Research on Geometrical Characteristics of Straight Bevel Gears with a Small Shaft Angle with a Non-Generated Gear and Generated Pinion

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Based on the precessional bevel gears with a small shaft angle, a number of drives for oil and gas equipment have been developed. They are characterized by high efficiency (about 0.9) and a small starting torque. Due to multi-pair tooth meshing they have a higher load capacity as compared to other types of gears, which provides a safe drive operation in the Far North. Currently, the most studied gears are bevel gears with a small shaft angle with double-concave-convex teeth. However, the process of gear cutting for such gears is extremely complicated – the machine has to be re-set four times. The cutting technology can be simplified by using gears with a non-generated gear and a generated pinion. The article provides calculations of the lines of action in meshing and the principal reduced curvature for a straight bevel gears with a small shaft angle with a non-generated gear and a generated pinion, geometrical characteristics necessary for determining the contact stress arising in tooth meshing and for developing methods of calculating the gear load capacity.

Keywords: *straight bevel gears, drive, non-generated gear, straight teeth, gear cutting, equation of meshing, principal reduced curvature, lines of action*

1. INTRODUCTION

Processes of bevel gear teeth formation [1, 2, 3, 4, 5, 6, 7, 8] are more complex compared to spur and worm spiroid [9] gears. To produce straight bevel and spiral bevel gears, in the beginning of the last century experts from Gleason, Klingelnberg, Oerlikon developed various cutting processes, specialized gear-cutting machines and cutting tools. Machines by Gleason use a circular cutter head to cut teeth by applying a single-indexing generating method (Face Milling Method) [4, 5, 6]. They can also use a multi-stage cutter head and apply a continuous cutting method (Face Hobbing Method) [5, 7, 10].

The cost of bevel gear mass-production on Gleason machines is reduced significantly when bevel gears with a small shaft angle are used [11, 12].

By this method, gear teeth are cut without the generating process. Gleason experts have developed such methods as FORMATE, HELIXFORM. Despite a more complex task of determining the optimal tooth modification [11, 12],], such gear technology is more progressive. Bevel gears with cyclo-poloidal teeth are cut using Klingelnberg and Oerlikon gear-cutting machines [13, 14, 15, 16, 17].

This process implies the use of either tapered hobs or specialized multi-stage cutter heads, and cyclo-poloidal teeth are cut by a continuous cutting method.

The Revacycle method is the most important step in improving the efficiency of cutting straight bevel gears [18].

It applies a single-indexing method and a circular broach-style cutter. This gear refers to bevel gears with a small shaft angle. The gear cutting process is highly efficient and therefore is used in mass-production.

Geometry of tooth surfaces of bevel gears cut by various methods varies significantly. Providing for the required geometrical parameters of meshing of contacting gears for each particular method is a complex mathematical task generally reducing to solving systems of transcendental equations or to minimizing functions of many variables with limited equality and inequality [11, 12, 16, 17, 19, 20].

Over the recent years, the problem of determining the optimal bevel gear geometry providing their required load capacity under operation conditions is addressed in multiple works [21, 22, 23, 24, 25, 26].

Most research works are investigating orthogonal bevel gears (shaft $\Sigma = 90°$), which pinion and gear teeth are cut using the generating process.

For some oil and gas equipment the essential unit is a coaxial gearbox with small radial dimensions (geared insert into the screw pump) having high performance efficiency, small starting torque (gearbox ball valve) at a high load capacity.

Gearboxes (Figure 1) complying with the listed requirements are based on a bevel precessional gear with a small shaft angle [27, 28, 29], which shaft angle $\Sigma = 3°...5°$ and the difference in the number of pinion and gear teeth is one or two.

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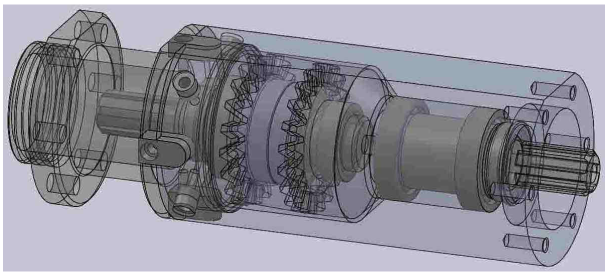

Figure 1. Coaxial gearbox based on a bevel gears with a small shaft angle

Such gearboxes when used in ball valve drives have higher efficiency (about 90%) compared to spiroid gearboxes [9] due to multi-pair tooth meshing, improved load capacity, small starting torque and a wide range of gear ratios (10-100) [28, 29]. The drives demonstrate a reliable operation in the Far North at infrequent actuations and short-term operation intervals. Currently, the most studied gears are bevel gears with a small shaft angle with double-concave-convex teeth in a longitudinal direction [19, 27, 28, 29].

Figure 2 shows the design scheme of the proposed coaxial gearbox with precessional bevel gears: pinion 1 with an initial conical surface; gear 2 with ring gears 2 and 3 spaced apart by a value B, initial surfaces of which is a plane; moving pinion 4 with an initial conical surface; carrier 5 with an eccentric shaft sector, located at an angle Σ to the common axis O - O of the coaxial gearbox.

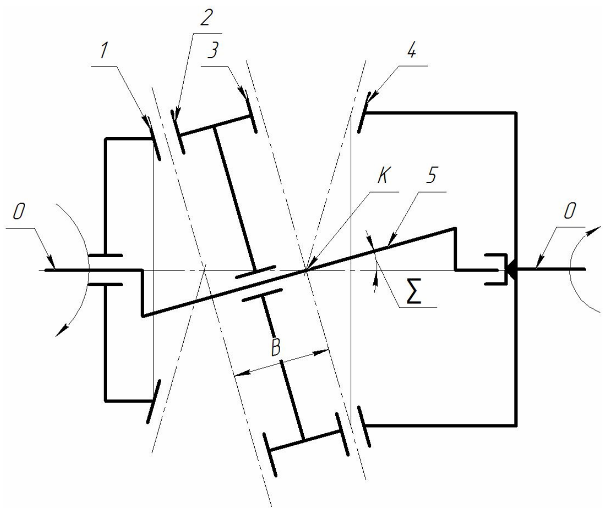

Figure 2. Scheme of coaxial gearbox based on prece–ssional bevel gear

During operation of the coaxial gearbox, pinion 1 is fixed; gear 2 performs a compound motion - rotation about its axis and together with the carrier 5 about the axis O - O. Vertices of the pitch cones of the pinion 1 and ring gear 2 (pitch angle 90°) do not coincide. To exclude planetary motion of the gears in the most loaded (slow-speed) bevel precessional gear (gear coupling), composed of gears with ring gears 3 and 4, it is necessary that the vertex of the pitch cone of ring gear 4 and the vertex of ring gear 3 (pitch angle 90°) not only coincide, but are at the crossing point of the axis of eccentric shaft sector and the axis of the coaxial gearbox O - O. This condition is met by only one value of B calculated by the formula [2]:

$$B = \frac{m_n}{2 \cdot \cos\beta}\left(\frac{z_2}{tg\varepsilon} - \frac{z_1}{\sin\varepsilon}\right) \qquad (1)$$

where m_n = normal module, mm; β = helix angle, °; z_1 = number of pinion teeth; z_2 = number of gear teeth; Σ = shaft angle, °.

Gear coupling, a precessional bevel gears of coaxial gearbox, which is composed of gears with ring gears 3 and 4 in accordance with work [2], is designed with number of teeth $z_3 = z_4$. The gear ratio is equal to one. In the gearbox this gear is a slow-speed and therefore is the most loaded. To increase its load capacity and service life the normal module of this gear is taken as equal, larger or smaller than module m_n for a bevel prece–ssional gear with gear rings 1 and 2, and $z_3 = z_4$ is taken as equal, larger or smaller than z_2 (Figure 3). Equality of modules and that $z_3 = z_4 = z_2$ provides minimal radial dimensions of coaxial gearbox.

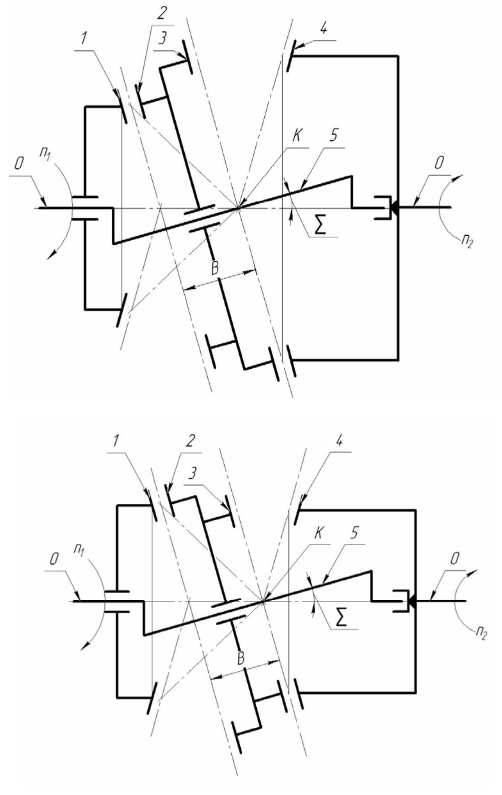

Figure 3. Coaxial gearbox scheme, gear coupling module is lager (a) or smaller (b) than bevel precessional gear

However, the process of gear cutting for such gears is extremely complicated. As each side of the pinion and gear tooth has to be cut individually, the machine has to be re-set four times. The use of spiral teeth in bevel gears with a small shaft angle is virtually eliminated because of undercutting during gear manufacturing.

The long-term experience in manufacturing bevel and hypoid gears demonstrates that we can simplify the gear cutting process by means of bevel gears with a small shaft angle [11, 12, 18]. Their distinguishing characteristics are that gears can be crowned in a profiled direction. Gear teeth surfaces are determined based on how they meet the required geometrical parameters of the gear. Gear teeth are cut by the single indexing method on standard gear-cutting machines or kinematically simple machines.

The first version of a bevel gear with a small shaft angle was proposed in [30], where the gear geometry and the process of generating pinion tooth surfaces are reviewed. The given paper provides dependences for calculating lines of action in the gear and determining the principal reduced curvatures in their points. These values are used when building techniques for calculating contact stresses of the bevel gear with a small shaft angle and assessing its load capacity.

2. ANALYSIS OF GEAR TEETH MESHING IN THE STRAIGHT BEVEL GEAR WITH A SMALL SHAFT ANGLE WITH A NON-GENERATED GEAR AND A GENERATED PINION

Figure 4 describes the design scheme of the straight bevel gears with a small shaft angle with a non-generated gear and a generated pinion.

The idea to create this process came from the phenomenon that the generating roll to form the gear member takes an increasingly longer time the larger the ratio between pinion and gear is. The design method takes advantage of this in forming a tooth profile that is straight, and therefore requires no time-consuming generating roll. The pinion in turn is generated with extra profile curvature (without a significant cutting time increase) such that it perfectly rolls with the non-generated gear. The results are good performing gearsets and greatly reduced manufacturing time. [5]

Cutting of the gear tooth space is done by a cutting tool with a straight-line cutting edge. The gear remains fixed during cutting. Prior to gear cutting each of the following gear tooth space, the gear rotates at an angle equal to the tooth pitch angle, i.e. single-indexing is realized. As cutting tools we can use cutters (cutting without generating on a gear-planing machine), end-mill type cutters or side milling cutters (cutting on a gear hobbing machine with a turntable).

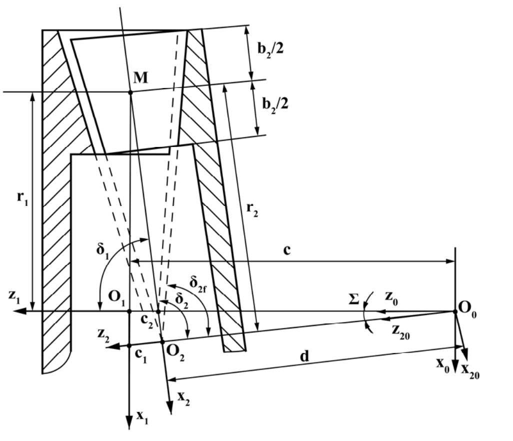

Figure 4. Design scheme of the straight bevel gears with a small shaft angle with a non-generated gear and a generated pinion and a scheme of coordinate transformation from the coordinate system S2 to the coordinate system S1

From the described gear cutting method it follows that the gear tooth surface is a plane. The reference surface of tooth is a plane that goes through the generating point M (center of action) parallel to the plane $x_2o_2y_2$ (perpendicular to the axis z_2). The coordinate system S_2 (x_2, y_2, z_2) is rigidly connected to the gear. In the coordinate system S_p (x_p, y_p, z_p) the tooth surface which axis y_p is directed along the normal towards the gear tooth surface is a plane and calculated by the equation (1):

$$x_p = u \; ; y_p = 0 \; ; z_p = h \, . \tag{1}$$

where u is the line coordinate along the longiduinal direction, h is the line coordinate along the profile direction.

Let us express the position vector $\overline{r_p}$ of the tooth surface ("plane") in the coordinate system S_p (x_p, y_p, z_p) as a row matrix $\tilde{r}_p = \left\| x_p ; y_p ; z_p ; 1 \right\|$, elements of which are coordinates x_p, y_p, z_p, and the position vector of the tooth surface $\overline{r_2}$ in the system S_2 (x_2, y_2, z_2), - as a row matrix $\tilde{r}_2 = \left\| x_2 ; y_2 ; z_2 ; 1 \right\|$, elements of which are defined by coordinates x_2, y_2, z_2. Then the tooth surface in the system S_2 (x_2, y_2, z_2) is described as the formula:

$$\tilde{r}_2 = \tilde{A}_{2p} \cdot \tilde{r}_p \tag{2}$$

where $\tilde{A}_{2p}$ is the square matrix (4 x 4) [4, 6], which defines coordinate transformation from the coordinate system S_p to the coordinate system S_2.

Solving the equation (2), we obtain:

$$\left. \begin{array}{c} x_2 = u \cdot \cos\theta_{f2} - h \cdot \sin\theta_{f2} \cdot \cos\alpha_n - r_2 \\ y_2 = -h \cdot \sin\alpha_n + t \\ z_2 = u \cdot \sin\theta_{f2} + h \cdot \cos\theta_{f2} \cdot \cos\alpha_n \end{array} \right\} \tag{3}$$

where θ_{f2} is the gear dedendum angle; α_n is the pressure angle; r_2 is the mean cone distance of the gear; t is a half of the tooth thickness.

In the studied straight bevel gears with a small shaft angle the pinion tooth surface is an envelope in a one-parameter process (motion). The imaginary generating surface is a gear tooth surface. Figure 4 also describes the coordinate systems between the imaginary gear and work gears. The coordinate system S_2 (x_2, y_2, z_2) is rigidly connected to the gear and the coordinate system S_1 (x_1, y_1, z_1) is rigidly connected to the pinion during generation of the pinion surface.

Due to the machine kinematics of gear cutting, when the gear tooth rotates around its axis at an angle φ_2 the pinion rotates around its axis at an angle (performs rotational motion about its axis) φ_1, related to the angle φ_2 by formulas:

$$\varphi_1 = \varphi_2 \cdot i \, ,$$

$$i = Z_2^* / Z_1^* \; ; \tag{4}$$

where i is the gear ratio of the machine generating train, Z_2^* is a number of gear teeth, Z_1^* is a number of pinion teeth.

The coordinates of the current point of the gear tooth surface (1) can be determined by two independent parameters: u and h, that is $\tilde{r}_2 = \tilde{r}_2(u,h)$. Due to the rolling motion when generating the pinion tooth surface, the matrix of relative motion $\widetilde{A}_{12}$ is a function of the parameter φ_1 : $\widetilde{A}_{12} = \widetilde{A}_{12}(\varphi_1)$. A position vector $\overline{r_1}$ of the pinion tooth surface in the coordinate system S_1 (x_1, y_1, z_1) (Figure 4), in the matrix form is determined as:

$$\widetilde{r}_1 = \widetilde{A}_{12} \cdot \widetilde{r}_2 \quad (5)$$

where $\widetilde{r}_1 = \|x_1, y_1, z_1, 1\|$ is a row matrix, composed of the coordinate projections of the position vector $\overline{r_1}$ of the pinion tooth surface.

According to Figure 4, elements a_{ij}, $i = \overline{1,4}$, $j = \overline{1,4}$ of the matrix $\widetilde{A}_{12}$ have the form:

$$a_{11} = \cos\varphi_1 \cdot \cos\Sigma \cdot \cos\varphi_2 + \sin\varphi_1 \cdot \sin\varphi_2 ;$$
$$a_{12} = -\cos\varphi_1 \cdot \cos\Sigma \cdot \sin\varphi_2 + \sin\varphi_1 \cdot \cos\varphi_2 ;$$
$$a_{13} = \cos\varphi_1 \cdot \sin\Sigma ;$$
$$a_{14} = d \cdot \cos\varphi_1 \cdot \sin\Sigma ;$$
$$a_{21} = -\sin\varphi_1 \cdot \cos\Sigma \cdot \cos\varphi_2 + \cos\varphi_1 \cdot \sin\varphi_2 ;$$
$$a_{22} = \sin\varphi_1 \cdot \cos\Sigma \cdot \sin\varphi_2 + \cos\varphi_1 \cdot \cos\varphi_2 ;$$
$$a_{23} = -\sin\varphi_1 \cdot \sin\Sigma ; \quad (6)$$
$$a_{24} = -d \cdot \sin\varphi_1 \cdot \sin\Sigma ;$$
$$a_{31} = -\sin\Sigma \cdot \cos\varphi_2 ;$$
$$a_{32} = \sin\Sigma \cdot \sin\varphi_2 ;$$
$$a_{33} = \cos\Sigma ;$$
$$a_{34} = d \cdot \cos\Sigma - c ;$$
$$a_{41} = a_{42} = a_{43} = 0 ; a_{44} = 1 .$$

The values c and d included in the elements a_{ij} are calculated according to the formulas:

$$c = r_1 \cdot (i - \cos\Sigma) \cdot (\sin\Sigma)^{-1} ;$$
$$d = r_1 \cdot (i \cdot \cos\Sigma - 1) \cdot (\sin\Sigma)^{-1} ; \quad (7)$$

where r_1 is the mean cone distance of pinion

Solving the formula (5), we obtain:

$$\widetilde{r}_1(u, h, \varphi_1) = \widetilde{A}_{12}(\varphi_1)\widetilde{r}_2(u, h) . \quad (8)$$

As the tooth surface can have only two independent parameters, for mathematical description of the pinion tooth surface it is necessary to relate an additional relation among the parameters φ_1, u and h. In the theory of gearing [4, 19] such relation is referred to as the equation of meshing:

$$f(u, h, \varphi_1) = 0 . \quad (9)$$

If the equation of meshing is known, the pinion tooth surface, as an envelope to the family of gear tooth surface, is described as follows [4, 6, 19]:

$$\widetilde{r}_1(u, h, \varphi_1) = \widetilde{A}_{12}(\varphi_1)\widetilde{r}_2(u, h) ;$$
$$f(u, h, \varphi_1) = 0 . \quad (10)$$

In this study, to determine the equation of meshing, we used the method suggested in the [4]. The pinion tooth surface will be an envelope to the family of gear tooth surfaces in the relative motion with the parameter φ_1 only when the relative velocity $\overline{V_\varphi}$ is perpendicular to the unit vector $\overline{m_2}$ of the generating surface (gear tooth surface). This condition corresponds to the fact that the scalar product $\overline{V_\varphi}$ and $\overline{m_2}$ is equal to zero [4]. Then, the equation of meshing may be derived as follows:

$$f(u, h, \varphi_1) = \overline{m_2} \cdot \overline{V_\varphi} =$$
$$m_{2x} \cdot V_{\varphi x} + m_{2y} \cdot V_{\varphi y} + m_{2z} \cdot V_{\varphi z} = 0 \quad (11)$$

Projections of m_{2x}, m_{2y}, m_{2z} on the unit normal (vector) $\overline{m_2}$ of the gear tooth surface (3), are determined as:

$$m_{2x} = -\sin\theta_{f2} \cdot \sin\alpha_n;$$
$$m_{2y} = \cos\alpha_n; \quad (12)$$
$$m_{2z} = \cos\theta_{f2} \cdot \sin\alpha_n.$$

The relative velocity $\overline{V_\varphi}$ with the parameter φ_1 is determined by the equation based on [4]:

$$\widetilde{V}_\varphi = \widetilde{C}(\varphi_1) \cdot \widetilde{r}_2 , \quad (13)$$

where $\widetilde{C}(\varphi_1) = \widetilde{A}_{21}(\varphi_1) \cdot \dfrac{d\widetilde{A}_{12}(\varphi_1)}{d\varphi_1}$; $\widetilde{A}_{21}(\varphi_1)$ is a square matrix (4 x 4), which is inverse to the matrix $\widetilde{A}_{12}(\varphi_1)$ (6); $\dfrac{d\widetilde{A}_{12}(\varphi_1)}{d\varphi_1}$ is a square matrix (4 x 4) determined by differentiation of the matrix $\widetilde{A}_{12}(\varphi_1)$ (6) with respect to φ_1 ; $\widetilde{r}_2$ is a column matrix composed by the coordinates of the gear tooth surface position vector (3).

Elements c_{ij}, $i = \overline{1,4}$, $j = \overline{1,4}$ of the matrix $\widetilde{C}(\varphi_1)$, after transformations are described as:

$$c_{11} = c_{22} = c_{33} = c_{34} = c_{41} = c_{42} = c_{43} = c_{44} = 0;$$
$$c_{12} = -c_{21} = \cos\Sigma - i^{-1};$$
$$c_{13} = -c_{31} = -\sin\varphi_2 \cdot \sin\Sigma;$$
$$c_{23} = -c_{32} = -\cos\varphi_2 \cdot \sin\Sigma; \quad (14)$$
$$c_{14} = -d \cdot \sin\varphi_2 \cdot \sin\Sigma;$$
$$c_{24} = -d \cdot \cos\varphi_2 \cdot \sin\Sigma;$$

Based on the equations (3) and (14), equation (13) yields expressions for the position vector $\overline{V_\varphi}$ projections:

$$V_{\varphi x} = -c_v \cdot d_v - a_v \cdot \sin\varphi_2 \sin\Sigma - d \cdot \sin\varphi_2 \sin\Sigma;$$
$$V_{\varphi y} = -b_v \cdot d_v - a_v \cdot \cos\varphi_2 \sin\Sigma - d \cdot \cos\varphi_2 \cdot \sin\Sigma;$$

$$V_{\varphi z} = b_v \cdot \sin\varphi_2 \sin\Sigma - c_v \cdot \cos\varphi_2 \cdot \sin\Sigma, \qquad (15)$$

where

$$a_v = u \cdot \sin\theta_{f2} + h \cdot \cos\theta_{f2} \cdot \cos\alpha_n ;$$

$$b_v = u \cdot \cos\theta_{f2} - h \cdot \sin\theta_{f2} \cdot \cos\alpha_n - r_2 ;$$

$$c_v = h \cdot \sin\alpha_n - t ;$$

$$d_v = \cos\Sigma - i^{-1} .$$

Using the expressions (15) and (12) based on (11) we obtain the equation of meshing which, after transformations, is described as:

$$f(u,h,\varphi_1) = \sin\theta_{f2} \sin\alpha_n (c_v \cdot d_v +$$

$$+a_v \cdot \sin\varphi_2 \sin\Sigma + d \cdot \sin\varphi_2 \sin\Sigma) - \cos\alpha_n \cdot \qquad (16)$$

$$\cdot(b_v \cdot d_v + a_v \cos\varphi_2 \sin\Sigma + d \cdot \sin\varphi_2 \sin\Sigma) +$$

$$+\cos\theta_{f2} \sin\alpha_n (b_v \sin\varphi_2 \sin\Sigma - c_v \cos\varphi_2 \sin\Sigma\big] = 0$$

This equation is solved analytically with respect to φ_2. The resulting dependences (10) determining the pinion tooth surface are described as:

$$x_1 = A_1 \cdot \cos\varphi_1 + B_1 \cdot \sin\varphi_1 ;$$

$$y_1 = -A_1 \cdot \sin\varphi_1 + B_1 \cdot \cos\varphi_1 \qquad (17)$$

$$z_1 = \sin\Sigma \cdot (f_3 \cdot \sin\varphi_2 - f_1 \cdot \cos\varphi_2) + ;$$

$$+\cos\Sigma \cdot (f_2 + d) - c ;$$

$$\varphi_2 = \arcsin\left[-C_\varphi (\sqrt{A_\varphi^2 + B_\varphi^2})^{-1} \right] - \xi ;$$

where Σ is the shaft angle; the last formula is the equation of meshing (11).

Then, transforming the equations, we obtain

$$A_1 = \cos\Sigma \cdot (f_1 \cdot \cos\varphi_2 - f_3 \cdot \sin\varphi_2) + \sin\Sigma \cdot (f_2 + d) ;$$

$$B_1 = f_3 \cdot \sin\varphi_2 - f_1 \cdot \cos\varphi_2 ;$$

$$A_\varphi = \sin\alpha_n \cdot \sin\Sigma \cdot (u + d \cdot \sin\theta_{f2} - r_2 \cdot \cos\theta_{f2}) ;$$

$$B_\varphi = -\sin\Sigma \cdot (f_2 \cdot \cos\alpha_n - f_3 \cdot \sin\alpha_n \cdot$$

$$\cdot \cos\theta_{f2} + d \cdot \cos\alpha_n) ;$$

$$C_\varphi = (i^{-1} - \cos\Sigma) \cdot (f_1 \cdot \cos\alpha_n + f_3 \cdot \sin\theta_{f2} \cdot \cos\alpha_n) ;$$

$$f_1 = u \cdot \cos\theta_{f2} - h \cdot \sin\theta_{f2} \cdot \cos\alpha_n - r_2 ;$$

$$f_2 = u \cdot \sin\theta_{f2} + h \cdot \cos\theta_{f2} \cdot \cos\alpha_n ;$$

$$f_3 = t - h \cdot \sin\alpha_n .$$

where the angle ξ is determined based on its trigonometric functions:

$$\sin\xi = B_\varphi \cdot (\sqrt{A_\varphi^2 + B_\varphi^2})^{-1} ;$$

$$\cos\xi = A_\varphi \cdot (\sqrt{A_\varphi^2 + B_\varphi^2})^{-1} ;$$

These formulas enable us, on the one hand, to perform the analysis of generating pinion tooth surfaces, and, on the other hand, to study the geometry of meshing in the straight bevel gear with a small shaft angle with a non-generated gear and a generated pinion, which is "matched" according to the way of its gear tooth surfaces generation.

By using formulas (17) and the MathCad software a computer program was developed to study the position and lines of action in gear meshing of the straight bevel gear with a small shaft angle with a non-generated gear and a generated pinion. The program showed its lines of action with the pinion tooth surface and the gear tooth surface of the straight bevel gear with a small shaft angle with a non-generated gear and a generated pinion for a number of fixed values of the angle of action (φ_1=-0.172; -0.115; -0.057; 0.0; 0.088; 0.177; 0.265) (Figure 5).

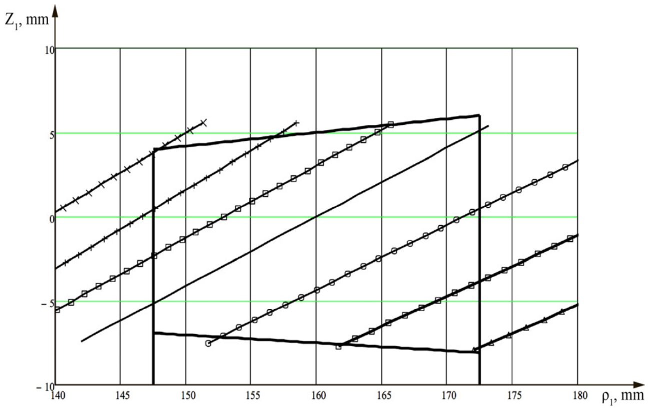

Figure 5. Lines of action on the pinion tooth surface

In the studied straight bevel gear with a small shaft angle with a non-generated gear and a generated pinion a change in the angle φ_1 from $\varphi_{1\min}$ to $\varphi_{1\max}$ = 0.265 is according to the maximum angle of action. Taking into account the value of the tooth pitch angle on the pinion ($t_1 = 2 \cdot \pi / z_1^* = 0{,}09817$), we determined that ($-\varphi_{1\min} + \varphi_{1\max})/t_1$ = 4.45 tooth pairs are simultaneously in gear meshing.

Table 1. Pinion and gear design parameters

Parameter	Pinion	Gear
Number of teeth	64	65
Normal module (mm)	5.0	
Pressure angle (°)	20	
Face width (mm)	25	
Pitch angle (°)	88	90
Addendum factor	1	
Clearance coefficient	0.5	

3. CALCULATION OF PRINCIPAL REDUCED CURVATURES IN MESHING OF THE STRAIGHT BEVEL GEAR WITH A SMALL SHAFT ANGLE WITH A NON-GENERATED GEAR AND A GENERATED PINION

Let us consider the task of calculating principal reduced curvatures in meshing of a bevel gear with a small shaft angle with a non-generated gear and a generated pinion, necessary for calculating contact stresses of the gear and assessing its load capacity. Based on [4, 6], the principal curvatures (K_1 and K_2) of an envelope to the one-parameter family of surfaces given as (10) are the solution to:

$$K_1 + K_2 = (M_1 + M_2)/M_3;$$

$$K_1 \cdot K_2 = M_4 / M_3. \quad (18)$$

Here we have:

$$M_1 = \begin{vmatrix} f_u; & f_h; & f_\varphi; \\ x_{2u}; & x_{2h}; & V_{\varphi x}; \\ m_{2yu}; & m_{2yh}; & w_{\varphi y} \end{vmatrix};$$

$$M_2 = \begin{vmatrix} f_u; & f_h; & f_\varphi; \\ m_{2xu}; & m_{2xh}; & w_{\varphi x}; \\ y_{2u}; & y_{2h}; & V_{\varphi y} \end{vmatrix};$$

$$M_3 = \begin{vmatrix} f_u; & f_h; & f_\varphi; \\ x_{2u}; & x_{2h}; & V_{\varphi x}; \\ y_{2u}; & y_{2h}; & V_{\varphi y} \end{vmatrix};$$

$$M_4 = \begin{vmatrix} f_u; & f_h; & f_\varphi; \\ m_{2xu}; & m_{2xh}; & w_{\varphi x}; \\ m_{2yu}; & m_{2yh}; & w_{\varphi y} \end{vmatrix};$$

x_{2u}, x_{2h}, y_{2u}, y_{2h} - derivatives with respect to u and h of the projections x_2 and y_2 (3); m_{2xu}, m_{2xh}, m_{2yu}, m_{2yh} - derivatives with respect to u and h of the projections m_{2x} and m_{2y} (12); f_u, f_h and f_φ - derivatives with respect to u, h and φ of the equation of meshing (16); $w_{\varphi x}$, $w_{\varphi y}$ - projections on the axes x_2 and y_2 of the coordinate system $S_2(x_2,y_2,z_2)$ of the analog vector $\overline{w}_\varphi$ of the relative velocity of the unit normal vector end. The projections $w_{\varphi x}$, $w_{\varphi y}$ are determined based on the matrix expression:

$$\widetilde{w}_\varphi(\varphi_1) = \widetilde{C}(\varphi_1) \cdot \widetilde{m}_2. \quad (19)$$

From (12) it follows that $m_{2xu} = m_{2xh} = m_{2yu} = m_{2yh} = 0$. As a result, the numerator product $K_1 \cdot K_2$ (18) equals zero, since at any angle of pinion rotation $\varphi_1 = const$ in the longitudinal direction the pinion and gear teeth are in contact along the line, that is $K_1 = 0$. To calculate the principal reduced curvature in the profile direction of the tooth (K_2) it is necessary to disclose the left-side dependence in the expressions (18).

Having the matrix $\widetilde{C}(\varphi_1)$ (14), from (19) considering (3) we find the projections of the position vector $\overline{w}_\varphi$:

$$w_{\varphi x} = d_v \cdot \cos\alpha_n - \sin\varphi_2 \sin\Sigma \cos\theta_{f2} \sin\alpha_n;$$

$$w_{\varphi y} = (d_v \cdot \sin\theta_{f2} - \cos\theta_{f2} \cos\varphi_2 \sin\Sigma) \sin\alpha_n; \quad (20)$$

$$w_{\varphi z} = (\cos\varphi_2 \cos\alpha_n - \sin\varphi_2 \sin\theta_{2f} \sin\alpha_n) \sin\Sigma.$$

Differentiation of the equation of meshing (16) with respect to u, h and φ for the derivatives f_u, f_h, and f_φ yields:

$$f_u = \sin\alpha_n \cdot \sin\varphi_2 \cdot \sin\Sigma - \cos\alpha_n \cdot$$

$$\cdot (d_v \cdot \cos\theta_{f2} + \sin\theta_{f2} \cdot \cos\varphi_2 \cdot \sin\Sigma);$$

$$f_h = d_v \cdot \sin\theta_{f2} - \cos\theta_{f2} \cdot \cos\varphi_2 \cdot \sin\Sigma; \quad (21)$$

$$f_\varphi = i^{-1} \sin\theta_{f2} \sin\alpha_n \sin\Sigma [a_v \cdot \sin\varphi_2 + d \cdot \cos\varphi_2] +$$

$$+ i^{-1} \cos\alpha_n \cdot \sin\Sigma \cdot [a_v \cdot \cos\varphi_2 + d \cdot \sin\varphi_2] +$$

$$+ \cos\theta_{f2} \cdot \sin\alpha_n \times\times i^{-1} \cdot \sin\Sigma \cdot [b_v \cdot \cos\varphi_2 + c_v \cdot \sin\varphi_2]$$

Expanding the dependence (18), after transformations we obtain the following expression:

$$K_2 = (H_1 - H_2 - H_3)/(H_4 - H_5 - H_6 - H_7); \quad (22)$$

where

$$H_1 = f_u \cdot x_{2h} \cdot w_{\varphi y};$$

$$H_2 = f_h \cdot x_{2u} \cdot w_{\varphi y};$$

$$H_3 = f_u \cdot y_{2h} \cdot w_{\varphi x};$$

$$H_4 = f_u \cdot x_{2h} \cdot V_{\varphi y};$$

$$H_5 = f_\varphi \cdot x_{2u} \cdot y_{2h};$$

$$H_6 = f_h \cdot x_{2u} \cdot V_{\varphi y};$$

$$H_7 = f_u \cdot y_{2h} \cdot V_{\varphi x}.$$

For calculation of the gear tooth point M (Figure 5), in which $u = 0$, $\varphi_1 = \varphi_2 = 0$, $h = 0$, the formula (22) is simplified and takes the form:

$$K_2 = \frac{E \cdot w_{\varphi y} + f_u \cdot w_{\varphi x} \cdot \sin\alpha_n}{r_2 \cdot E \cdot d_v + f_\varphi \cos\theta_{f2} \sin\alpha_n}, \quad (23)$$

where

$$E = f_u \sin\theta_{f2} \cos\alpha_n + f_h \cos\theta_{f2}.$$

Let us consider the algorithm of calculating the principal reduced curvature in meshing of a bevel gear with a small shaft angle with a non-generated gear and a generated pinion at any point of the line of action corresponding to the predetermined rotation angle of the gear $\varphi_1 = const$. According to the method outlined in [30] we calculate the geometric parameters of the bevel gear with a small shaft angle with a non-generated gear and a generated pinion. Fixing $h = h^*$ in the range $-h_f \leq h \leq h_a$, where h_f, h_a - the height of the tooth dedendum and addendum, correspondingly, by solving the square with respect to u equation of meshing (16) at $\varphi_1 = const$ and $h = h^*$ we determine the value $u = u^*$. Then, using the relationships (3), (12), (15), (20) and (21), we calculate the parameters included in the formula (23), according to which we establish the desired value K_2.

Based on the obtained dependences in the MathCad environment a computer program was developed to calculate K_2 in the bevel gear with a small shaft angle with a non-generated gear and a generated pinion for any given meshing phase. As an illustration of the program operation, for a number of fixed values of the gear rotation angle (φ_1 = -0.172; -0.115; -0.057; 0.0; 0.088; 0.177; 0.265) Figure 6 shows changes of K_1 with respect to the length of the lines of action of in the bevel gear with a small shaft angle with a non-generated gear and a generated pinion.

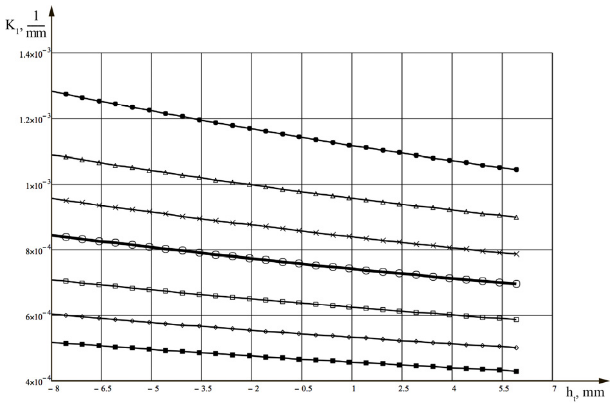

Figure 6. Results of calculation of the principal reduced curvature in meshing

By using formulas in the MathCad software a computer program was developed to study the position and lines of action in gear meshing of the straight bevel gear with a small shaft angle with a non-generated gear and a generated pinion. The program showed its lines of action with the pinion tooth surface and the gear tooth surface of the straight bevel gear with a small shaft angle with a non-generated gear and a generated pinion for a number of fixed values of the angle of action (φ_1=-0,172; -0,115; -0,057; 0,0; 0,088; 0,177; 0,265).

Using presented values and dependences was manufactured coaxial gearbox with a bevel gear with a small shaft angle with a non-generated gear and a generated pinion. Its gear ratio is 32.

For the particular gears it was established that over four pairs of teeth are simultaneously in meshing. Within the meshing phase along the tooth height the principal reduced curvature varies within ±10 %, and the principal reduced curvature change relative to the curvature at the design point is from 66% (as the tooth enters meshing) to - 46% (as the tooth exits meshing).

The resulting dependences are the basis for the development of a methodology for calculating the load capacity of the straight bevel gear with a small shaft angle with a non-generated gear and a generated pinion.

Figure 7. Model of the coaxial gearbox with a precessional bevel gear with a small shaft angle

4. CONCLUSION

An original coaxial gearbox containing straight bevel gears with a small shaft angle with a non-generated gear and a generated pinion is considered. To simplify the process of cutting the teeth of pinion and gear, it is suggested that gears produced with a non-generated gear.

The new scientific results of the article are:

- dependencies for calculating the coordinates of the points of the surfaces of the pinion and gear teeth for straight bevel gears with a small shaft angle;
- dependencies for calculating the coordinates of the lines of action in meshing surfaces of the pinion and gear teeth;
- dependencies for calculating the main reduced curvature at the points of the contact lines in the gears mesh.

The geometrical characteristics of the gears mesh of the straight bevel gears with a small shaft angle are investigated. In the studied gears, more than four pairs of teeth are simultaneously engaged. In the bevel orthogonal transmission, the number of pairs of teeth can not be more than two simultaneously in meshing. The geometric characteristics of the straight bevel gears with a small shaft angle with a non-generated gear and a generated pinion obtained in this paper are necessary and sufficient for constructing a method for calculating the loading capacity of a transmission.

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ИСТРАЖИВАЊЕ ГЕОМЕТРИЈСКИХ КАРАКТЕРИСТИКА ПРАВОЗУБИХ КОНУСНИХ ЗУПЧАНИКА СА МАЛИМ УГЛОМ ГОРЊЕГ КОНУСА БЕЗ ОЗУБЉЕНОГ

ВЕЋЕГ А СА ОЗУБЉЕНИМ МАЊИМ ЗУПЧАНИКОМ

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Коришћењем конусних зупчаника са малим углом прецесије горњег конуса развијен је погон за један део опреме на гас и уље. Зупчаници се одликују високим степеном искоришћености (око 0,9) и малим почетним обртним моментом. Захваљујући спрезању већег броја парова зубаца ови зупчаници имају већи капацитет оптерећења у поређењу са другим типовима зупчаника, што обезбеђује безбедан рад на далеком северу. Данас се највише проучавају конусни зупчаници са малим углом горњег конуса са двоструким конкавно-конвексним зупцима. Међутим, процес израде зубаца таквих зупчаника је изузетно компликован – машина мора да се ресетује четири пута. Технологија израде се може поједноставити коришћењем зупчаника без озубљеног већег и са озубљеним мањим зупчаником. Чланак приказује израчунавања праваца дејства код спрезања и редуковану основну закривљеност правозубих конусних зупчаника са малим углом горњег конуса без озубљеног већег и са озубљеним мањим зупчаником, геометријске карактеристике потребне за одређивање напона у контакту који настаје спрезањем зубаца и за развој метода за израчунавање капацитета оптерећења зупчаника.