Symbolic Regression Metamodel Based Multi-Response Optimization of EDM Process

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1. INTRODUCTION

The electric discharge machining (EDM) was initially found by an English chemist J. Priestly in the year 1770 to cause an erosive effect over the work material. After many years later in 1943 the great scientist Lazarenko invented an EDM to cut the extremely hard material like tungsten [1]. In EDM spark is used to make the intrinsic shape in any electrically conducting material irrespective of their hardness and strength [2]. By using the ordinary tool any complex shape can be made with high precision [3]. In EDM the tool and work material submerged in a dielectric fluid and the material removal takes place due to the spark erosion. By the application of voltage, the potential difference developed between the tool and work piece. Due to the potential difference the high velocity of electron moves towards the workpiece which helps to erode it [4]. The dielectric fluid is neutral in nature and it evaporates due to the leakage current by the moving electron. Due to moving electron, a plasma channel created between the workpiece and tool, and it creates a crater in both tool and work piece. Once the plasma channel extinguishes the flow of dielectric fluid removed material is flushed [5].

The EDM performances can be analysed by material removal rate (MRR), surface roughness (SR), electrode wear rate (EWR), tool wear rate (TWR) etc. Ideally the desire of the experiment is to maximize the MRR to enhance the productivity with a least surface finish. The discharge energy released during sparking is directly related to the SR and MRR [6]. At higher discharge energy the MRR is more but it severely affects the SR [7]. The discharge energy is directly related to the input parameters like discharge current, pulse on time, pulse off time gap voltage etc [8]. Generally, increasing the discharge current, gap voltage, pulse frequency, pulse duration leads to increase the MRR but it lowers the surface finish [4]. Like MRR the TWR is also an important parameter for the calculation of machining cost in EDM. The discharge current, gap voltage, pulse on-time and pulse off-time are the fundamental machining parameters involved in EDM. Gap voltage is the voltage applied between the electrode and the workpiece during the EDM process. The current applied to the electrode during pulse on-time is referred to as discharge current. Pulse on-time is the time for which the current is applied to the electrode during each EDM cycle [5]. Pulse off-time is the waiting time between two pulse on-times and during that time the particles are removed from the setup.

The effect of process parameters on the performance of EDM is very vital for the researchers and for its analysis a parametric study is very much essential. The parametric analysis can be done by changing the input parameter while keeping the other variable constant. However, to make a clear analysis a huge number of experiments have to be carried out which increase the cost and time. Additionally, it is also vital to find out the combination of input parameters to get an optimum result. A metamodel can be used to find out the combination of all the input parameters to achieve the aimed response parameters. The most widely used metamodeling technique for the optimization of manufacturing/machining process is the Response Surface Methodology (RSM), which is essentially a polynomial regression approach [9-13]. Some authors have also used

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some advanced metamodeling techniques like artificial neural network (ANN) [14-16], ANFIS [17], gene expression programming (GEP) [3] in the machining operation. By developing the metamodel it can be coupled with a global optimization algorithm to find the optimum machining parameters. The Metaheuristic algorithm is the most commonly used optimization algorithm and out of all metaheuristic algorithm genetic algorithms (GA) are perhaps the most popular and have been applied to all classes of optimization problems with a better success. Ragavendran et al. [4] developed an RSM model by using a Box-Behnken design (BBD) set to optimize the current, pulse off-time and pulse on-time. Subsequently, they used a GA to minimize SR and maximize the MRR Kumaran et al. [18] relied on Grey fuzzy optimization to optimize deburring in CFRP. Hourmand et al used the RSM technique to metamodeling to express MRR and EWR as a function of voltage, duty factor current and pulse on time [19]. Chiang et al. [20] also used RSM to find the influence of pulse on time, current, duty factor and discharge voltage on SR, MRR and EWR. RSM was used in combination with NSGA-II by Baraskar et al. [21]. Hewidy [22] too used RSM to optimize the process parameters of wire electrical discharge machining (WEDM) process. Świercz et al. [23] also took the help of RSM metamodeling to build empirical relationships between current, pulse on time, pulse off time with MRR, SR and white layer thickness. Maity and Mishra [24] used an ANN to build a metamodel to predict the MRR, Recast Layer thickness and overcut effect. An ANN metamodel was also used by Yusoff et al. [25] for predicting SR MRR, sparking gap and cutting speed.

Due to the excellent mechanical properties, Inconel based alloys are widely used in different areas like aeronautical, chemical, nuclear and medical industries. Inconel has excellent resistance to corrosion, creep and fatigue at high temperatures. Thakur et al. [26] carried out an observation which confirms that more than 50% of the materials used in the aeroplane engines are made of Inconel alloys. Out of all the Inconel alloys Inconel 718 is the most popular and widely used material for the manufacturing of turbine disks, blades, combustors, etc. [27]. Due to the low thermal conductivity of Inconel alloys a very high temperature is induced while grinding and cutting the material which is a serious problem [28]. It is really very tough to machine the Inconel alloys by using the conventional machining process due to its high strength to weight ratio. EDM can be extensively used for the machining Inconel alloys due to its non-contact material removal mechanisms and the low process forces of EDM produce burr-free intricate shape with a very high aspect ratio [29].

In the present research, copper is taken as an electrode material for the machining of Inconel 718. Further, it is seen that researchers so far have relied mainly on RSM for building empirical relationships between input and output parameters of EDM process. Thus, in this work, a novel metamodeling approach-genetic programming (GP) based on advanced machine learning features is used. By comparing with a traditionally applied metamodeling technique- RSM, the comparative advantages of GP are highlighted. MRR and TWR are experimentally studied based on a BBD design. The GP metamodels for MRR and TWR are coupled with a GA to carry out multi-objective Pareto optimization. As compared to a straightforward single objective optimization, multi-response Pareto optimization are more beneficial because this presents the user with a host of candidate solutions where a suitable compromise between the maximized MRR and minimized TWR is obtained.

### Table 1. Factors and levels used in the research.

<table>
<thead>
<tr>
<th>Factors</th>
<th>Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current (I)</td>
<td>4</td>
</tr>
<tr>
<td>Pulse on-time (T_on)</td>
<td>50 125 200</td>
</tr>
<tr>
<td>Pulse off-time (T_off)</td>
<td>50 85 120</td>
</tr>
</tbody>
</table>

### 2. MATERIALS & METHODS

#### 2.1 Design of Experiments

For the purpose of building predictive models or metamodels a Box-Behnken design (BBD) is used in this study to conduct a set of EDM experiments that would serve as the training data. The EDM experiments are conducted at three different levels of each input parameter (current (I), pulse-on-time (T_on) and pulse-off-time (T_off)). Table 1 lists the factors and levels used in this research. Material removal rate (MRR) and tool wear rate (TWR) are considered as the response variables (i.e. the outputs). The experimentation data as per BBD design is reported in Table 2.

#### 2.2 Experimental Details

In this research, a Die sinking EDM is used for the machining of the Inconel 718. A cylindrical copper tool with a diameter of 2 mm is used as an electrode for the machining of the work piece. The work piece and electrode are separated by a moving dielectric fluid EDM oil. The Inconel 718 has a chemical composition of C-0.8%, Mn-0.35%, Ni- 54%, Cr- 20%, Ti- 0.75% with balanced Fe.

Material Removal Rate (MRR) and Tool Wear Rate (TWR) in the experiments are calculated by the weight loss method using precision electronic balance weighing...
machine with a least count of 1 mg. MRR is calculated by measuring the weight loss of work piece as per the eqn. (1),

$$MRR = \frac{w_b - w_a}{t}$$  \hspace{1cm} (1)$$

where \(w_b\) and \(w_a\) are the weight of work-piece before and after machining respectively. \(t\) is the machining time in minutes.

TWR is calculated by measuring the weight loss of tool as per the eqn. (2),

$$TWR = \frac{t_{wb} - t_{wa}}{t}$$  \hspace{1cm} (2)$$

\(t_{wb}\) and \(t_{wa}\) are the weights of the tool before and after machining respectively.

Each experiment is carried out thrice and its average is considered as the working value, to account for uncertainties in experimentation. The average experimental values of MRR and TWR along with the standard deviations of the experiments are presented in Figure 1.

![Figure 1. Average experimentally recorded values along with their standard deviations for (a) MRR (b)TWR.](image)

2.3 Response Surface Methodology

Response Surface Methodology (RSM) is a combination of statistical and mathematical techniques that reduce a complex input-output system into easily understandable polynomial equations. Using the training dataset RSM fits a curve of the a priori selected polynomial form. In general, the RSM fitted empirical equation may be represented as,

$$y = f(x_1, x_2, x_3, ..., x_k) + \varepsilon$$  \hspace{1cm} (3)$$

where \(f\) denotes the approximate response surface and \(\varepsilon\) is the normally distributed statistical error. \(x's\) represent each independent parameter (inputs) while \(k\) is the maximum number of independent parameters.

In this research, a second order polynomial model of the following form is selected a priori,

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \sum_{j=1}^{k} \beta_{ij} x_i x_j + \varepsilon$$ \hspace{1cm} (4)$$

Here \(\beta's\) are the coefficients of regression. These coefficients of regression help in describing the response \((y)\) as a function of predictor variables \((x's)\).

Using the BBD design points listed in Table 2, an RSM model is fitted based on multiple regression fitting scheme. The difference between the predicted value \((\hat{y}_i)\) and the actual experimental value \((y_i)\) is called residue.

$$e_i = y_i - \hat{y}_i$$  \hspace{1cm} (5)$$

The coefficients of regression in eqn. (4) are selected such that the sum of squared residuals (SSR) is minimum. Since residuals are the errors in the fitting, sometimes SSR is also referred to as the sum of squared error in predictions (SSE).

$$SSE = \sum_{i=1}^{n} e_i^2$$  \hspace{1cm} (6)$$

ANOVA is then performed to identify and eliminate the non-significant terms of the fitted RSM model [30-31].

2.4 Genetic Programming

Genetic Programming (GP) is a powerful computational intelligence technique that can perform symbolic regression to build explanatory models based on the provided training datasets. It is a highly automated process that requires no manual intervention once the algorithm is started. On the contrary, a polynomial regression carried out by RSM may require the use of additional statistical tests like ANOVA to determine the significance of the polynomial terms in the model. The GP on the other hand self-prunes the insignificant terms because of the inherent evolutionary traits.

GP starts with a randomly generated population of candidate solutions called individuals or GP trees. This population is called as generation zero. Each GP tree is made up of two key ingredients - functions and terminals. Mathematical operators like +, -, *, /; trigonometric functions like sin, cos etc., exponential functions, logarithmic function etc. form the functions. Terminals, on the other hand, are comprised of constants and variables. Fitness for each GP tree of the population is calculated. The fitness is calculated using some predefined metric like mean squared error or coefficient of determination. In case of using mean squared error as the metric lower values are considered better. Similarly, in the case of the coefficient of determination higher the values, the better. Next, the population in generation zero is improved by using three key genetic operators called selection, crossover and mutation. An additional genetic property called elitism is also used in the current work. Crossover is the process of randomly grafting chosen parts from one GP to another GP tree. This is done by associating a higher probability of selection for the crossover of higher fitness GP trees. Mutation is used to create new GP trees for the new population by randomly altering a small part or node of the selected GP tree. Elitism is the act of copying a small proportion of the fittest GP trees, unchanged into the next generation. This improved population makes up the subsequent generation. This process is repeated until a predefined accuracy on the metric scale or the maximum number of generations is reached.
2.5 Optimization using Genetic Algorithm

In this work, multi-objective Pareto optimization is carried out by using Genetic Algorithm (GA). GA and GP work on the same concept i.e. Darwin’s laws of natural selection [32]. Instead of trying to build a function relation like GP, GA is engaged in exploring the search space to find a suitable combination of input parameters that would maximize or minimize the target response.

Like GP, GA also starts by initiating a random population and calculating its fitness. Based on the probability associated with each fitness, individuals are selected for operation by various genetic operators. New individuals are created for the next generation by crossover (random combination of parts from two parents to form two children) and mutation (random small modifications in individuals created by the flipping of the binary encoded genes). These operations are carried out until the maximum iterations are reached [33]. While in any single objective optimization, the search for optima is straightforward; in case of multi-objective Pareto optimization a non-dominated sort algorithm is used to rank the optimal solutions such that each solution in Pareto set is not dominated by any other solution.

Table 2. Experimental results and metamodel predictions for training dataset.

<table>
<thead>
<tr>
<th>Exp. No</th>
<th>MRR (mm3/min)</th>
<th>TWR (mm3/min)</th>
<th>Exp. RSM</th>
<th>GP</th>
<th>Exp. RSM</th>
<th>GP</th>
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<tbody>
<tr>
<td>1</td>
<td>2.3621</td>
<td>2.3887</td>
<td>0.9986</td>
<td>0.9981</td>
<td>0.5270</td>
<td>0.9981</td>
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<td>2</td>
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<tr>
<td>3</td>
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<td>0.0643</td>
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</table>

3. RESULTS & DISCUSSION

3.1 Comparative Assessment of Metamodels

The EDM experiments listed in Table 2 are used for training the RSM and GP based MRR and TWR metamodels. The RSM is trained using DesignExpert™, a popular statistical package and the GP is realized by using an author compiled FORTRAN program. The predicted values of the MRR and TWR are listed in Table 3. The iterative improvement of the GP metamodel across the training generations is seen in Figure 2. In Figure 2(a), it is seen that the GP best fitness for MRR increases sharply till 10th generation and achieves about 90% R². Beyond these a gradual increase in its best fitness is seen till it achieves a near ideal fitness of 99.27%. The RSM metamodel on the other hand achieves a maximum R² of 97.4%. Similarly, for the TWR GP metamodel, stepwise improvement in the best fitness is seen. For the TWR, RSM and GP metamodels have a R² of 83.85% and 99.74% respectively. In Figure 2(b) it is seen that for both MRR and TWR, the GP average fitness increases rapidly up to generation 20, beyond which it becomes very the increase becomes gradual. The iterative improvement of average fitness eventually becomes sluggish. This means that the selected number of 50 generation limit is appropriate as training beyond this would perhaps lead to over fitting. A comparative plot of predictions made by RSM and GP metamodels against the experimental points is shown in Figure 3. It is seen that for MRR prediction both metamodels have similar prediction capability, with perhaps GP being marginally better. However, for TWR, GP is seen to be significantly better than RSM.

The prediction power of the metamodels is further assessed using two different error metrics—MAE (mean absolute error) and RMSE (root mean squared error). It is seen from Table 4 that the GP has marginal improvement of 1.78% over RSM on MRR metamodel, however in case of TWR the improvement is about 21%. Similar drastic improvements in MAE and RMSE are also seen.

Table 4. Relative improvement of GP over RSM metamodels.

<table>
<thead>
<tr>
<th>Response</th>
<th>Metric</th>
<th>RSM</th>
<th>GP</th>
<th>Relative % improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRR</td>
<td>R²</td>
<td>0.9742</td>
<td>0.9915</td>
<td>1.78</td>
</tr>
<tr>
<td></td>
<td>MAE</td>
<td>0.0454</td>
<td>0.0230</td>
<td>49.29</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>0.0561</td>
<td>0.0312</td>
<td>44.39</td>
</tr>
<tr>
<td>TWR</td>
<td>R²</td>
<td>0.8270</td>
<td>0.9973</td>
<td>20.59</td>
</tr>
<tr>
<td></td>
<td>MAE</td>
<td>0.1332</td>
<td>0.0143</td>
<td>89.26</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>0.2562</td>
<td>0.0190</td>
<td>92.59</td>
</tr>
</tbody>
</table>
3.2 Effect of current, pulse on-time and pulse off-time on MRR and TWR

Due to the non-linear relation between the input and response parameters in the experiments it is difficult to predict the response parameter for a particular set of input parameters. In this work, current ($I$), pulse-on-time ($T_{on}$) and pulse-off-time ($T_{off}$) are the input parameters. The combined effect of these three input parameters on MRR and TWR in the form of surface plots is shown in Figure 4 and Figure 5. The surface plots are drawn by varying any two process parameters between their experimental range, while the third parameter is held constant at its mean level. As seen from Figure 4(a) and 4(b), the increase in temperature, in general, increases the MRR. Pulse off time seems to have the least influence on the MRR among the three selected parameters. Similarly, in Figure 5, higher TWR are seen to be associated with higher current values. Also, higher current on time is seen to aid in the tool wear. For machining of any material TWR is considered as one of the significant response parameters in EDM process. The TWR is directly related to the MRR and practically those experiment having higher MRR will in general have higher TWR and vice-versa.

3.3 Mult-Objective Optimization

A multi-objective genetic algorithm is used for finding a set of Pareto optimal solutions to maximize the MRR and minimize the TWR simultaneously. It is clear from the study of the involved independent parameters that both these objectives are contradictory. For example, while the high current could increase the MRR, it would also increase the TWR. Because of such conflicting agendas, single-objective optimization may not always be the best approach. Pareto optimal solution or the Pareto front represents a set of non-dominated solutions, mathematically each of which is a viable compromise. The Pareto front for MRR versus TWR is shown in Figure 6. It is clear from the Pareto front that the absolute maximum MRR and absolute minimum TWR cannot be achieved simultaneously, because with increasing MRR, TWR increases. Thus, the user is left with an option to choose what is best for the particular application. Using this Pareto curve, the user can choose any combination of input parameters, that he/she feels works best for the problem without violating any set constraints. The selection of the appropriate Pareto solution can be done based on prior experience or the help of multi-criteria decision-making approaches like TOPSIS, PSI etc. can also be employed.

4. CONCLUSION

In this research, a traditionally hard-to-machine material is machined using EDM. To maximize the productivity of the EDM process and minimize the tool costs, a metamodel coupled with global optimization approach is employed. To maximize productivity, material removal rate (MRR) is sought to be maximized. On the other hand, to minimize the tool costs, tool wear rate (TWR)
is sought to be minimized. Using genetic programming (GP) metamodeling approach, MRR and TWR are expressed as functions of current, pulse on-time and pulse off-time. As compared to fixed-form polynomial RSM metamodels the genetically searched form-free GP metamodels were seen to perform about 2% and 21% better for MRR and TWR respectively. The evaluation of the error metrics also showed significant improvement for the GP metamodels. Thus, the present GP based metamodeling approach can be suitably used for predictive modeling of machining processes.

REFERENCES


ОПТИМИЗАЦИЈА ВИШЕСТРУКОГ ОДГОВОРА КОД ЕДМ ПОСТУПКА КОРИШЋЕЊЕМ МЕТА-МОДЕЛА СИМБОЛИЧКЕ РЕГРЕСИЈЕ

Р.К. Гхадаи, К. Калита, К-З. Гао

Електроерозивна обрада (EDM) је популаран поступак обраде који има широку употребу код тешко обрадљивих и критих материјала. Није потребан резни алат и може да се користи код обрадака сложене геометрије. Међутим, недостаци су мала брзина скидања материјала и претерано хабање алате. Рад покушава да реши наведене слабости применом мета-модела заједно са своебухватном оптимизацијом у циљу предвиђања одговарајућих комбинација улазних параметара (струја, успостављање и гашење електричног лука), што би довело до повећања брзине скидања материјала и хабања алате свело на минимум. Мета-модели су развијени коришћењем нове симболичке регресије базиране на генетском програмирању. После компаративне евалуације у односу на конвенционалне мета-моделе методологије одговора површине, мета-модели генетског програмирања показују бољи и прецизнији прорачун. Мета-модели генетског програмирања су затим повезани са генетским алгоритмом у циљу вишеструку ономатеизације EDM поступка.