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## Influence of the Wall Roughness on a Linear Shear Flow

*In this paper, we will be interested in the viscous flow fluid over a rough wall which has a periodic roughness and small amplitude. The Reynolds number for the flow over a rough wall is low and the creeping flow equations apply. By using an asymptotic expansion, the analytic expressions are obtained for the pressure and the components f velocity of the flow of second order due to the roughness, then concluding the expressions of pressure and velocity of the flow on the rough wall.*

**Keywords:** Rough wall, Stokes flow, roughness, low Reynolds number.

### 1. INTRODUCTION

Usually the walls are assumed to be smooth. Although this assumption is successful in describing fluid flow on macroscopic length scale, but it is not valid for microscopic scales due to the importance of the influence of roughness on the flow, which will have of course the repercussions on a lot of practical applications, such as in separation techniques in analytical chemistry (field-flow fractionation and hydrodynamic chromatography) [1], small scale transport phenomena, microfluidics, biological flow fields [2,3].

There is a large literature on roughness influence in various industrial fields from the macro scale [4] to micro scale [5]. The most of this literature concerns turbulent flow, or of an experimental nature in fluid mechanics. The question is often how microscopic roughness parameters influence macroscopic flow. Huh and Mason [6] studied theoretically by using mechanistic arguments, the effect of roughness of a solid surface on its wettability by a liquid. By calculating the equilibrium shape of a liquid drop resting on a rough surface, they obtained the relation between the true (or microscopic) equilibrium contact angle at the three phase contact line and the apparent contact angle observed macroscopically at the geometrical contour plane of the solid.

Many studies showed that rough surfaces limited a flow, with a no slip boundary condition, behaves as plan surface with a slip condition [7, 8, 9, 10].

Jansons [11] showed that very small amounts of roughness can well approximate a no-slip boundary condition macroscopically.

Priezjev [12] by using the (MDS), investigated the influence of molecular-scale surface roughness on the slip behavior in thin liquid films. He has shown that the slip length increases almost linearly with the shear rate for atomically smooth rigid walls and incommensurate structures of the liquid/solid interface. The majorities of these works are interested in the influence of the wall

roughness on the slip length, without calculating the flow of second order due to this roughness, in a case of a rough wall with a sinusoidal roughness.

In this paper, we will calculate the flow of second order due to a roughness in the case of rough wall with periodic roughness. We consider the steady shear flow over a periodically corrugated surface. We will use the asymptotic approach to derive the analytical expression of the stream function of flow generated by the roughness, then we derive the analytic expressions of the pressure and components velocity due to the roughness of wall. The slip length for a no-symmetric periodic roughness is calculated.

### 2. MATERIALS AND METHODS

We consider a shear flow limited by a rough wall Fig.1. The flow away from the wall is defined by the linear velocity field  $\tilde{\mathbf{V}}_s^\infty = k_s \tilde{Z} \mathbf{i}_x$ , but near the rough wall their form is no longer linear, that due to the perturbation generated by the roughness.

The roughness considered here is two-dimensional and the flow sought is perpendicular to the roughness Fig.1. Let  $(\tilde{\mathbf{V}}_s, \tilde{P}_s)$  be the velocity and pressure field of the flow in the vicinity of the wall In the right-handed system of rectangular Cartesian coordinates  $(\tilde{X}, \tilde{Y}, \tilde{Z})$ , the velocity field can be expressed as

$$\tilde{\mathbf{V}}_s = \tilde{U}_s \mathbf{i}_x + \tilde{W}_s \mathbf{i}_z \quad (1)$$

The field flow  $(\tilde{\mathbf{V}}_s, \tilde{P}_s)$  is governed by the Stokes equations by assuming that their Reynolds number is small.

$$\begin{cases} \mu_f \tilde{\nabla}^2 \tilde{\mathbf{V}}_s^{(0)} = \tilde{\nabla} \tilde{P}_s^{(0)} \\ \text{With} \\ \tilde{\nabla} \cdot \tilde{\mathbf{V}}_s^{(0)} = 0 \end{cases} \begin{cases} \tilde{\mathbf{V}}_s = 0, \text{ on the rough wall } \tilde{Z} = 0. \\ \tilde{\mathbf{V}}_s^{(0)} \rightarrow \tilde{\mathbf{V}}_s^\infty, \tilde{Z} \rightarrow \infty, \text{at infinity} \end{cases}$$

from the wall.

(2)

In the case of small roughness amplitude ( $\epsilon \ll 1$ ), the velocity and pressure fields can develop asymptotically in the form:

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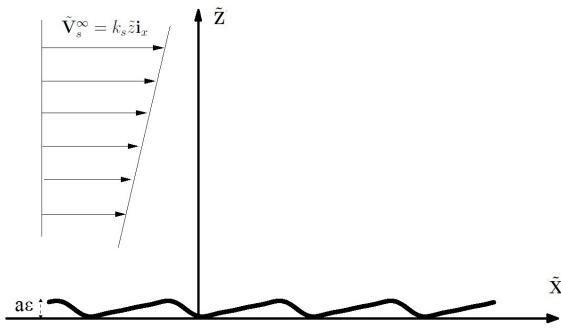
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$$\tilde{\mathbf{V}}_s = \tilde{\mathbf{V}}_s^{(0)} + \varepsilon \tilde{\mathbf{V}}_s^{(1)} + O(\varepsilon^2) \quad (3a)$$

$$\tilde{P}_s = \tilde{P}_s^{(0)} + \varepsilon \tilde{P}_s^{(1)} + O(\varepsilon^2) \quad (3b)$$



**Figure 1. Linear shear flow over a periodic wall.**

with

- $(\tilde{\mathbf{V}}_s^{(0)}, \tilde{P}_s^{(0)})$  are the velocity and pressure field of a linear shear flow limited by a smooth (virtual) wall located in  $\tilde{Z} = 0$  with a no-slip condition.
- $\varepsilon (\tilde{\mathbf{V}}_s^{(1)}, \tilde{P}_s^{(1)})$  are the velocity and the pressure of the field flow of order  $\varepsilon$  generated by the wall roughness.

The flows  $(\tilde{\mathbf{V}}_s^{(0)}, \tilde{P}_s^{(0)})$  and  $\varepsilon (\tilde{\mathbf{V}}_s^{(1)}, \tilde{P}_s^{(1)})$  are governed by the Stokes equations. By substituting the expressions (3) of velocity and pressure field of the global flow in the Stokes equations (2), and by linearity of these equations and the separation of terms of order 1 and of order  $\varepsilon$ , we obtain:

The flow of order (0):

$$\begin{cases} \mu_f \tilde{\nabla}^2 \tilde{\mathbf{V}}_s^{(0)} = \tilde{\nabla} \tilde{P}_s^{(0)} \\ \text{With} \\ \tilde{\nabla} \cdot \tilde{\mathbf{V}}_s^{(0)} = 0 \end{cases} \begin{cases} \tilde{\mathbf{V}}_s = 0, \text{ on the rough wall } \tilde{Z} = 0. \\ \tilde{\mathbf{V}}_s^{(0)} \rightarrow \tilde{\mathbf{V}}_s^\infty, \tilde{Z} \rightarrow \infty, \text{ at infinity} \end{cases} \quad (4)$$

from the wall.

The solutions of the velocity and pressure fields of the order (0), according to the boundary conditions (4), are trivial:

$$\tilde{\mathbf{V}}_s^{(0)} = \tilde{\mathbf{V}}_s^\infty = k_s \tilde{Z} \mathbf{i}_x \quad (5a)$$

$$\tilde{P}_s^{(0)} = 0 \quad (5b)$$

For the flow of order (1), we must first find the boundary conditions. On the rough wall, whose profile is described by the equation  $Z_p = a\varepsilon \mathcal{R}(\tilde{X})$  we use the Taylor expansion in  $\tilde{Z} = 0$  to express the velocity vector  $\tilde{\mathbf{V}}_s$ .

$$[\tilde{\mathbf{V}}_s]_{\tilde{Z}=0} + \varepsilon a \mathcal{R}(\tilde{X}) \left[ \frac{\partial \tilde{\mathbf{V}}_s}{\partial \tilde{Z}} \right]_{\tilde{Z}=0} + O(\varepsilon^2) = 0 \quad (6)$$

By substituting the expression (3) of  $\tilde{\mathbf{V}}_s$  in Taylor development (6), and with boundary conditions (4) of the flow  $(\tilde{\mathbf{V}}_s^{(0)}, \tilde{P}_s^{(0)})$ , the boundary condition of the flow  $(\tilde{\mathbf{V}}_s^{(1)}, \tilde{P}_s^{(1)})$  on a virtual plane  $\tilde{Z} = 0$ :

$$[\tilde{\mathbf{V}}_s^{(1)}]_{\tilde{Z}=0} = -a \mathcal{R}(\tilde{X}) \left[ \frac{\partial \tilde{\mathbf{V}}_s^{(0)}}{\partial \tilde{Z}} \right]_{\tilde{Z}=0} \quad (7)$$

The boundary conditions (2) and (4) at infinity from the wall of the perturbed velocity field  $\tilde{\mathbf{V}}_s$  and unperturbed velocity field  $\tilde{\mathbf{V}}_s^{(0)}$  give the expression of the velocity field of order (1)  $\tilde{\mathbf{V}}_s^{(1)}$  at infinity ( $\tilde{Z} \rightarrow \infty$ ):

$$\tilde{\mathbf{V}}_s^{(1)} \rightarrow 0, \tilde{Z} \rightarrow \infty \quad (8)$$

The condition (7) becomes with the expression (5a):

$$[\tilde{\mathbf{V}}_s^{(1)}]_{\tilde{Z}=0} = -a \mathcal{R}(\tilde{X}) \tilde{k}_s \mathbf{i}_x \quad (9)$$

Where  $\mathcal{R}(\tilde{X})$  is a periodic, normalized function that describes the roughness of the wall (10).

$$\mathcal{R}(\tilde{X}) = c_0 + \sum_{n=1}^{+\infty} (c_n \cos(n\omega \tilde{X}) + s_n \sin(n\omega \tilde{X})) \quad (10)$$

The condition (9) appears as a tangential velocity which varies with the position  $\tilde{X}$  on the virtual plane wall localized in  $\tilde{Z} = 0$ , and it is analogous to a Navier slip condition with length slip.

## 2.1 Stream function of order ( $\varepsilon$ )

The order (1) solution with conditions (8) and (9) is searched for in terms of a stream function  $\tilde{\psi}_s^{(1)}$ . Following [7], an appropriate solution is written in the form:

$$\tilde{\psi}_s^{(1)} = d_0 \tilde{Z} + f_0 + \sum_{n=1}^{\infty} \left[ (d_n \tilde{Z} + f_n) \cos(n\omega \tilde{X}) + (e_n \tilde{Z} + g_n) \sin(n\omega \tilde{X}) \right] \exp(-n\omega \tilde{Z}) \quad (11)$$

By using the boundary conditions, we find the expressions of the coefficients  $d_0, f_0, f_n, d_n, e_n$  and  $g_n$ :

$$\begin{aligned} d_0 &= -c_0 a k_s \\ d_n &= -c_n a k_s \\ f_n &= 0 \\ e_n &= -s_n a k_s \\ g_n &= 0 \end{aligned}$$

According to Hocking [5], the value  $\beta = \varepsilon d_0 / a k_s = -c_0 \varepsilon$  represent the dimensionless slip length, which that the no slip velocity condition on the deformable wall turn into the slip velocity condition on the plan wall located at  $\tilde{Z} = 0$ .

$$\tilde{U}_s = a \beta \frac{\partial \tilde{U}_s}{\partial \tilde{Z}} \quad (12)$$

And then, the flow away from the rough surface can be written as follow

$$\tilde{U}_s \sim (\tilde{Z} + a \beta) k_s \quad (13)$$

The above expression of  $\beta$  show that the dimensionless slip length is negative, this sign depends on the

reference plane  $\tilde{Z} = 0$  where it is located with respect to the wall deformation. In our model, it is at the bottom of the deformation, so the minus sign indicates that the equivalent plane surface with adhesion condition is located above the plane wall  $\tilde{Z} = 0$ , between peaks and valleys, with adimensional separated distance  $c_0\varepsilon$ .

## 2.2 Velocity field of the flow of ( $\varepsilon$ ) order

After determining the stream function of the flow generated by the roughness, the velocity field of order  $\varepsilon$  can be expressed by:

$$\tilde{\mathbf{V}}_s^{(1)} = \tilde{U}_s^{(1)}\mathbf{i}_x + \tilde{W}_s^{(1)}\mathbf{i}_z$$

With  $\tilde{U}_s^{(0)}$ ,  $\tilde{W}_s^{(0)}$  are the components, in  $\tilde{X}$  and  $\tilde{Z}$  direction, of velocity field  $\tilde{\mathbf{V}}_s^{(1)}$ . These components are linked to the stream function by:

$$\tilde{U}_s^{(1)} = \frac{\partial \tilde{\psi}_s^{(1)}}{\partial \tilde{Z}} ; \quad \tilde{W}_s^{(1)} = -\frac{\partial \tilde{\psi}_s^{(1)}}{\partial \tilde{X}}$$

From the stream function expression (11), we find:

$$\begin{aligned} \tilde{U}_s^{(1)} &= -ac_0k_s + a\tilde{k}_s \sum_{n=1}^{+\infty} [c_n \cos(n\omega\tilde{X}) + s_n \sin(n\omega\tilde{X})] \\ &\quad [\varepsilon n\omega\tilde{Z} - 1] \exp(-n\omega\tilde{Z}) \end{aligned} \quad (14a)$$

$$\begin{aligned} \tilde{W}_s^{(1)} &= -ac_0k_s + a\tilde{k}_s \sum_{n=1}^{+\infty} [s_n \cos(n\omega\tilde{X}) - c_n \sin(n\omega\tilde{X})] \\ &\quad n \exp(-n\omega\tilde{Z}) \end{aligned} \quad (14b)$$

## 2.3 Pressure field of the flow of order ( $\varepsilon$ )

The flow  $(\tilde{\mathbf{V}}_s^{(1)}, \tilde{P}_s^{(1)})$  is governed by the Stokes equation, so:

$$\frac{\partial \tilde{P}_s^{(1)}}{\partial \tilde{X}} = \mu_f \frac{\partial^2 \tilde{U}_s^{(1)}}{\partial \tilde{X}^2} + \mu_f \frac{\partial^2 \tilde{U}_s^{(1)}}{\partial \tilde{Z}^2} \quad (15a)$$

$$\frac{\partial \tilde{P}_s^{(1)}}{\partial \tilde{Y}} = 0 \quad (15b)$$

$$\frac{\partial \tilde{P}_s^{(1)}}{\partial \tilde{Z}} = \mu_f \frac{\partial^2 \tilde{W}_s^{(1)}}{\partial \tilde{X}^2} + \mu_f \frac{\partial^2 \tilde{W}_s^{(1)}}{\partial \tilde{Z}^2} \quad (15c)$$

From the expressions (14) of  $\tilde{U}_s^{(1)}$  and  $\tilde{W}_s^{(1)}$ , we may write:

$$\begin{aligned} \frac{\partial^2 \tilde{U}_s^{(1)}}{\partial \tilde{X}^2} &= a\tilde{k}_s \sum_{n=1}^{+\infty} n^2 \omega^2 [-c_n \cos(n\omega\tilde{X}) - s_n \sin(n\omega\tilde{X})] \\ &\quad [\varepsilon n\omega\tilde{Z} - 1] \exp(-n\omega\tilde{Z}) \end{aligned} \quad (16a)$$

$$\begin{aligned} \frac{\partial^2 \tilde{U}_s^{(1)}}{\partial \tilde{Z}^2} &= a\tilde{k}_s \sum_{n=1}^{+\infty} n^2 \omega^2 [s_n \cos(n\omega\tilde{X}) + c_n \sin(n\omega\tilde{X})] \\ &\quad [\varepsilon n\omega\tilde{Z} - 3] \exp(-n\omega\tilde{Z}) \end{aligned} \quad (16b)$$

$$\frac{\partial^2 \tilde{W}_s^{(1)}}{\partial \tilde{X}^2} = a\tilde{k}_s \sum_{n=1}^{+\infty} n^3 \omega^3 [-s_n \cos(n\omega\tilde{X}) + c_n \sin(n\omega\tilde{X})] \quad (16c)$$

$$\tilde{Z} \exp(-n\omega\tilde{Z})$$

$$\begin{aligned} \frac{\partial^2 \tilde{U}_s^{(1)}}{\partial \tilde{Z}^2} &= a\tilde{k}_s \sum_{n=1}^{+\infty} n^2 \omega^2 [-s_n \cos(n\omega\tilde{X}) - c_n \sin(n\omega\tilde{X})] \\ &\quad [\varepsilon n\omega\tilde{Z} - 2] \exp(-n\omega\tilde{Z}) \end{aligned} \quad (16d)$$

The pressure  $\tilde{P}_s^{(1)}$  depends on two variables  $\tilde{X}$  and  $\tilde{Z}$ , its differential

$$d\tilde{P}_s^{(1)} = \frac{\partial \tilde{P}_s^{(1)}}{\partial \tilde{X}} i_x + \frac{\partial \tilde{P}_s^{(1)}}{\partial \tilde{Z}} i_z$$

After an analytical calculation used in the symbolic software, the following result is obtained:

$$\frac{\partial}{\partial \tilde{Z}} \left( \frac{\partial \tilde{P}_s^{(1)}}{\partial \tilde{X}} \right) = \frac{\partial}{\partial \tilde{X}} \left( \frac{\partial \tilde{P}_s^{(1)}}{\partial \tilde{Z}} \right)$$

Therefore the expression of the pressure  $\tilde{P}_s^{(1)}$

$$\begin{aligned} \tilde{P}_s^{(1)} &= 2a\mu_f k_s \omega \sum_{n=1}^{+\infty} n^3 \omega^3 [s_n \cos(n\omega\tilde{X}) - c_n \sin(n\omega\tilde{X})] \\ &\quad n \exp(-n\omega\tilde{Z}) \end{aligned} \quad (17)$$

## 2.4 Global Stokes flow $(\tilde{\mathbf{V}}_s, \tilde{P}_s)$

By substituting the expressions (5), (14) and (17) of the velocities and pressures fields of the flows  $(\tilde{\mathbf{V}}_s^{(0)}, \tilde{P}_s^{(0)})$  and  $(\tilde{\mathbf{V}}_s^{(1)}, \tilde{P}_s^{(1)})$  in the expressions of the velocity and pressure fields (3) of the global flow in the vicinity of the rough wall, we find:

$$\begin{aligned} \tilde{U}_s^{(1)} &= k_s \tilde{Z} - ac_0 k_s + a\varepsilon \tilde{k}_s \sum_{n=1}^{+\infty} [c_n \cos(n\omega\tilde{X}) + s_n \sin(n\omega\tilde{X})] \\ &\quad [\varepsilon n\omega\tilde{Z} - 1] \exp(-n\omega\tilde{Z}) \end{aligned} \quad (18a)$$

$$\begin{aligned} \tilde{W}_s^{(1)} &= a\varepsilon k_s \tilde{Z} \sum_{n=1}^{+\infty} [c_n \cos(n\omega\tilde{X}) + s_n \sin(n\omega\tilde{X})] \\ &\quad [\varepsilon n\omega\tilde{Z} - 1] \exp(-n\omega\tilde{Z}) \end{aligned} \quad (18b)$$

$$\begin{aligned} \tilde{W}_s^{(1)} &= \varepsilon 2a\mu_f k_s \tilde{Z} \sum_{n=1}^{+\infty} [s_n \cos(n\omega\tilde{X}) - c_n \sin(n\omega\tilde{X})] \\ &\quad n \exp(-n\omega\tilde{Z}) \end{aligned} \quad (18c)$$

The stream function of the flow is also written in the form

$$\tilde{\psi}_s = \tilde{\psi}_s^{(0)} + \varepsilon \tilde{\psi}_s^{(1)} + O(\varepsilon^2) \quad (19)$$

where,  $\tilde{\psi}_s^{(0)} = \frac{1}{2} k_s \tilde{Z}^2$  is the stream function of the flow of order (0). By taking into account the expressions of

$\tilde{\psi}_s^{(0)}$  and  $\tilde{\psi}_s^{(1)}$ , the expression of stream function of the flow along the rough wall becomes:

$$\tilde{\psi}_s^{(1)} = \frac{1}{2} k_s \tilde{Z}^2 + \varepsilon \left[ (-a \tilde{k}_s c_n \tilde{Z}) \cos(n\omega \tilde{X}) + (-a \tilde{k}_s s_n \tilde{Z}) \sin(n\omega \tilde{X}) \right] \exp(-n\omega \tilde{Z}) \quad (20)$$

To make the system non-dimensional, the following quantities are used:

$$\tilde{U}_s = a k_s U_s; v; \tilde{P}_s = a \mu_f k_s P_s; \tilde{\psi}_s = a^2 k_s \psi_s.$$

With  $U_s$ ,  $W_s$ ,  $P_s$ ,  $\psi_s$  are the dimensionless quantities of the velocity components, the pressure and the stream function of the stokes flow in the vicinity of a rough wall.

### 3. RESULTS AND DISCUSSION

In this section, we calculate the numerical values of the stream function, the velocity and the pressure field, on the range of two periods, by taking two types of roughness, the symmetric roughness Fig 2 and the no-symmetric one Fig 7. The results show that the roughness and its parameters (period, amplitude and form) have a huge influence on the flow over the rough wall.

Fig 3 and Fig 8 show that the particles trajectories of the fluid no longer follow their linear trajectories as away from the rough wall, but they flow the trajectories resemble to profile of roughness. This influence of roughness on the particles of fluid decays as we move away from the rough wall.

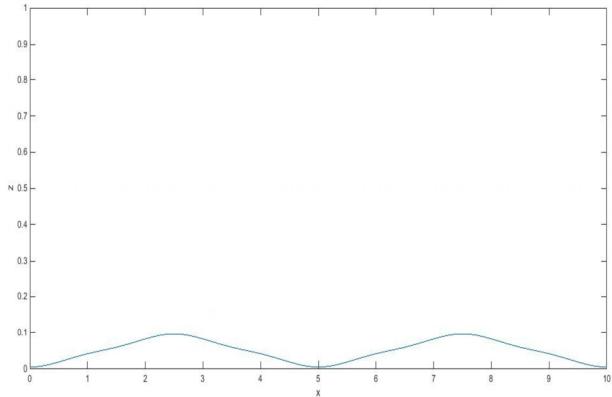


Figure 2: The symmetric roughness profile with  $\delta = 1/2$ ,  $\varepsilon = 0.1$  and  $L = 5$ .

Figs 4 and 9 show the pressure fields for both kinds of roughness. These results show that the pressure field depends on the roughness form. Take for example the region  $2.5 < X < 7.5$  in Fig 2, we observe from Fig 4 that The pressure contours indicate the presence of an adverse pressure gradient in the region  $2.5 < X < 5$  on the right side of the peak, whereas on the left side of the peak the pressure reaches its maximum value in the region  $5 < X < 7.5$ . These results are in good agreement with those found by Niavarani and Priezjev [13] for the case of sinusoidal roughness. This variation of the pressure field near a rough wall can explain that the fluid particles move easily on the right side of the peak, but as soon as they reach the point  $X = 5$ , they find in front of them the growing part of the roughness which

will decrease their motion, that is why we observe the increasing of the pressure in this region. These influences of roughness is remarked also for the velocity components as showed in Figs. 5, 10, 6, 11.

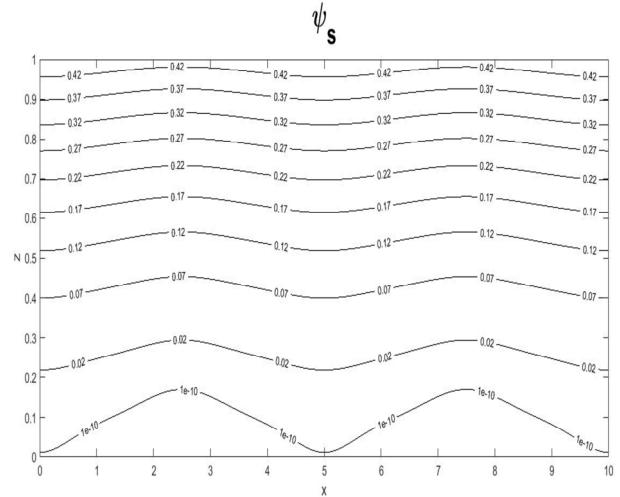


Figure 3: Stream function of the flow near a rough wall, symmetric roughness with  $\delta = 1/2$ ,  $\varepsilon = 0.1$  and  $L = 5$ .

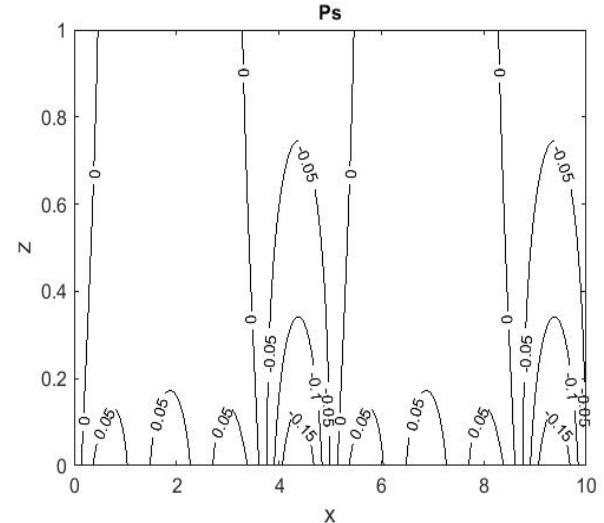


Figure 4: Pressure field of the flow near a rough wall, with symmetric roughness,  $\delta = 1/2$ ,  $\varepsilon = 0.1$  and  $L = 5$ .

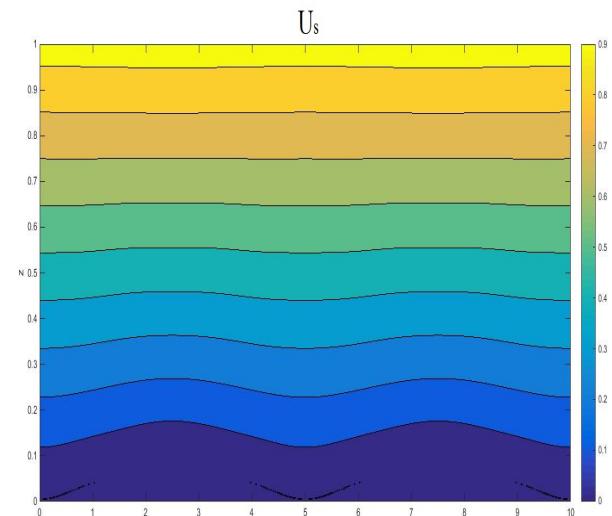
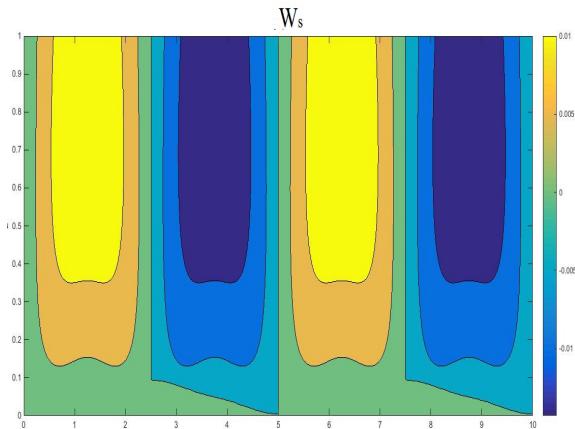
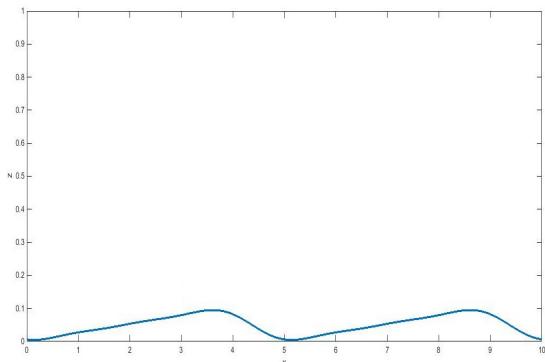


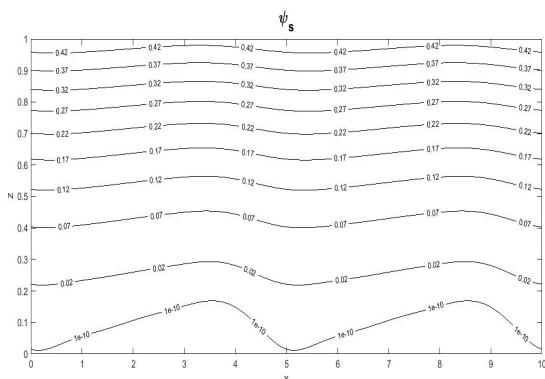
Figure 5: The field of the horizontal component of the velocity of the flow near a rough wall, symmetric roughness with  $\delta = 1/2$ ,  $\varepsilon = 0.1$  and  $L = 5$ .



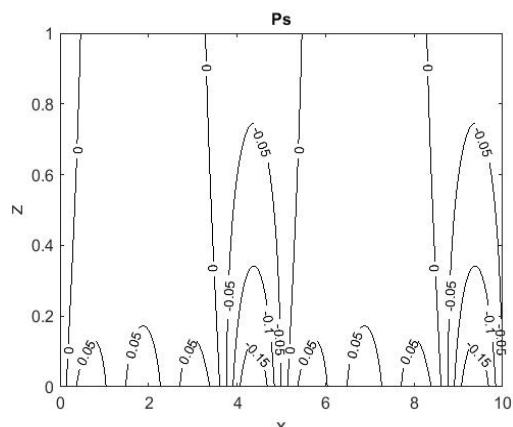
**Figure 6:** The field of the vertical component of the velocity of the flow near a rough wall, symmetric roughness with  $\delta = 1/2$ ,  $\epsilon = 0.1$  and  $L = 5$ .



**Figure 7:** The no-symmetric roughness profile with  $\delta = 3/4$ ,  $\epsilon = 0.1$  and  $L = 5$ .



**Figure 8:** Stream function of the flow near a rough wall, no-symmetric roughness with  $\delta = 3/4$ ,  $\epsilon = 0.1$  and  $L = 5$ .



**Figure 9:** Pressure field of the flow near a rough wall, with no-symmetric roughness,  $\delta = 3/4$ ,  $\epsilon = 0.1$  and  $L = 5$ .

#### 4. CONCLUSIONS

The study shows that the wall roughness generates a flow of second order, which has a big influence on the global flow over a rough wall. The numerical results show, in the case of symmetric or asymmetric roughness, that the streamlines function of the flow is influenced by the perturbation due to the roughness. The fluid particles follow the trajectories imposed by the roughness form. This influence is important near a rough wall, whereas we observe the disappearance of this influence due to the roughness on the fluid particles that are far from the wall, where the flow becomes again linear. In addition to this, the pressure and velocity components varies in the periodic way. As shown in the numerical result of the pressure, for the two cases of roughness, the flow near a rough wall creates the adverse pressure gradient in the right side of the peak, whereas on the lift side the pressure reaches its maximum value.

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#### NOMENCLATURE

$R_e$	Reynolds number
$\varepsilon$	Dimensionless roughness amplitude
$L$	Period of roughness
$\tilde{V}$	Dimensional velocity
$\tilde{P}$	Dimensional pressure
$\tilde{\psi}$	Dimensional stream function

$(\tilde{X}, \tilde{Y}, \tilde{Z})$	Cartiesian coordinates
$\mathcal{R}(\tilde{X})$	normalized function
$K_s$	Shear rate

#### УТИЦАЈ ХРАПАВОСТИ ЗИДА НА ЛИНЕАРНО СТРУЈАЊЕ УСЛЕД СМИЦАЊА

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Рад се бави струјањем вискозног флуида на периодично храпавом зиду мале амплитуде. Рејнолдсов број за струјање на храпавом зиду је мали а важе и једначине пузећег струјања. Такође важи стање без граница на храпавом зиду. Коришћењем асимптомског проширења добијају се аналитички изрази за притисак и компоненте брзине струјања другог реда, а потом и крајњи изрази за притисак и брзину струјања на храпавом зиду.