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Prediction of a Fertilizer Particle Motion Along a Vane of a Centrifugal Spreader Disc Assuming its Pure Rolling

The paper analyzes the motion of idealized spherical homogeneous fertilizer particle along the straight vane attached to flat rotating disc. The analysis, based on the assumption on the pure rolling of the particle along the vane (without sliding), has been performed in the non-inertial reference coordinate system, which rotates together with the spreader disk. The particle motion along the vane is described by hyperbolic cosine function, which is the solution of the ordinary in-homogenous second-order differential equation having constant coefficients. Solution of this kind represents an approximation of the real motion of fertilizer particle along the radial vane fixed to horizontal disc rotating at constant angular velocity. However, it can be very useful for optimization of centrifugal spreader design and working parameters, as well as for further analysis of the whole fertilizer spreading process that also includes particle flight.

Keywords: *fertilizer,particle, motion, radial vane, analytical solution, centrifugal disc spreader, non-inertial reference system.*

1. INTRODUCTION

Contemporary agricultural crop production cannot be imagined without chemical fertilizers usage. The main purpose of their use is to provide the right rate of nutrients to the crops [1] necessary for their growth and for providing planned yield under acceptable costs and environmental influence. However, it is aimed to improve the utilization of biological yield potential of each specific plant. It is believed that an average contribution of fertilizer application with respect to the cost and energy share, as well as to increasing the crop yield reaches between the 50% and 60%, [2,3].

Due to the large environmental impact of fertilizer use and their share in the total energy needed in the crop production [4], it is important to optimize the spreading process [5] respecting the time of application and prescribed application rates. If not, the labor, crop yield and energy looses [6], followed by less or more negative ecological effects [7], will be the result of inadequate soil fertilization. Authors [8,9] stated that both over-dosage as well as the under-dosage of fertilizer results in the yield losses.Depending on their aggregate condition, there are two main groups of chemical fertilizers: liquid and solid. Solid chemical fertilizers are one of the most important available sources for crops nutrition [10]. Different kinds of solid fertilizer spreading machines have been designed but centrifugal fertilizer spreaders stayed as most commonly used due to their robustness, simplicity, low cost and large working widths [11, 12-14]. During the last few years, centrifugal spreaders have been modified to enable even larger operational widths that additionally increase their high productivity and decrease costs [15]. The spreading width can now reach to over 45 m [16].

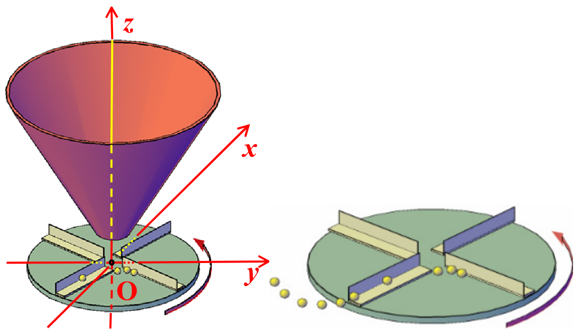

Figure 1. A centrifugal fertilizer spreader, having a flat disc with straight radial vanes.

Centrifugal fertilizer spreaders are equipped with hoper and one or two rotating discs with vanes. During the operation, gravitation moves fertilizer particles down–ward in the hopper, toward the bottom orifice placed beyond the disk spinner at specified distance from its rotation axis. After leaving the hopper, particles fall down on the spinner and establish the contact with one of attached vanes,Figure 1. The vane rotates with disk and traces the particle trajectory until it's ejection into the air. The fertilizer spread pattern and thus the distribution uniformity depends on many factors [17]:fertilizer parti–cle physical properties, field application conditions, spre–ader configuration, the setting tables, etc. [5]. It is hardly possible to account for all these factors, and introduction of more or less accurate hypothesis is necessary to resolve the problem of fertilizer spreading uniformity. Furthermore, many models are focused only to some specific segment, instead of the whole spreading process.

Having in mind that twenty-first century engineering is at the epicentre of an explosion in new knowledge and that new and revolutionary discoveries in science,

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engineering, medicine, mathematics and the social sciences have influenced and contributed towards erasure of borders among academic disciplines [18], urging the more intensive and deeper analysis in all of the research fields in the World. In that sense, various models on fertilizer application were further developed taking into account different parameters. So,the models for predicting the particle motion along the vaneof fertiliser disk spreader have been reported by [19-23].Some researchers analyzed the ballistic flight of the fertilizer particle [22,24,25,26]. Using their results, [27] made a complete model enabling more accurate prediction of the fertilizer particles spreading. Further ideas in defining and modelling the particle movement on the spreading dick can be found in an example of model that is defining the motion of the round-shaped fruits on the rotating sorting machines [28].

More advanced models were presented by [12,13, 16,29,30]. However, they were also based on the assumption that particles do not interact mutually, what may cause prediction errors. To resolve this problem, models given by [31-33] included the particle interac–tions in the algorithm based on the numerical discrete element methods. Unfortunately, quantitative simula–tions of this kind need physical fertilizer characteristics as input data, which are at present difficult to measure [13].

In this work an analytical solution of the mechanical model, which describes motion of fertilizer particle along the straight radial vane of centrifugal spreader disc is presented. The motion of fertilizer particle is analyzed in the non-inertial reference system fixed to the spinning disc. Therefore, the carrying motion is accounted for by introducing the appropriate inertial forces. A number of assumptions have been introduced in the dynamic model of a particle motion in order to facilitate the problem solving. Consequently, the resul–ting equations and their solution represent an appro–ximation of real motion, which enables analysis of working parameters that govern the spreading process. Further confirmation of the presented model must be done the aim of having improved similarity between simulation and experiment [34].

2. MATERIAL AND METHODS

In the present work is presented an analytical model of a single fertilizer particle motion along a straight radial vane attached to a horizontal spreader disc. This is the first phase of a fertilizer spreading process, described using the laws of theoretical mechanics in non-inertial reference system [35,36]. The second phase, left out from this paper, comprehends the particle free flight.

Following methods of mathematical analysis [37], particle motion is described in this particular case by the non-homogeneous second order differential equation, which has the analytical solution. Calculations have been performed starting from the solution of this differential equation, using specific test data and MS Excel 2007. Therefore, presented approach does not impose the application of highly sophisticated software.

The second phase of spreading process, ballistic flight of the fertilizer particle through the atmospheric air until its lending, is left out from the consideration. However, the presented model provides an additional insight in the fertilizer particle moving over the spinning disc and its velocity vector at the leaving point from the vane. The information on this velocity vector is crucial for further calculations and model development related to the flight of the fertilizer particle through the air and, consequently, the fertilizer spreading pattern.

3. DYNAMIC MODEL

3.1 Assumptions

Presented equations have been developed under appropriate assumptions, which facilitated formulation of analytical model of the fertilizer particle motion.

Motion of individual fertilizer particle along the radial straight vane of horizontal rotating disk is analyzed assuming its pure rolling along the vane. Thus, interactions between particles are neglected, although they can have impact on the spreading process [13].

Spreading disc is rotating at constant angular velocity ω_D [s^{-1}] = const.

Fertilizer particles are homogeneous ideal spheres.

Particle rolls along the vane without slipping.

Following [23,27,29,30], bouncing of particles over the disc and its vanes was neglected. This way, it is assumed that the particle is carried out by the rotating vane along the whole travel on the disc.

3.2 Differential equation of the particle motion

The motion of the fertilizer particle is considered in two coordinate systems. The first, namely the absolute coordinate system *Oxyz*, is fixed to the machine chassis Figure 1, while the second is the relative reference system *Oξηζ*. Both reference systems have the so-called "right-handed" orientation, see Figure 2.

The relative reference system is fixed to the spreading disc and rotates together with the vane along which the observed fertilizer particle moves. Therefore, the coordinate system *Oξηζ* is a non-inertial reference system. This means that the Newton's second law can be applied only by introducing the additional imaginary "inertial" forces: centrifugal $\vec{F}_{CF}[N]$ and Coriollis force $\vec{F}_{COR}[N]$. Beside "inertial" forces, on the parti–cle also act the following real forces (Figure 2): the fric–tion forces exerted by rotating disc $\vec{F}_{FD}[N]$ and vane $\vec{F}_{FV}[N]$, the particle weight $\vec{G}[N]$,and orthogonal reactions of the disc and vane, $\vec{N}_D[N]$ and $\vec{N}_V[N]$.

Besides the vertical forces $\vec{N}_D[N]$ and $\vec{G}[N]$, all other forces exerting on the observed fertilizer particle lie in the horizontal plane, having radial direction, Figure 2. This statement is based on the assumption that disk lies in the horizontal plane. Taking into account the all forces acting on the particle gives the vector differential equation of particle motion in the non-inertial reference system of rotating disc:

$$m\frac{d^2\vec{r}}{dt^2} = \vec{F}_{FD} + \vec{F}_{FV} + \vec{F}_{CF} + \vec{G} + \vec{N}_D + \vec{N}_V + \vec{F}_{COR} \quad (1)$$

where m[kg] is the particle mass and $\vec{a^r} = \frac{d\vec{v^r}}{dt} = \frac{d^2\vec{r^r}}{dt^2}[m \cdot s^{-2}]$. is the vector of relative linear acceleration, which represents the first derivative of the relative particle velocity vector $\vec{v^r}$ and the second derivative of position vector $\vec{r^r}$ of the particle in the non-inertial (relative) reference coordinate system $Oxyz$.

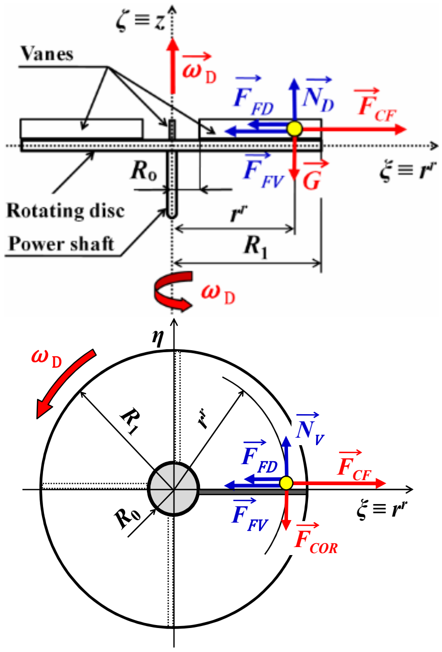


Figure 2. Forces exerting on the single spherical particleof fertilizer, during its movement along the rotating disk.

Vector differential equation (1) can be represented in relative coordinate system $O\xi\eta\zeta$ by scalar equations:

$$\xi \equiv r^r,\; m \cdot = \frac{d^2 r^r}{dt^2} = -F_{FD} - F_{FV} + F_{CF} \quad (2)$$

$$\eta:\; 0 = N_V - F_{COR} \Rightarrow N_V = F_{COR} \quad (3)$$

$$\xi \equiv z:\; 0 = N_D - G \Rightarrow N_D = G \quad (4)$$

The particle weight is the multiple of mass m [N] and the intensity of gravitational acceleration g [m · s^{-2}]. It is vertically aligned, oriented downward, oppositely to the axis $O\zeta$. Therefore vector of gravitational force is:

$$\vec{G} = m \cdot \vec{g} = -m \cdot g \cdot \vec{\zeta_0} \quad (5)$$

Coriollis force, arisen from combined particle moti–on, depends on the carrying angular velocity ω^c, which is the disk rotation velocity $\vec{\omega_D}$, and relative linear velocity $\vec{v^r}$ of the particle along the vane:

$$\vec{F}_{COR} = -2 \cdot m \cdot v^r \cdot \omega_D \cdot \vec{\eta_0} \quad (6)$$

where $\vec{\eta_0}$ is the unity ort-vector of axis $O\boldsymbol{\eta}$.

Force $\vec{N}_D$ is the orthogonal vane reactive force on the particle, arisen under influence of Coriolis force $\vec{F}_{COR}$ of the particle against the vane. Therefore, these forces are collinear but of opposite direction, Figure 3b:

$$\vec{N}_V = -\vec{F}_{COR} = -\left(-2v^r m\omega_D \vec{\eta_0}\right) = 2v^r m\omega_D \vec{\eta_0} \quad (7)$$

Similarly, vector $\vec{N}_D$ is the orthogonal reaction force of spreading disc on the fertilizer particle, arisen under influence of the particle weight $\vec{G}$ on this disc. These two forces are collinear, at opposite directions, i.e.:

$$\vec{N}_D = -\vec{G} = -\left(-m \cdot g \cdot \vec{\zeta_0}\right) = m \cdot g \cdot \vec{\zeta_0} = mg\vec{k} \quad (8)$$

Centrifugal force exerted to a fertilizer particle

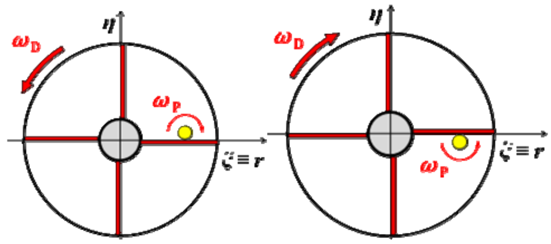
$$\vec{F}_{CF} = m \cdot \omega_D^2 \cdot r^r \cdot \vec{\xi_0} \quad (9)$$

depends on the particle mass m[kg], its current radius r^r [m] along the vane (i.e. the distance from the axis of disc rotation $O\zeta \equiv Oz$, and square of the disc rotational velocity ω_D [s^{-1}].

It is assumed that the particle rolls against the vane (around the vertical spinning axis) and slides over the spreading disc at the contact point. Friction force F_{FD} caused by particle sliding over the spreading disc is a function of dynamic friction coefficient μ[-], and normal reaction N_D[N] of the disc toward the particle:

$$\vec{F}_{FD} = -\eta \cdot N_D \cdot \vec{\xi_0} = -\eta \cdot m \cdot g \cdot \vec{\xi_0} \quad (10)$$

Based on the assumption that the spherical particle of fertilizer rolls against the vane, there are two possible directions of spreading disc rotation: counter clockwise and clockwise direction. However, Figure 3, directions of $\vec{\omega_p}$ and $\vec{\omega_D}$ are opposite in both cases.



(a) (b)

Figure 3. Relation between directions of the disc angular velocity $\vec{\omega_D}$ and angular velocity $\vec{\omega_p}$ of the rolling particle: (a) counter clockwise and (b) clockwise rotating disc.

Therefore, for clockwise rotating disc, the angular velocity vector $\vec{\omega_p}$ is aligned with the positive direction of $\zeta \equiv z$ axis. However, for the counter clockwise rotation of the disk, $\vec{\omega_p}$ is oriented in the opposite direction with respect to $\zeta \equiv z$ axis. Thus, the angular velocity vector of the particle can be represented as:

$$\vec{\omega_P} = \left(0, 0, \pm\omega_P\right)\left[s^{-1}\right] = \pm\omega_P \cdot \vec{\zeta_0} = \pm\omega_P \cdot \vec{k} \quad (11)$$

The assumption on pure fertilizer particle rolling motion along the vane, gives the following relationship:

$$r_P \cdot \omega_P = v^r = \frac{dr^r}{dt} = \dot{r}^r \quad (12)$$

Rolling of the particle along the vane is generated by torque of two parallel forces of opposite directions:

sliding friction force $\vec{F}_{FV}\,[N]$ exerted by the vane on the particle and centrifugal inertial force exerted on the particle $\vec{F}_{CV}\,[N]$, Figure 4. The torque arm equalsthe particle radius r_P. The rolling moment is:

$$M_P = r_P \cdot F_{FV} \tag{13}$$

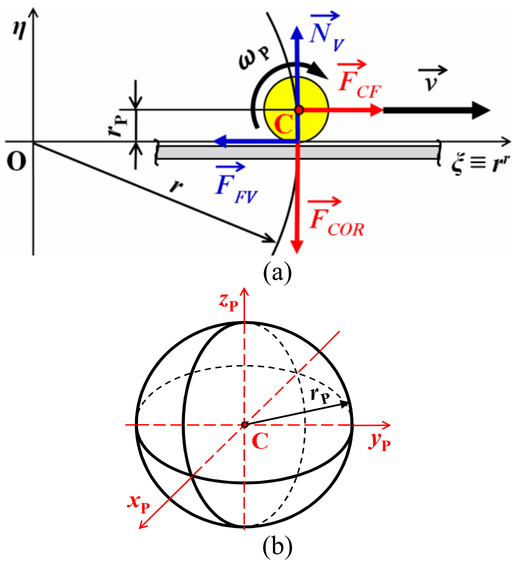


Figure 4. (a) Forces of the spinning torque of fertilizer particle: $\left(\vec{F}_{FV}, \vec{F}_{CF}\right)$; (b) Ideal spherical model particle.

Where F_{FV} is the intensity of the force $\vec{F}_{FV}$. To prevent sliding (i.e. to ensure pure rolling) of the fertilizer particle along the vane, intensity of the sliding friction force $\vec{F}_{FV}$ must not exceed the critical value $F_{\mu CRIT}$:

$$F_{FV} \le F_{\mu CRIT} = \eta \cdot N_V = \eta \cdot F_{COR} \tag{14}$$

Spinning of the particle around own centroidal axis Cz_P, which travels together with that particle along the vane, follows the well known law of Dynamics: *the rate of change of the axial angular momentum* L_{Cz_P} *of solid body equals the appropriate moment of force* $M_{Cz_P}\left(\vec{F}_{FV}\right)$ *defined with respect the same axis* Cz_P:

$$r^r = C_1 y_1(t) + C_2 y_1(t) = C_1 e^{\sqrt{\frac{5}{7}}\cdot\omega_D\cdot t} + C_2 e^{-\sqrt{\frac{5}{7}}\cdot\omega_D\cdot t} \tag{15}$$

For ideal spherical particle, having radius r_P:

$$I_{Cz_P} = \frac{2}{5} m r_P^2 \left[kgm^2\right] \Rightarrow F_{FV} = \frac{2}{5} m \frac{d\omega_P}{dt} \cdot r_P \tag{16}$$

If the fertilizer particle does not disintegrate, what means $r_P = const$, combining (16) with (12) gives:

$$F_{FV} = \frac{2}{5} \cdot m \cdot \frac{d^2 r^r}{dt^2} = \frac{2}{5} \cdot m \cdot \ddot{r}^r = \frac{2}{5} \cdot m \cdot a^r \tag{17}$$

Finally, including (9), (10) and (17) in (2) gives the second-order linear inhomogeneous ordinary differential equation with constant coefficients that describes relative motion of fertilizer particle along the horizontal radial vane of a spreading disc:

$$\frac{d^2 r^r}{dt^2} - \frac{5}{7} \cdot \omega_D^2 \cdot r^r = -\frac{5}{7} \cdot \mu \cdot g \tag{18}$$

3.3 Solution of the differential equation of motion

The homogeneous part of differential equation (18) is

$$\frac{d^2 r^r}{dt^2} - \frac{5}{7} \cdot \omega_D^2 \cdot r^r = 0 \tag{19}$$

having the following general solution

$$r^r = C_1 y_1(t) + C_2 y_2(t) = C_1 e^{\sqrt{\frac{5}{7}}\cdot\omega_D\cdot t} + C_2 e^{-\sqrt{\frac{5}{7}}\cdot\omega_D\cdot t} \tag{20}$$

Unknown integration constants C_1 and C_2 can be evalu– ated from the initial conditions of motion.

The solution of inhomogeneousordinary differential equation (18) can be found by the *constants variation method*, representing its general solution by expression:

$$r^r = C_1(t) \cdot \underbrace{e^{\sqrt{\frac{5}{7}}\cdot\omega_D\cdot t}}_{y_1(t)} + C_2(t) \cdot \underbrace{e^{-\sqrt{\frac{5}{7}}\cdot\omega_D\cdot t}}_{y_2(t)} \tag{21}$$

It includes the coefficients $C_1(t)$ and $C_2(t)$, as functions of time evaluated using the following two expressions:

$$\dot{C}_1(t) \cdot y_1(t) + \dot{C}_2(t) \cdot y_2(t) = 0 \tag{22}$$

$$\dot{C}_1(t) \cdot \dot{y}_1(t) + \dot{C}_2(t) \cdot \dot{y}_2(t) = F(t) \tag{23}$$

In the present case, they have the forms:

$$\dot{C}_1(t) \cdot \underbrace{e^{\sqrt{\frac{5}{7}}\cdot\omega_D\cdot t}}_{y_1(t)} + \dot{C}_2(t) \cdot \underbrace{e^{-\sqrt{\frac{5}{7}}\cdot\omega_D\cdot t}}_{y_2(t)} = 0 \tag{24}$$

$$\dot{C}_1 \underbrace{e^{\sqrt{\frac{5}{7}}\cdot\omega_D e^{\sqrt{\frac{5}{7}}\cdot\omega_D\cdot t}}}_{\dot{y}_1(t)} - \dot{C}_2(t) \cdot \underbrace{e^{-\sqrt{\frac{5}{7}}\cdot\omega_D e^{\sqrt{\frac{5}{7}}\cdot\omega_D\cdot t}}}_{\dot{y}_2(t)} = \tag{25}$$

$$= \underbrace{-\frac{5}{7}\mu g}_{F(t)}$$

giving

$$C_1(t) = \frac{\mu \cdot g}{2 \cdot \omega_D^2} \cdot e^{-\sqrt{\frac{5}{7}}\cdot\omega_D t} + k_1 \tag{26}$$

$$r^r(t) = \left(R_0 - \frac{\mu \cdot g}{2 \cdot \omega_D^2}\right) \cdot \cosh\left(\sqrt{\frac{5}{7}}\omega_D \cdot t\right) + \frac{\mu \cdot g}{2 \cdot \omega_D^2} \tag{27}$$

After introducing expressions (26) and (27) in (21), and rearranging, the general solution of the inhomogeneous ordinary differential equation that governs relative motion of the fertilizer particle along the vane arises:

$$r^r(t) = k_1 \cdot e^{\sqrt{\frac{5}{7}}\cdot\omega_D t} + k_2 \cdot e^{-\sqrt{\frac{5}{7}}\cdot\omega_D t} + \frac{\mu \cdot g}{2 \cdot \omega_D^2} \tag{28}$$

Derivation of this equation gives the relative radial velocity of the particle $v^r(t)$:

$$\dot{r}^r(t)=\sqrt{\frac{5}{7}}\cdot\omega_D\left(k_1 e^{\sqrt{\frac{5}{7}}\cdot\omega_D t}-k_2 e^{-\sqrt{\frac{5}{7}}\cdot\omega_D t}\right) \tag{29}$$

Fertilizer particle starts relative radial motion along the vane from its inner edge, accelerating from the zero radial velocity. Combining the initial conditions

$$\dot{r}^r(t_0=0)=\dot{r}^r(0)=\dot{r}_0^r=v_0^r=0 \tag{30}$$

$$\dot{r}^r(t_0=0)=\dot{r}^r(0)=\dot{r}_0^r=v_0^r=0 \tag{31}$$

and formulas (28-29) gives the integration constants k_1 and k_2:

$$k_1=k_2=\frac{1}{2}\cdot\left(R_0-\frac{\mu\cdot g}{2\cdot\omega_D^2}\right) \tag{32}$$

Therefore, the particular solution of second-order linear inhomogeneous ordinary differential equation with constant coefficients (18) is:

$$\overrightarrow{v^a}(t)=\overrightarrow{v^c}(t)+\overrightarrow{v^r}(t) \tag{33}$$

Expression for velocity is the first derivation of (33):

$$v^a(t)=\sqrt{\left[v^c(t)\right]^2+\left[v^r(t)\right]^2} \tag{34}$$

Ejection time t_k is identical to traveling time of fertilizer particle along the vane. This period begins at the moment $t_0 = 0$, when the particle touches down the vane (assuming no bouncing), and ends at the moment $t = t_K$ when the particle leaves the spreader. This time can be evaluated by setting $r^r(t_K) = R^1$ in (33):

$$t_K=\sqrt{\frac{5}{7}}\cdot\omega_D\cdot a\cosh\left(\frac{R_1-\dfrac{\mu\cdot g}{2\cdot\omega_D^2}}{R_0-\dfrac{\mu\cdot g}{2\cdot\omega_D^2}}\right) \tag{35}$$

3.4 Absolute velocity vector of fertilizer particle

Absolute velocity $\overrightarrow{v^a}$ of the particle is a vector sum of carrying velocity $\overrightarrow{v^c}$ induced by vane and disc spinning, and relative velocity along the vane $\overrightarrow{v^r}$, Figure 5:

$$\overrightarrow{v^a}(t)=\overrightarrow{v^c}(t)+\overrightarrow{v^r}(t) \tag{36}$$

Relative velocity vector is aligned with the positive direction of Oξ axis and the carrying velocity is aligned with the positive orientation of axis Oη. The magnitude of relative velocity, as function of time, is defined by (34). Thus, its magnitude at particle ejection point can be calculated by substituting t = tK, from (35), in (34):

$$v^r(t_K)=\sqrt{\frac{5}{7}}\omega_D\left(R_0-\frac{\mu\cdot g}{2\cdot\omega_D^2}\right)\sinh\left(\sqrt{\frac{5}{7}}\omega_D\cdot t_K\right) \tag{37}$$

Magnitude v^c of the carrying velocity vector is equal to the multiple of particle radius r^r and angular velocity ω_D = const of carrying motion (disc rotation):

$$v^c(t)=r^r(t)\cdot\omega_D \tag{38}$$

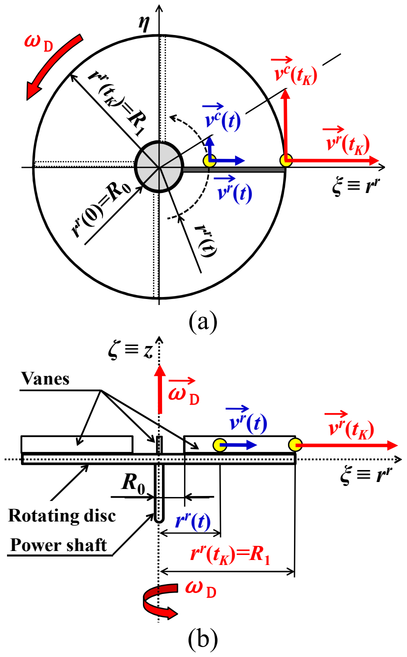


Figure 5. Components of the absolute velocity vector of the fertilizer particle at the ejection point when it leaves the spinning disc: (a) overhead view; (b) side view.

It follows:

$$v^c(t_K)=r^r(t_K)\omega_D=R_1\omega_D \tag{39}$$

Absolute velocity vector is defined by its magnitude:

$$v^a(t)=\sqrt{\left[v^c(t)\right]^2+\left[v^r(t)\right]^2} \tag{40}$$

The absolute velocity at ejection point is:

$$v^a(t_K)=\sqrt{\left[v^c(t_K)\right]^2+\left[v^r(t_K)\right]^2} \tag{41}$$

having the orientation angle

$$\alpha_v=a\tan\left\{\frac{v^c(t_K)}{v^r(t_K)}\right\}[-]=\frac{360}{2\pi}\cdot a\tan\left\{\frac{v^c(t_K)}{v^r(t_K)}\right\}[^\circ] \tag{42}$$

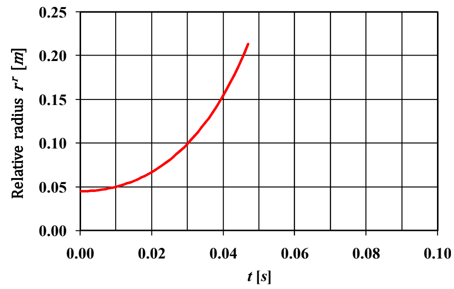
Knowing the disc angular velocity $\omega_D[s^{-1}]$ = const, the angle of disk rotation $\theta(t_K)[^\circ]$ during the particle travelling time t_K [s] can be evaluated:

$$\theta(t_K)=\frac{360}{2\pi}\cdot\omega_D\cdot t_K[^\circ]=6\cdot n_\perp D\left[\mathbf{o}\cdot\min^{\mathrm{T}}(-1)\right] \tag{43}$$

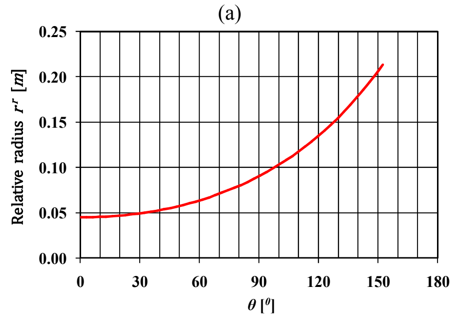
where $n_\perp D\left[\mathbf{o}\cdot\min^{\mathrm{T}}(-1)\right]$ is the disk rotation rate. This way, the position of the particle drop-out point from the hoper can be optimized.

4. NUMERICAL RESULTS

Particle kinematic parameters werecalculated using pre–sented analytical model and the following input data: n = 540 [min^{-1}]; R_0 = 0,045 [m]; R_1 = 0,21 [m]; μ = 0,3 [-]; g = 9,81 [m·s^{-2}]. Figures 6, 7 and 8 present variations of these kinematic parameters of the fertilizer particle during its motion along the vane.



(a)

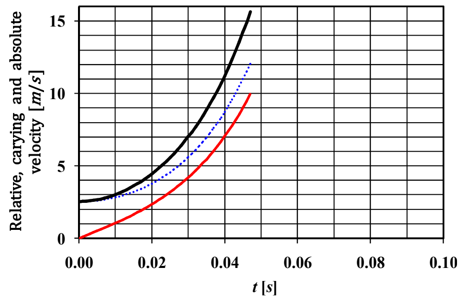


(b)

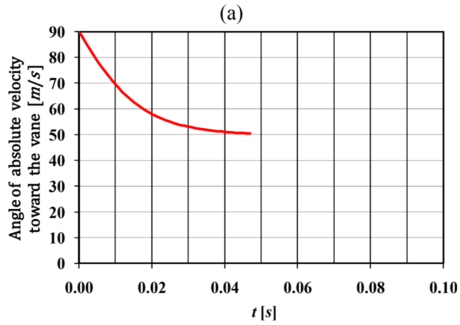
Figure6. Radial position of the fertilizer particle ($r^r(t)$), as function of: (a) the travelling time and (b) the angle of disc (vane) rotation in the absolute reference system.

Changes of the radial distance r^r [m] of the particle, with respect to travelling time t [s] and the angle of disk rotation θ [°], are shown in Figures 6a and 6b, respectively. The particle reaches the outer disk edge at radius R_1 = 0,21 [m] and leaves the disk after t_K = 0.047 [s]. During the particle travelling, spreader disc spins around the vertical axis outlining the angle of $\theta[(t)]_k$ = 152.3 [°].

Figures 7a and 8a present variations of the magni–tudes of relative $v^r(t)$, carrying $v^c(t)$ and absolute velocity $v^a(t)$ of the fertilizer particle, as functions of the travelling time t and the particle radial position r^r, respectively. As it can be seen in these figures, absolute velocity changes its magnitude starting from 2,54 [m/s] and up to final value of 15,64 [m/s].

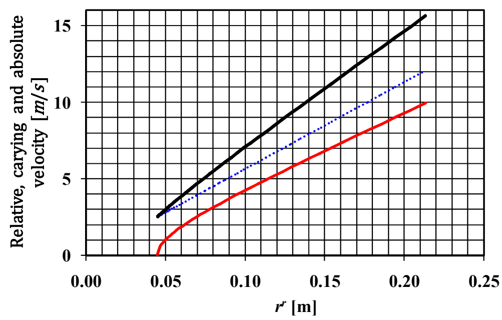


(a)

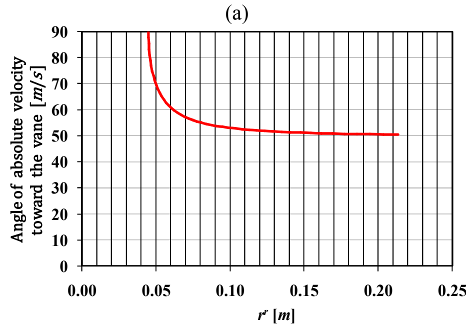


(b)

Figure 7. Variations of kinematic parameters of the particle motion along the vane as functions of travelling time: (a) relative $v^r(t)$——, carrying $v^c(t)$······· and absolute velocity $v^a(t)$——; (b) orientation angle $\alpha_v(t)$ of absolute velocity toward the vane ——.



(a)



(b)

Figure 8. Variations of kinematic parameters of the fertilizer particle motion along the vane as functions of the radial position of the particle: (a) relative $v^r(t)$——, carrying $v^c(t)$······· and absolute velocity $v^a(t)$——; (b) orientation angle $\alpha_v(t)$ of absolute velocity toward the vane ——.

Dependence of the orientation angle α_v of absolute velocity toward the vane, on the travelling time t and the particle position r^r, are presented in figures 7b and 8b. At the point of initial contact of the particle with the vane, vector of absolute velocity $\vec{v^a}$ is orthogonal to the vane, because it has the only one non-zero component, the carrying velocity vector $\vec{v^c}$. As the particle accelerates in time, α_v decreases to the final value of $\alpha_v(t_K)$ = 50.5 [°] at the edge of disc and vane.

5. CONCLUSION

In this paper, the motion of a solitary fertilizer particle along a horizontal straight radial vane of a centrifugal disc spreader is analyzed. Under the introduced assumption that particle rolls without sliding along the vane, presented final analytical expressions enable estimation of the basic kinematic parameters of the particle motion while it is in contact with disc. This way, information on the radial position of the fertilizer particle r^r along the vane is provided through two forms: with respect to travelling time (Figure 6a), and with respect to the angle of disc (vane) rotation in the absolute reference system(Figure 6b). Variations of the magnitudes of the relative v^r, carrying v^c and absolute v^a velocity, against travelling time t (Figure 7a), and relative radius r^r (Figure 8a) were also evaluated. Orientation of the absolute velocity vector of the particle, based on its angle α_v toward the vane, is shown in Figures 7b and 8b.

For testinput data of n = 540 [min^{-1}]; R_0 = 0,045 [m]; R_1 = 0.21 [m]; μ = 0.3 [-]; g = 9.81 [m·s^{-2}], travelling time was found to be t_K = 0.047 [s] and spreader disc spinning angle around the vertical axis was $\theta[(t)]_k$ = 152.3 [°]. During this temporal interval, fertilizer particle accelerates up to the absolute velocity magnitude of 15,64 [m/s] at ejection point. During the acceleration period, the angle α_v starts from α_v (t = 0) = 90 [°], when $\overrightarrow{v^a}$ is orthogonal toward the vane, and decreases nearly asymptotically to the value of $\alpha_v[(t)]_K = 50.5$ [°].

The authors believe that presented procedure may facilitate design and working regime optimization of centrifugal disk spreaders having radial vanes, as well as the planning of experiments related to centrifugal disc spreaders. It can be also incorporated into automatic control systems of centrifugal disc spreaders, in order to improve their efficiency under variable site-specific conditions and demands.

It should be also noted that presented approach does not impose the application of highly sophisticated software. The application of widely used low-cost program MS Excel 2007 (or later version) is quite appropriate for this purpose.

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NOMENCLATURE

$\vec{F}_{FD}$, $\vec{F}_{FV}$	friction forces exerted by the disc and by the vane on the particle [N];
$\vec{F}_{CF}$, $\vec{F}_{COR}$	centrifugal and Coriollis force, exerted on the particle [N];
$\vec{g}$, $\vec{G}$	gravitational acceleration [m·s^{-2}] and weight [N] of the fertilizer particle;
m	mass of fertilizer particle [kg];
$\vec{N}_D$, $\vec{N}_V$	orthogonal reaction forces of the disc and vane on the particle [N];
R_0, R_1	entrance and exit radius of the particle from the disc rotation axis [m];
r^r	radial distance from disc centre in the relative ("***r***") system O$\xi\eta\zeta$[m];
x,y,z	particle coordinates in the coordinate system of spreader chassis[m];

Greek symbols

ξ,η,ζ	coordinates in the non-inertial rotating (relative) system fixed to disk [m];
θ	disc spinning angle [-], [^{0}]
ω_D	angular velocity of rotating disc [s^{-1}].

ПРЕДВИЂАЊЕ КОТРЉАЊА ЧЕСТИЦЕ ЂУБРИВА ДУЖ ЛОПАТИЦЕ ЦЕНТРИФУ–ГАЛНОГ РАСИПАЧА СА ДИСКОМ

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У раду је анализирано кретање идеализоване, лоптасте, хомогенечестице минералног ђубрива дуж радијалне лопатице постављене на ротирајућем диску центрифугалног расипача ђубрива. Анализа, заснована на претпоставци о котрљању честице дуж лопатице без клизања, изведена је у неинерцијалном референтном координатном систему који ротира заједноса диском за расипање ђубрива. Кретање честице дуж лопатице је описано хиперболичном косинусном функцијом, која представља решење обичне хомогене диференцијалне једначине другог реда са константним коефицијентима, која описује динамику честице. Решење ове врсте представља апроксимацију стварног кретања честице ђубрива дуж лопатице диска који ротира константном угаоном брзином. Међутим, може бити веома корисно за оптимизацију дизајна центрифугалног распршивача и његових радних параметара, као и за даљу анализу читавог процеса расипања ђубрива који обухвата и лет честице у ваздуху по напуштању лопатице.