point the tangent operator is singular

$$\mathbf{K_T}\left(\mathbf{D}^c,\gamma^c\right)\phi_j=0 \quad (5)$$

where the superscript c denotes the critical point and ϕ_j the buckling mode. To avoid a direct singularity check of the tangent stiffness, it is easier to utilize tangent information at some converged load step n and extrapolate it to the critical point. The one-point approach only utilizes information at the current step and extrapolates by only one point. The stress stiffness part of the tangent stiffness at the critical point is approximated by extrapolating the nonlinear stress stiffness from the current configuration as a linear function of the load factor γ.

$$\mathbf{K}_\sigma\left(\mathbf{D}^c,\gamma^c\right)\approx\lambda\mathbf{K}_\sigma\left(\mathbf{D}^n,\gamma^n\right)=\lambda\mathbf{K}_\sigma^n \quad (6)$$

It is assumed that the part of the tangent stiffness consisting of $\mathbf{K}_L^n$ and $\mathbf{K}_0$ does not change with additional loading, which holds if the additional displacements are small. The tangent stiffness at the critical point is approximated as:

$$\mathbf{K_T}\left(\mathbf{D}^c,\gamma^c\right)\approx\mathbf{K}_0+\mathbf{K}_L^n+\lambda\mathbf{K}_\sigma^n \quad (7)$$

and by inserting into (5) we obtain a generalized eigenvalue problem

$$\left(\mathbf{K}_0+\mathbf{K}_L^n\right)\phi_j=-\lambda\mathbf{K}_\sigma^n\phi_j \quad (8)$$

where the eigenvalues are assumed as ordered by the magnitude such that λ_1 is the lowest eigenvalue and ϕ_1 the corresponding eigenvector. The solution to (8) yields the estimate for the critical load factor at load step n as:

$$\gamma_j^c=\lambda_j\gamma^n \quad (9)$$

If $\lambda_1<1$ the first critical point has been passed and in contrast $\lambda_1>1$ the critical point is upcoming. The one-point procedure works well for both bifurcation and limit points. The closer the current load step gets to the critical point, the better the approximation becomes, and it converges to the exact result in the limit of the critical load [16].

3. DESCRIPTION OF SPECIMEN AND EQUIPMENT

The specimen for the experiment (Figure 2.) was used a cylindrical tin-walled shell, with dimensions ø65x135 mm. The wall thickness was 0.11 mm. The specimen was subjected to the steadily increasing axial load.

The load was applied by means of hydraulic press Armavir, type PSU-50. The equipment and the specimen mounted for experiment is given in the Figure 3.

For making a dent in the middle of the shell wall, it was used mechanism shown at the Figure 4. The bottom end of the vertical bar is fixed to the lower plate of press, and at the upper end of the bar there is a treaded hole and the screw passing through it. The screw should be tightened enough to lean against the cylindrical shell. With the further tightening of the screw, the wall deforms and so the dent is formed. The depth of the dent is easily calculated by counting the number of revolutions during the tightening, and multiplying it by the pitch of screw.

Figure 2. Tin-walled shell – specimen of the experiment

Figure 3. Equipment with mounted specimen

Figure 4. Mechanism for making dents

4. COURS, RESULTS AND ANALYSIS OF EXPERIMENT

In order to fulfil the requirement for the shell to carry the entire load, something had to be done to exclude the specimen basis from carrying a part of it. To do so, it was implemented two iron rings, at the both sides – upper and lower. These rings were intermediates between press plates and specimen shell, 'and so the basis was excluded from carrying the load.

The main idea of the experiment was to increase the applied axial force gradually (3 kg per step) and to measure the strain of the specimen for every load step.

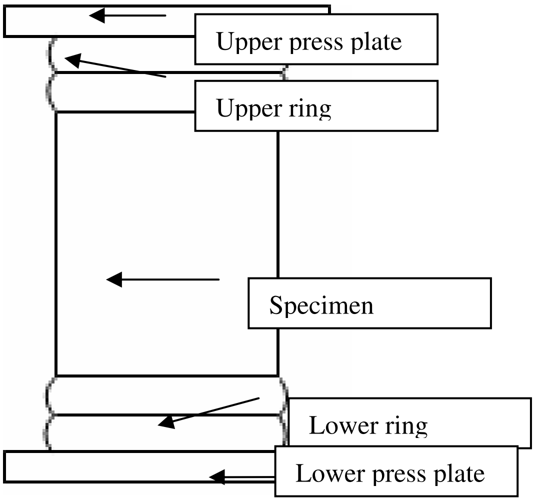


Figure 5. Pre-experimental postulate

In every step it was obtained two values: current applied load and the vertical deformation of the specimen. These data pairs were inserted into he diagram. Three cases were subjected to the experiment:

1. Specimen with no dents applied,
2. Specimen with the dent of 2 mm,
3. Specimen with the dent of 4 mm.

It has been carried out 4 series of measurements (4 specimens) for each case. The following figures, present averaged results for each of those cases (average value is made of four measurements).

Analysing the following diagrams, it is come to the conclusion that influence of dent positioned at the middle of shell and perpendicular to it is significant.

In Table 1, there are average values of critical buckling load for three of the analysed cases of dent inclination. Average critical load in case 1 and case 2 differs approximately 12 kg, but difference between case 2 and case 3 is significantly less (3 kg).

Also, analysing diagrams of experimental results, it is observed decrease of the resistance of shell in the force interval 22 to 26 kg. It is observed for every depth of dent.

Table 1: Average buckling load

Case	Dent (mm)	Load (kg)
1	0	60
2	2	48
3	4	45

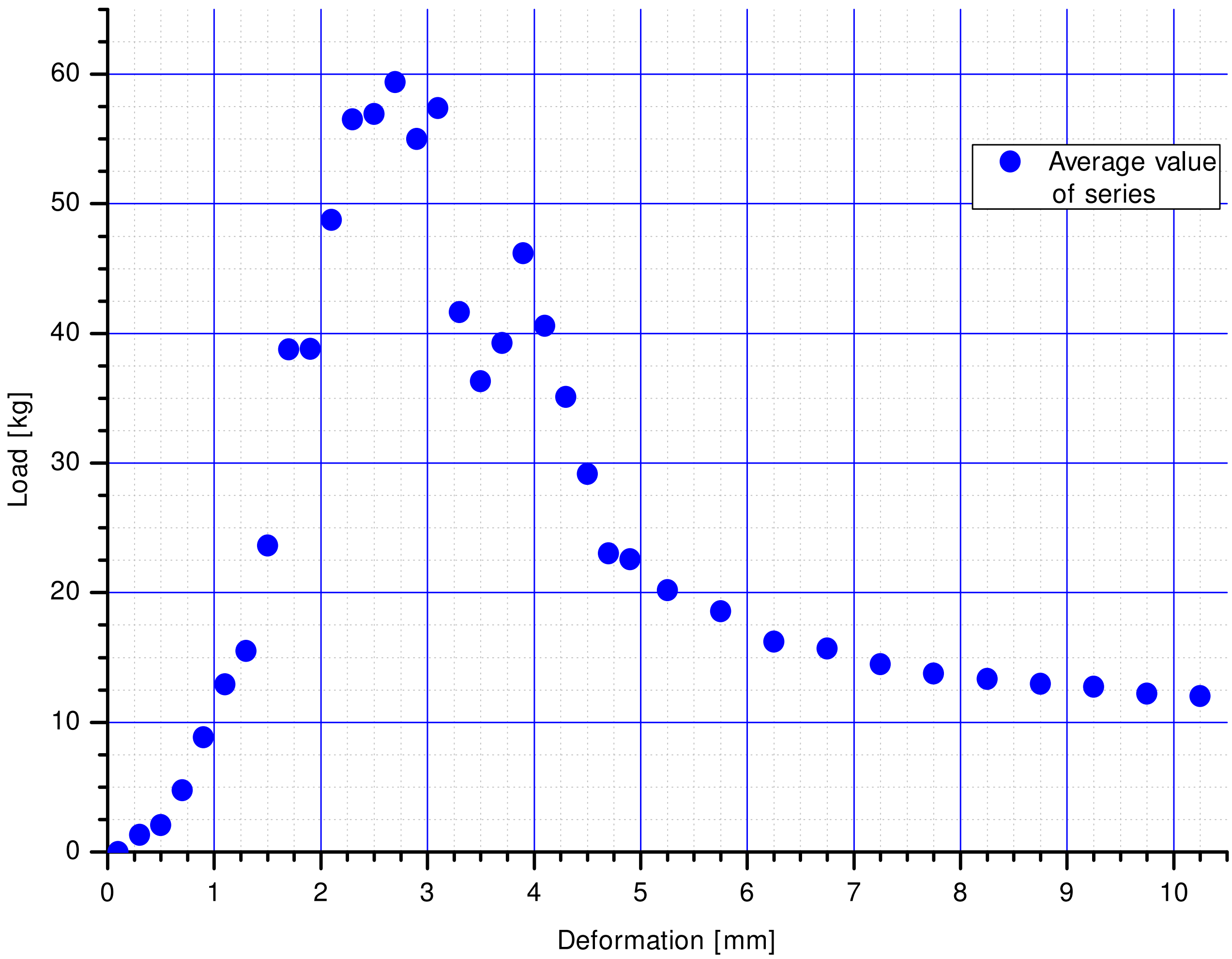


Figure 6. Results of experiment for 1. Case – Dent=0 mm

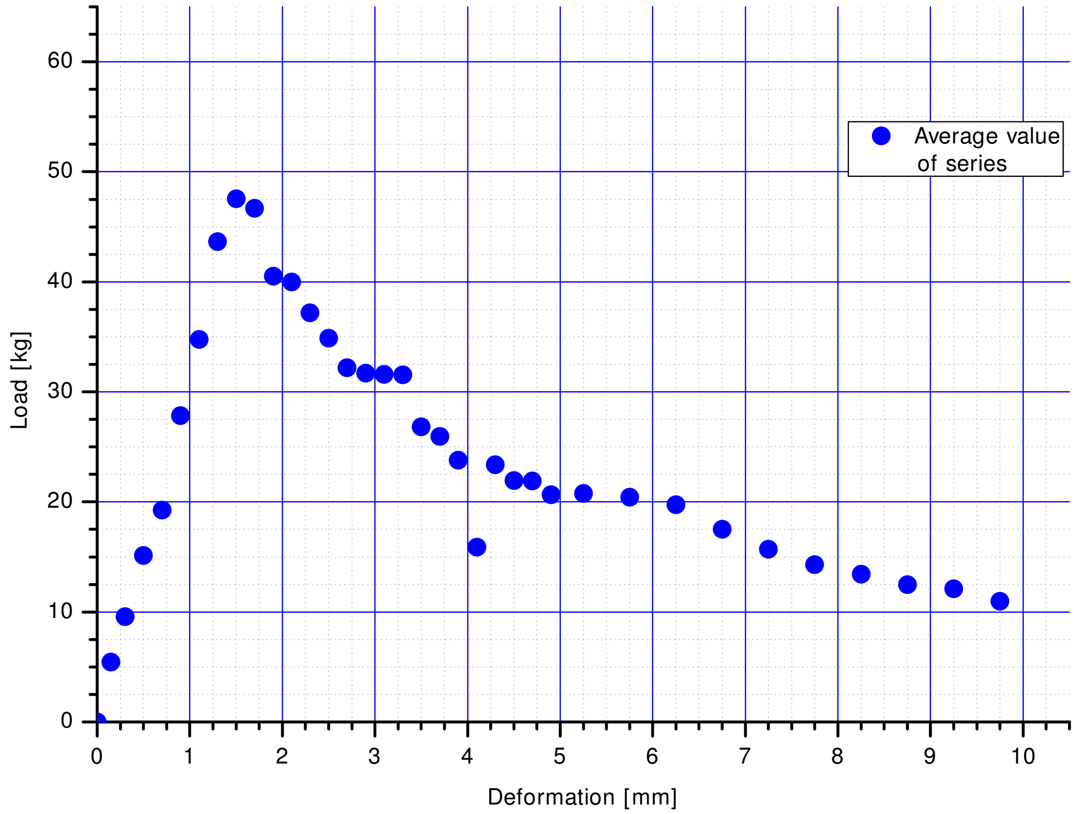


Figure 7. Results of experiment for 2. Case – Dent=2 mm

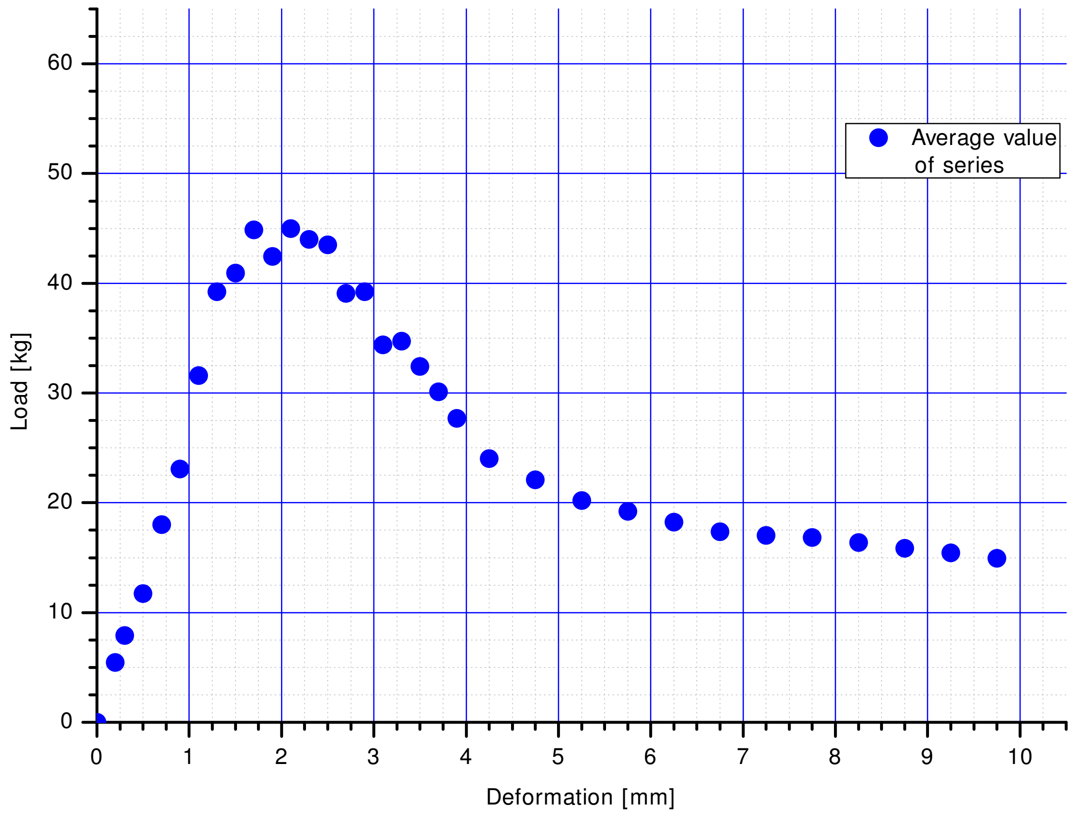


Figure 8. Results of experiment for 3. Case – Dent=4 mm

5. NUMERICAL SIMULATION OF BUCKLING OF THIN-WALLED CYLINDRICAL SHELL

The FE methods are considered to be the most appropriate tool in cases dealing with structural mechanics problems involving complications in behaviour that are analytically intractable. Buckling of the cracked shells is just such type of the problem. The FE methods have been successfully applied in problems involving shell buckling [6].

3D model corresponds to the specimen from the experiment. It is modelled as thin-walled shell, 0.11 mm kick, 135 mm high and with diameter of 65 mm.

Figure 9 3D models of specimens (dent= 0mm, 2mm and 4mm, respectively)

The dented model is formed by creating 3 cross sections, upper, middle and bottom section. The dent is defined in the middle section with the constraint to maintain the length of the circumference.

The type of the element used for buckling analysis is hexahedron 8-nodded 3D element.

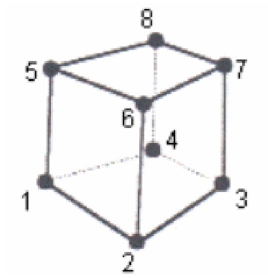


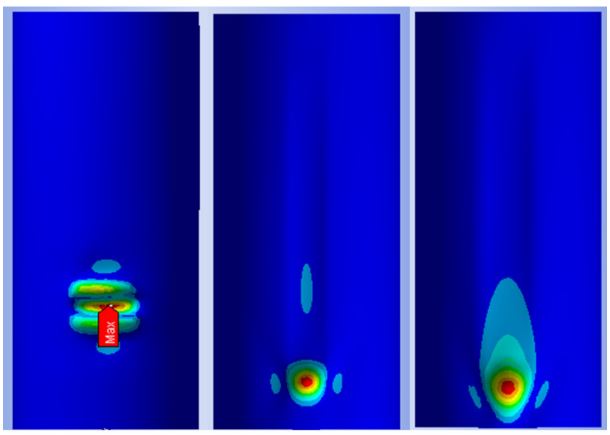
Figure 10 HEX8 element, used to mesh model

Table 2 Element statistic

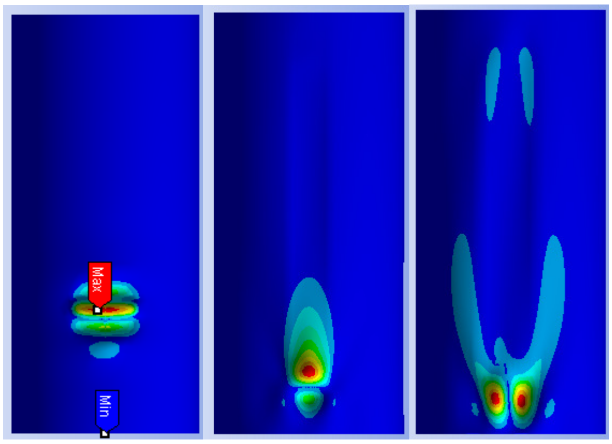
Number of nodes	18856
Number of elements	4256
Element size	Coarse

The 3 modes of total deformation (Figure 11) are presented for every analysed case of dent. In case of undented model, the side of buckling changes simultaneously. On the other hand, when simulating buckling of dented model, deformations always occur on the same side where the dent is.

First buckling mode:



Second buckling mode:



Third buckling mode

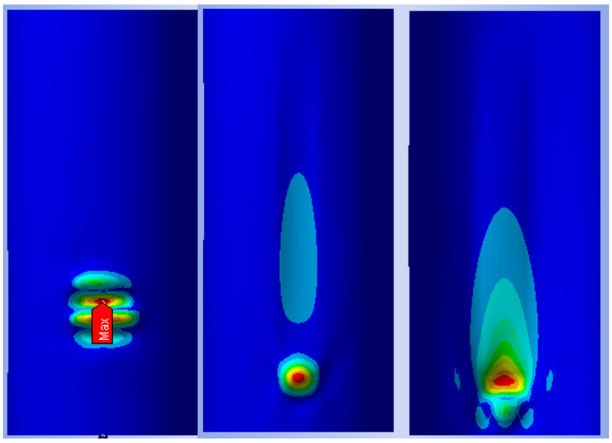


Figure 11 Modes of total deformation

6. CONCLUSION

The following conclusions are made based on the experiment and FE analysis carried out on thin cylindrical shells with a dent of different sizes of inclination located at half of the height of cylindrical shell taken for examination.

1. There is a significant decrease of the buckling resistance if the values of the specimen with no dent, and the specimen with 2 mm deep dent are compared. Reduction is approximately 20%. However, if the specimen with 2 mm dent to specimen with 4 mm dent is compared, the decrease of the buckling resistance is remarkably lower (6%).
2. The decrease of resistance of the shell in the interval of load 22 to 26 kg for every case of dent inclination is observed.
3. Quite good concurrence regarding to the position and shape of the deformations occurred due to buckling is observed, if numerical and experimental results are compared.

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NOMENCLATURE

$[K]$	structural stiffness matrix
$[S]$	stress stiffness matrix.
k_x, k_y, k_{xy}	middle surface curvatures of the shell
U, V	shell displacements along axial and circumferential directions
W	radial displacement
R	radius of the cylindrical shell
$\delta\mathbf{D}$	incremental global displacement vector
$\mathbf{F}^n$	global internal force vector
$\mathbf{R}^{n+1}$	global applied load vector
$\mathbf{K}_\mathbf{T}^n$	global tangent stiffness
$\mathbf{K}_0$	global initial stiffness
$\mathbf{K}_\sigma^n$	global stress stiffness
$\mathbf{K}_\mathbf{L}^n$	global displacement stiffness
$\mathbf{R}_n$	applied load vector
$\mathbf{R}$	applied reference load vector
$\{\phi_i\}$	the Eigen vector
λ_i	the Eigen value
$\varepsilon_x, \varepsilon_y, \gamma_{xy}$	strain components
γ^n	stage control parameter (load factor)

ИЗВИЈАЊЕ ЦИЛИНДРИЧНЕ ТАНКОЗИДЕ СТРУКТУРЕ ИЗРАЂЕНЕ ОД ЛЕГУРЕ АЛУМИНИЈУМА СА ГРЕШКОМ У ГЕОМЕТРИЈИ У ВИДУ УДУБЉЕЊА, ПОД ДЕЈСТВОМ АКСИЈАЛНОГ ПРИТИСКА

М. Станковић, М. Ристић, А. Симоновић, М. Јовановић

Танкозиде цилиндричне структуре имају веома високу примену у индустрији. Пошто су обично оптерећене на аксијални притисак, најчешћи узрок њиховог отказа је појава извијања.

У овом чланку се приказује нумеричка анализа извијања танкозиде цилиндричне структуре израђене од легуре алуминијума са имперфекцијом у виду удубљења на средини структуре и експериментална верификација резултата добијених том анализом.

Нумеричка анализа је одрађена помоћу софтвера ANSYS 16.2, а експериментална испитивања помоћу хидрауличне пресе Армавир, ПСУ-50, на којој је узорак подвргнут аксијалном притиску чији се интензитет постепено повећавао до појаве извијања. Поређењем резултата експеримента уочено је значајно смањење вредности критичног напона на извијање између епрувета без имперфекције и епрувета са имперфекцијом од 2 мм, док су вредности критичног напона за епрувете са имперфекцијама од 2 мм и 4 мм приближне.