Some Applications of a Novel Desirability Function in Simultaneous Optimization of Multiple Responses

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In the framework of multi-response optimization techniques, the optimization methodology based on the desirability function is one of the most popular and most frequently used methodologies by researchers and practitioners in engineering, chemistry, technology and many other fields of science and technique. Numerous desirability functions have been introduced to improve the performance of this optimization methodology. Recently, a novel desirability function for multi-response optimization is proposed, which is smooth, nonlinear, and differentiable, and thus more suitable for applying some of the more efficient gradient-based optimization methods. This paper evaluates the performance of the proposed method through six real examples. After a comparative analysis of the results, it is shown that the proposed method in a certain measure outperforms the other competitive optimization methods.

Keywords: *desirability function, multi-response optimization, design of experiments*

1. INTRODUCTION

In general, there are two categories of optimization techniques. The first deals with single optimization problems, while the second solves multi-response optimization (MRO) problems.

However, in practice, each product/process/system consists of many quality characteristics (responses) that need to be optimized. In such circumstances, it is not reasonable to optimize each response separately, since an improvement in one response, as a rule, degrades at least one or several of the remaining responses.

Therefore, in real world problems, MRO involve the simultaneous optimization of many different and often conflicting responses. In such problems, there is no unique optimal solution, but rather a set of alternative solutions.

There are many different MRO optimization techni–ques, but most of them are very complex and sophisti–cated [1-5]. Therefore, researchers and practitioners have a lot of difficulty in selecting the appropriate technique for a specific problem at hand.

Desirability function methodology is an easy-to-use and well-established approach and, as such, quite acceptable to many researchers and practitioners.

The desirability function-based optimization appro–ach combines desirability function (DF) and design of experiments (DOE). The application of this metho–dology is not limited to specific optimization problems; on the contrary, it is one of the most widely used methodologies in many fields of science, research and development [5-8].

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It is commonly known that there is no efficient general-purpose universal optimization method. Among the many optimization techniques presented for desirability functions that can be employed to solve MRO problems, the direct search (DS) method and its modifications are still the "first resort methods" [9], and sometimes the only options for solving a large class of optimization problems.

In many cases the DS algorithms can provide acceptable approximation solutions, but it has the propensity to fall into the trap of a local optimum (i.e. it is prone to converge to a non-stationary point), so it is necessary that the algorithm is starting in multiple initial points into the feasible space in an attempt to find the best (global) optimum. However, in the desirability function methodology such a danger is not large, since the subregion of interest for optimization is relatively narrow, certainly less than the experimental region considered. Some of the DS methods (e.g. Nelder-Mead simplex method) are very popular nowadays within the research and scientific community because they are simple, flexible, easy to understand and relatively easy to implement.

Commercially available software products usually do optimization of desirability functions by derivate free search methods, or gradient-based methods.

It is necessary to note that some alternatives to desirability function-based approach have been develo–ped, such as gray rational analysis, physical progra–m–ming, vectorial optimization, quality loss function app–roach, principal component analysis, process capa–bility index-based aproach, and their hybrid variants [9-13].

In recent years, considerable attention has been paid by researchers and practitioners towards employing some successful metaheuristic search techniques for global optimization, such as genetic algorithm (GA), simulated annealing (SA) algorithm, particle swarm optimization (PSO) algorithm , ant colony optimization

(ACO) algorithm, tabu search (TS) algorithm, artificial bee colony (ABC) algorithm, differential evolution (DE), including the later developed cuckoo search (CS) algorithm, imperialist competitive algorithm (ICA) teaching-learning-based optimization (TLBO) method, gray wolf optimizer (GWO), and many others, which can provide the best feasible solution of an optimization problem [14-20]. At the present time, among these algorithms, the classical genetic algorithm and its hybrid variants are still the most popular techniques for various optimization problems [15], [21], especially in material processing technologies [16], [22], [23]. Many of the above mentioned algorithms are used for single and/or multi-response optimization.

In some multi-response optimization problems, the graphical approach can be very useful when two design factors (input variables) are considered and the number of responses (output variables) is not too large. In such cases, the contour plot methodology allows to find visually the optimal conditions that simultaneously satisfy all the involved responses. For three design factors, utilizing a large number of successful iterations, the near-optimal solution can be found. When the number of design factors is greater than three, the graphical approach becomes impractical.

The remainder of this paper is organized as follows. In the second Section is presented a novel desirability function in detail. In the third section, the optimization procedure is formulated. To verify the proposed approach, six examples are considered in the fourth Section. In the fifth Section different optimization results are compared and discussed. Finally, brief concluding remarks are given in the sixth Section.

2. DESIRABILITY FUNCTION

Obviously, any type of free-form transformation (exponential, power, logarithmic, logistic) may be employed as a desirability function [5, 24-29].

Hitherto, many desirability functions have been introduced to improve the performance of the desirability-based optimization method.

Recently, a novel desirability function for multi-response optimization was recommended [29], which is a smooth, nonlinear, and differentiable function at target points. This desirability function was defined as:

$$d_j(\psi_j) = \begin{cases} (1-\psi_j^2)^{r_j}, & if\ -1 \le \psi_j \le 1 \\ 0, & otherwise \end{cases} \ ; \ d_j \in [0,1] \quad (1)$$

where $\psi_j = f(\hat{y}_j(\mathbf{x}))$ is the dimensionless converting function, which linearly converts the predicted (fitted) responses into coded variables $j = (1, 2, ..., m)$, $\hat{y}_j(\mathbf{x})$ is the j th estimated response function, $\mathbf{x}$ is the vector of the coded input variables $(x \in R^n)$, r_j is the shape parameter $(r_j > 0)$.

The individual desirability functions are defined according to the nature of the responses to be optimized, which are usually classified in three main categories:

1. The nominal-the-best (NTB);
2. The larger-the-better (LTB);
3. The smaller-the-better (STB).

Note: For the one-sided desirability function $r_j = r_j^s$ (LTB) or $r_j = r_j^t$ (STB), and for the two-sided symmetrical desirability function (NTB) $r_j = r_j^s = r_j^t$.

A graphical representation of these functions is given in Figure 1.

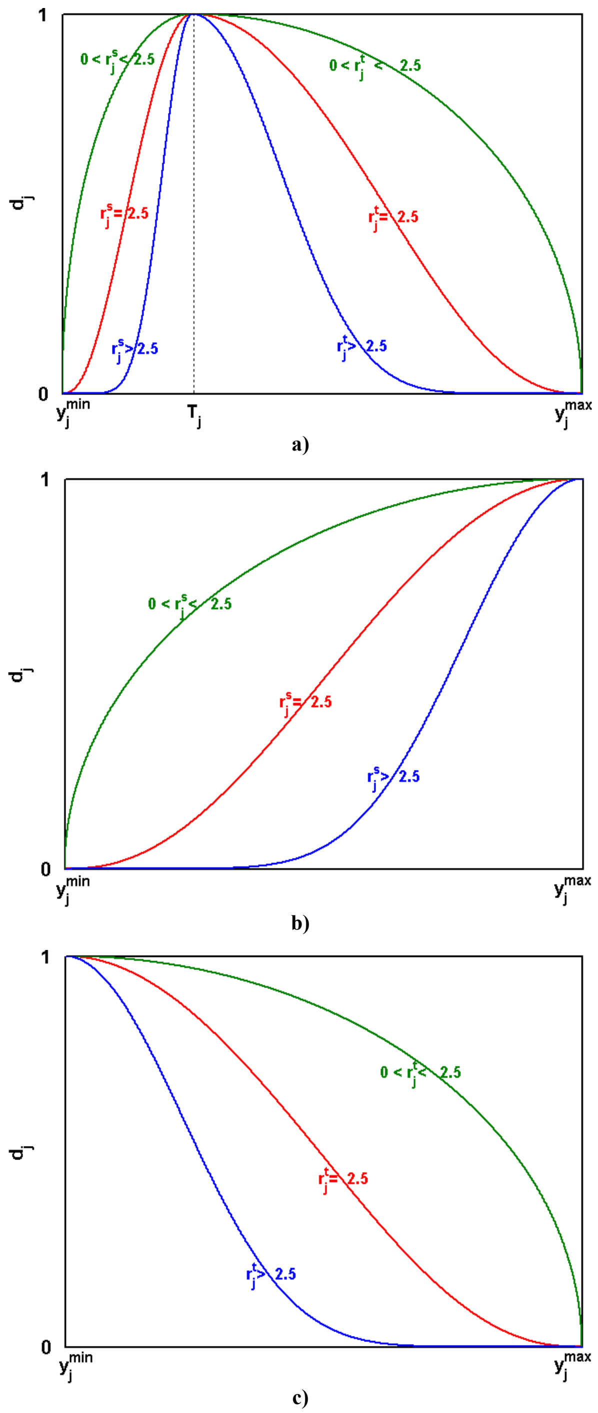

Figure 1. Graphical representation of desirability function: a) NTB, b) LTB, c) STB

Depending on whether a particular response has to be minimized, maximized or assigned to a certain target value, the desirability function from equation (1) can be expressed as follows:

- for the NTB type response,

$$d_j(\psi_j) = \begin{cases} 0, & if\ \psi_j < -1 \\ (1-\psi_j^2)^{r_j^s}, & if\ -1 \le \psi_j \le 0 \\ (1-\psi_j^2)^{r_j^t}, & if\ \ 0 \le \psi_j \le 1 \\ 0, & if\ \psi_j > 1 \end{cases} \quad (2)$$

with,

$$\psi_j\left(\hat{y}_j(\mathbf{x})\right) = \begin{cases} \dfrac{\hat{y}_j(\mathbf{x}) - T_j}{T_j - y_j^{\min}}, & if\ y_j^{\min} \le \hat{y}_j(\mathbf{x}) \le T_j \\ \dfrac{\hat{y}_j(\mathbf{x}) - T_j}{y_j^{\max} - T_j}, & if\ T_j \le \hat{y}_j(\mathbf{x}) \le y_j^{\max} \end{cases}$$

- for the LTB type response,

$$d_j(\psi_j) = \begin{cases} 0, & if\ \psi_j < -1 \\ (1-\psi_j^2)^{r_j^s}, & if\ -1 \le \psi_j \le 0 \\ 1, & if\ \psi_j > 0 \end{cases} \quad (3)$$

with,

$$\psi_j\left(\hat{y}_j(\mathbf{x})\right) = \frac{\hat{y}_j(\mathbf{x}) - y_j^{\max}}{y_j^{\max} - y_j^{\min}}, \quad if\ y_j^{\min} \le \hat{y}_j(\mathbf{x}) \le y_j^{\max}$$

- for the STB type response,

$$d_j(\psi_j) = \begin{cases} 1, & if\ \psi_j < 0 \\ (1-\psi_j^2)^{r_j^t}, & if\ 0 \le \psi_j \le 1 \\ 0, & if\ \psi_j > 1 \end{cases} \quad (4)$$

with,

$$\psi_j\left(\hat{y}_j(\mathbf{x})\right) = \frac{\hat{y}_j(\mathbf{x}) - y_j^{\min}}{y_j^{\max} - y_j^{\min}}, \quad if\ y_j^{\min} \le \hat{y}_j(\mathbf{x}) \le y_j^{\max}$$

where $y_j^{\min}$ and $y_j^{\max}$ are the lower and the upper bound (specification limits) on the j th response, res–pectively, r_j^s and r_j^t are the j th adjustable shape para–meters, and T_j is the target value of the j th response which is a NTB- type one.

For some reason, the decision maker (DM) may decide that a response has to be "in range". In such a case, the individual desirability function is the simplest, namely $d_j \equiv 1$ for $y_j^{\min} \le \hat{y}_j(\mathbf{x}) \le y_j^{\max}$ (otherwise, $d_j \equiv 0$).

The upper bound $y_j^{\max}$ denotes a large enough value for the LTB type response, whereas the lower bound $y_j^{\min}$ denotes a small enough value for the STB type response. In other words, these bounds are target values for LTB- and STB-type responses. If r_j^s or r_j^t increases, the desirability function becomes more convex with more emphasis to the target; on the contrary, if r_j^s or r_j^t decreases, the desirability function becomes more concave with less emphasis to the target.

In general, in both the one-sided desirability functions (STB or LTB) and the two-sided desirability function (NTB) is $r_j^s \ne r_j^t$.

2.1 Determination of bounds

The bounds of design factors x_i are defined when selecting the experimental matrix, so they are known during the optimization process.

The bounds (and targets) of each response $y_j(\mathbf{x})$ should be determined in advance to define individual desirability functions. These bounds may be chosen arbitrarily in a different way, for instance, on the basis of product/process operating limits, the decision maker's subjective choice, consensus of experts, etc.

Instead of using the preference information about responses a priori (or in absence of such information), the physical range of individual response can be used to determine bounds as follows:

$$y_j^{\min} \approx \min(\hat{y}_j(\mathbf{x})),\ j = 1,2,3,....,m \quad (5)$$

$$y_j^{\max} \approx \max(\hat{y}_j(\mathbf{x})) \quad (6)$$

The bounds determined by equations (5) and (6) can represent the extreme values of the estimated (or experimental) responses within the entire investigated space Ω .

In that case, after the calculation phase, the DM will decide whether the current solution with the adopted bounds is satisfactory or not [30].

2.2 Determination of shape parameters

The shape parameters in desirability functions r_j^s and/or r_j^t can be chosen arbitrarily as described in pre–vious chapter for the bounds. However, these para–meters can be determined in a less arbitrary manner.

Namely, the individual desirability function is completely defined by just one point on the desirability curve (see Figure1). That point, say $a_0\{\psi_{j,0}, d_{j,0}\}$ represents the critical value of a response and its corresponding degree of desirability (satisfaction).

The unknown shape parameter is computed easily from the equation:

$$r_j = \frac{\ln d_{j,0}}{\ln(1-\psi_{j,0}^2)} \quad (7)$$

A researcher or a group of experts may choose any desirable value for each involved response, assign to it an adequate desirability, and then apply equation (7).

Table 1 shows a simplified desirability scale with qualitative (linguistic) interpretation of desirability values, according to Harrington and other researchers [7, 27, 31].

The rule is simple; a high value of the response (close to the target value) corresponds to a high value of desirability, and vice versa. In this way, it is most likely that a different shape parameter for each individual desirability function will be obtained.

Table 1. Desirability scale

d	Qualitative interpretation	
1.0-0.8	very high	fully acceptable (excellent)
0.8-0.6	high	acceptable
0.6-0.4	middle	satisfactory
0.4-0.2	low	acceptable/unacceptable
0.2-0.0	very low	completely unacceptable

Certainly, the shape parameter must be chosen carefully, since its different values produce different forms of desirability curve as can be seen from Figure 1.

For instance, large shape parameters $(r_j > 2.5)$ indicate that only response values near its target value provide a specified high desirability, while small shape parameters $(r_j < 2.5)$ provide high desirability in a wide range of response values around the target value.

In that sense, a shape parameter $r_j = 2.5$ can be accepted as the “default'' shape parameter.

2.3 Overall desirability

Harrington [27] defined an average desirability, called composite or overall desirability, as the geometric mean of the individual desirability functions:

$$D(\mathbf{y}) = \left[\prod_{j=1}^{m} d_j(y_j)\right]^{\frac{1}{m}} \tag{8}$$

where m is the number of responses.

Using this aggregated function, the multiple quality characteristics are converted into an equivalent single-quality characteristic, which is to be optimized. On the other hand, it should be taken into account that the overall desirability is a complex, nonlinear, and some–times, multimodal function (because of curvature in the response surfaces and their combination into the desi–rability function).

Obviously, the overall desirability also ranges between zero and one. If any individual desirability d_j of the corresponding response $\hat{y}_j$ becomes zero, then the overall desirability D also becomes zero, inde–pendently of the values of the remaining individual desirability functions.

For the product or process development it means that if only one of several quality characteristics is outside of the specified limits, the considered process or product is unacceptable to users (“all or nothing”).

Principally, the higher value of D, the better compromise exists among the multiple responses.

In real circumstances, it is quite likely that there is a difference in the importance levels (priorities) of dif–ferent responses, i.e. that one response has a greater impact on the product or process in comparison with the others.

For that reason, Derringer and Suich [28] proposed an extended overall desirability as the weighted geo–metric mean of the individual desirability functions:

$$D(\mathbf{y}) = \left[\prod_{j=1}^{m} d_j(y_j)^{w_j}\right]^{\frac{1}{\sum w_j}} \tag{9}$$

where w_j is the user-specified weight of the j-th predicted response.

If all the weights are the same, equation (9) is reduced to equation (8). This suggests that both functions have similar characteristics. Usually, the exponents of the individual desirability functions in the equation (9) are given in the normalized form, which satisfies the conditions $\sum w_j = 1$ and $w_j \in (0,1)$.

Equation (9) should be used when at least one response is of greater importance than the others. The weights can be specified by the DM next to the shape parameters in the desirability functions. In doing so, the DM must take into account the relative importance of the responses with respect to each other.

3. MULTI-RESPONSE OPTIMIZATION PROCEDURE

As noted above, using this methodology, the comp–licated multiple optimization problem is transformed into a simplified single optimization problem. In such a case, the aim of optimization procedure is to find the optimal vector of design factors $\mathbf{x}^*$ which maximize the overall desirability D.

There are many numerical techniques that can be used to solve this nonlinear constrained optimization problem. Sometimes these techniques are referred to as nonlinear programming methods.

In this case, the standard nonlinear constrained optimization procedure can be formally expressed as follows:

Maximize, $D\{\mathbf{y}(\mathbf{x})\}$

$$\text{subject to: } y_j^L \le y_j(\mathbf{x}) \le y_j^U \quad (j = 1, 2, ..., m) \tag{10}$$

$$x_i^L \le x_i \le x_i^U \quad (i = 1, 2, ..., n)$$

where the superscripts L and U denote the lower and upper bounds of the functions $y_j(\mathbf{x})$ and variables x_i, respectively.

The constraints $y_j(\mathbf{x})$ in equation (10) can be linear or nonlinear functions, and, as a rule, they are given in explicit form. In the general case, these constraints can be one-sided or two-sided constraints.

According to the priority-based approach, instead of overall desirability, the most important response (primary response) is used as the objective function and the rest of $(m-1)$ responses (secondary responses) are considered as inequality constraints.

This is an essential different concept from the desi–rability function methodology. Certainly, this approach offers less chance of achieving a good compromise among the predicted responses.

4. ILLUSTRATIVE EXAMPLES

The applicability, efficiency and accuracy of the proposed method will be demonstrated on six examples that have been widely studied in the literature [28,30, 32-43].

4.1 Example 1

Firstly, the famous 'tire tread compound problem', originally discussed by Derringer and Suich [28], will be presented. In this example the tire tread performance were characterized via four responses (output variables).

These responses are PICO abrasion index (y_1), 200% modulus (y_2), elongation at break (y_3), and hardness (y_4). As process factors (input variables) three chemical ingredients were selected, namely, hydrated silica (x_1), silane coupling agent (x_2), and sulfur (x_3).

The central composite design (with six center points and six star points) was employed for experimentation. The cubical experimental region Ω was given as $-1.633 \le x_i \le 1.633 (i=1,2,3)$. All the data are shown in Table 2.

Table 2. Design matrix and experimental results

Coded design factors			Responses			
x_1	x_2	x_3	y_1	y_2	y_3	y_4
-1	-1	1	102	900	470	67.5
1	-1	-1	120	860	410	65.0
-1	1	-1	117	800	570	77.5
1	1	1	198	2294	240	74.5
-1	-1	-1	103	490	640	62.5
1	-1	1	132	1289	270	67.0
-1	1	1	132	1270	410	78.0
1	1	-1	139	1090	380	70.0
-1.633	0	0	102	770	590	76.0
1.633	0	0	154	1690	260	70.0
0	-1.633	0	96	700	520	63.0
0	1.633	0	163	1540	380	75.0
0	0	-1.633	116	2184	520	65.0
0	0	1.633	153	1784	290	71.0
0	0	0	133	1300	380	70.0
0	0	0	133	1300	380	68.5
0	0	0	140	1145	430	68.0
0	0	0	142	1090	430	68.0
0	0	0	145	1260	390	69.0
0	0	0	142	1344	390	70.0

For each response the same experimental region was employed to fit the full second-order regression model as follows:

$$\hat{y}_1(\mathbf{x}) = 139.12 + 16.49x_1 + 17.88x_2 + 10.91x_3 - 4.01x_1^2 - 3.45x_2^2 - 1.57x_3^2 + 5.13x_1x_2 + 7.13x_1x_3 + 7.88x_2x_3$$

$$\hat{y}_2(\mathbf{x}) = 1261.11 + 268.15x_1 + 246.50x_2 + 139.48x_3 - 83.55x_1^2 - 124.79x_2^2 + 199.17x_3^2 + 69.38x_1x_2 + 94.13x_1x_3 + 104.37x_2x_3$$

$$\hat{y}_3(\mathbf{x}) = 400.38 - 99.67x_1 - 31.40x_2 - 73.92x_3 + 7.93x_1^2 + 17.31x_2^2 + 0.43x_3^2 + 8.75x_1x_2 + 6.25x_1x_3 + 1.25x_2x_3$$

$$\hat{y}4(\mathbf{x}) = 68.91 - 1.41x_1 + 4.32x_2 + 1.63x_3 + 1.56x_1^2 + 0.06x_2^2 - 0.32x_3^2 - 1.63x_1x_2 + 0.13x_1x_3 - 0.25x_2x_3$$

Whenever there is no significant difference between a full and incomplete mathematical model, the full model should be selected. In any case, a polynomial of higher degree (such as a second-order model) is a reasonable approximation of the true response surface over the relatively small (feasible) region of the design factors (which is a typical case in the optimization methodology applying the desirability function). In any case, polynomial models are often preferable, because these models are much easier to fit and work with than complicated nonlinear models.

Based on the decision maker's preference infor–mation, the following limitations for each response are given, as follows:

$$\begin{aligned} & y_1 > 120, \\ & y_2 > 1000, \\ & 400 < y_3 < 600, \\ & 60 < y_4 < 75. \end{aligned} \tag{11}$$

In the investigated experimental space Ω, this ine–qualities define nonempty feasible region Ω' $(\Omega' \subset \Omega)$.

Instead the preference requirements, the bounds of the four responses $(y_j^{\min}, y_j^{\max})$ were adopted based on the physical ranges (according the aforementioned concept), this is, (90, 190) for y_1, (1000, 1500) for y_2, (350, 650) for y_3, and (60, 75) for y_4.

Note: If no explicit specifications from the DM are available in advance, then this (no-preference) approach remains the only starting option.

The same shape parameters were selected for each of the individual desirability functions $r_j^s = r_j^t = 2.5$.

Following the conditions and limitations adopted previously, the optimization procedure was carried out. After optimization procedure, the initial solution at the optimal point $\mathbf{x}^* = \{x_1^*, x_2^*, x_3^*\} = \{0.126, 0.406, -1.241\}$, with the corresponding estimated response values $\hat{\mathbf{y}}^* = \{y_1^*(\mathbf{x}), y_2^*(\mathbf{x}), y_3^*(\mathbf{x}), y_4^*(\mathbf{x})\} = \{127.1, \quad 1443.0, \quad 469.3, 68.0\}$, and the overall desirability $D^* = 0.702$was found.

All the estimated response values satisfy inequalities (11).

Sometimes the solution obtained by applying the procedure described above does not meet the desired specifications. On the other hand, regardless of the satisfactory solution, the DM may require a better and more reliable solution. In such a case, the problem can be resolved using the shape-based and/or bound-based interactive desirability function method proposed by Jeong and Kim [26, 30].

By using this method the researchers attempt to find the most favorable compromise solution (the best one) in a small number of iterations. However, if the number and magnitude of preference parameters and their combinations that must be specified are large, then the number of iterations may be very large. Because it is actually a trial-and-error procedure, it is not easy for DM to find the most appropriate solution. In practice, the DM usually relies only on a free assessment, own intuition and some known solutions from the literature.

The simplest way to change the shape of individual desirability functions is to vary the shape parameters. In this paper, a systematic method for determining the shape parameter values in the individual desirability functions will be presented.

This procedure is based on a simulated experiment and response surface methodology (RSM) [44], and in this paper will be demonstrated on the Example 1.

For further analysis (to maintain consistency in the comparison of competitive methods), the original bounds and targets of each response were accepted.

Table 3 shows the design matrix and computing results for the simulated experiment (see Appendix). In this experiment the design factors are the shape parameters r_1, r_2, r_3, and r_4 $(0.5 \le r_k \le 10)$, and the response is the overall desirability D_r.

The data collected in Table 3 were employed for fitting the pure quadratic regression model (with original design factors) of the form:

$$y = 0.9602 - 0.1616\, r_1 + 0.01\, r_2 - 0.0334\, r_3 + 0.0049\, r_4 + 0.0087\, r_1^2 - 0.0002\, r_2^2 + 0.0018\, r_3^2 - 0.0005\, r_4^2 \quad (12)$$

The optimization task is to find the point, in which the response (overall desirability) attains the maximum.

The optimal solution was found at the point $r^* = \{r_1^*, r_2^*, r_3^*, r_4^*\} = \{0.5,\ 10.0,\ 0.5,\ 4.9\}$, with response $y^* \equiv D_r^* = 0.957$.

Table 3. Taguchi L₉ (3⁴) orthogonal array with response

Coded (Original) factors				Response
$A(r_1)$	$B(r_2)$	$C(r_3)$	$D(r_4)$	$y=D_r$
1(0.5)	1(0.5)	1(0.5)	1(0.5)	0.87258
1(0.5)	2(5.25)	2(5.25)	2(5.25)	0.81508
1(0.5)	3(10)	3(10)	3(10)	0.80757
2(5.25)	1(0.5)	2(5.25)	3(10)	0.22572
2(5.25)	2(5.25)	3(10)	1(0.5)	0.24913
2(5.25)	3(10)	1(0.5)	2(5.25)	0.42725
3(10)	1(0.5)	3(10)	2(5.25)	0.07515
3(10)	2(5.25)	1(0.5)	3(10)	0.23734
3(10)	3(10)	2(5.25)	1(0.5)	0.17013

For the new shape parameters, after repeated computing procedure, the new optimal setting $\mathbf{x}^*$ = {0.020, 0.116, -0.806}, with the corresponding esti–mated responses $\hat{\mathbf{y}}^* = \{130.8, 1299.2, 454.6, 67.9\}$, and the overall desirability $D^* = 0.860$ were obtained.

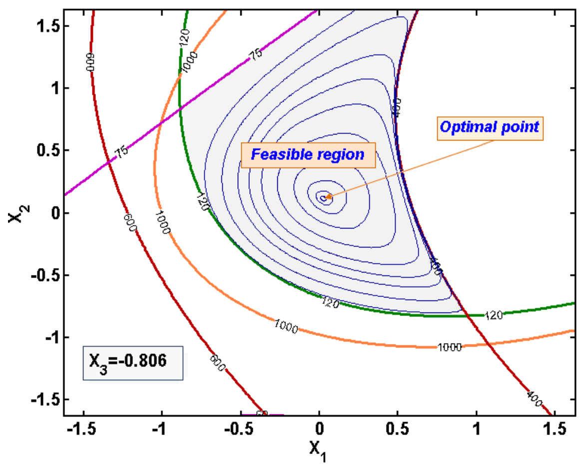

Figure 2. Overlaid contour plot for three design factors and four response constraints with x3: sulfur at optimal level

It can be said that the new solution gives a better balanced relationship between the responses.

For this improved solution, Figure 2 shows the contour plot for four responses with a (shaded) feasible region and contour lines of constant overall desirability around the optimum point.

This graphic representation may be helpful for better understanding of the situation.

4.2 Example 2

This example, which was originally discussed by Del Castillo et al. [32], refers to a wire-bonding process in the semiconductor industry.

The manufacturer must put together a module in a pre-molded package by bonding wires between the leads (position A) and the silicon chips (position B).

The design (control) factors that influence the tem–perature at the wire bond are the N2 flow rate (x_1), the N2 temperature (x_2), and the heater block temperature (x_3).

First three responses are maximum temperature (y_1), beginning bond temperature (y_2), and finish bond temperature (y_3) - at position A. Three other responses are maximum temperature (y_4), beginning bond temperature (y_5), and finish bond temperature (y_6) - at position B.

For more details, readers are referring to the original paper [32].

The design factors and their chosen levels used in the experiment are listed in Table 4. The bounds of design factors define the region of exploration.

Table 4. Factors and their levels

Design factors		Units	Factor levels		
			Lower (-1)	Middle (0)	Upper (+1)
x_1	Flow rate	SCFM	40	80.0	120
x_2	Flow temper.	°C	200	325	450
x_3	Block temper.	°C	150	250	350

The design used to collect the experimental data was a non-composite Box-Behnken design (BBD) with three replicated runs at the center point, for all six responses. The data are summarized in Table 5.

Table 5. Design matrix (BBD) and experimental results

Coded factors			Responses					
x_1	x_2	x_3	y_1	y_2	y_3	y_4	y_5	y_6
-1	-1	0	139	103	110	110	113	126
1	-1	0	140	125	126	117	114	131
-1	1	0	184	151	133	147	140	147
1	1	0	210	176	169	199	169	171
-1	0	-1	182	130	122	134	118	115
1	0	-1	170	130	122	134	118	115
-1	0	1	175	151	153	143	146	164
1	0	1	180	152	154	152	150	171
0	-1	-1	132	108	103	111	101	101
0	1	-1	206	143	138	176	141	135
0	-1	1	183	141	157	131	139	160
0	1	1	181	180	184	192	175	190
0	0	0	172	135	133	155	138	145
0	0	0	190	149	145	161	141	149
0	0	0	180	141	139	158	140	148

Table 6 depicts the minimum, maximum, and cor–responding target values for the observed responses.

It is interesting to observe that in this case all six responses belong to the NTB-type response. Also, for the given target values for the responses y_2, y_3, y_5, and y_6, the two-sided desirability functions d_2, d_3, d_5, and d_6, are the asymmetrical curves. It is observed from Table 6 that the specification limits of all the response variables are tight as compared to experimental response space (given in Table 5).

Table 6. Type of response and their specifications

Responses	Type	y_j^{min}	T_j	y_j^{max}
y_1	NTB	185	190	195
y_2	NTB	170	185	195
y_3	NTB	170	185	195
y_4	NTB	185	190	195
Y_5	NTB	170	185	195
y_6	NTB	170	185	195

Also, the second-order polynomial model was fitted to each of the six responses as follows:

$$\hat{y}_1(\mathbf{x}) = 180.67 + 2.50x_1 + 23.38x_2 + 3.63x_3 - 5.58x_1^2 - 6.83x_2^2 + 1.67x_3^2 + 6.25x_1x_2 + 4.25x_1x_3 - 19.00x_2x_3$$

$$\hat{y}_2(\mathbf{x}) = 141.67 + 6.00x_1 + 21.63x_2 + 14.13x_3 - 2.58x_1^2 - 0.33x_2^2 + 1.67x_3^2 + 0.75x_1x_2 + 0.25x_1x_3 + 1.00x_2x_3$$

$$\hat{y}_3(\mathbf{x}) = 139.00 + 6.63x_1 + 16.00x_2 + 20.38x_3 - 6.13x_1^2 + 1.63x_2^2 + 4.88x_3^2 + 5.00x_1x_2 + 0.25x_1x_3 - 2.00x_2x_3$$

$$\hat{y}_4(\mathbf{x}) = 158.00 + 8.50x_1 + 30.63x_2 + 7.88x_3 - 13.25x_1^2 - 1.50x_2^2 - 4.00x_3^2 + 11.25x_1x_2 + 2.25x_1x_3 - 1.00x_2x_3$$

$$\hat{y}_5(\mathbf{x}) = 139.67 + 4.25x_1 + 19.75x_2 + 16.50x_3 - 5.83x_1^2 + 0.17x_2^2 - 0.83x_3^2 + 7.00x_1x_2 + 1.00x_1x_3 - 1.00x_2x_3$$

$$\hat{y}_6(\mathbf{x}) = 147.33 + 4.50x_1 + 15.63x_2 + 27.38x_3 - 4.42x_1^2 + 0.83x_2^2 - 1.67_3^2 + 4.75x_1x_2 + 1.75x_1x_3 - 1.00x_2x_3$$

The ''default'' shape parameters were set in this example, for all the individual desirability functions in equations (2). Also, Harrington equation (8) for overall desirability was used.

The objective of multi-response optimization in this example is to make the responses as close as possible to their predefined target values (by maximizing the overall desirability).

After optimization procedure, the optimal solution at the point $\mathbf{x}^* = \{0.591, 0.794, 1.000\}$, with the corre–sponding estimated response values $\hat{\mathbf{y}}^* = \{190.1, 178.4, 180.7, 191.5, 174.7, 189.6\}$, and the overall desirability $D^* = 0.589$ was found.

All the estimated response values satisfy the requirements defining in Table 6.

Figure 3 shows the contour plot for six responses and contour lines of constant overall desirability around the optimum point inside the feasible region.

As in Example 1, the same improvement procedure could be repeated for Example 2. Obviously, it is not necessary in this case.

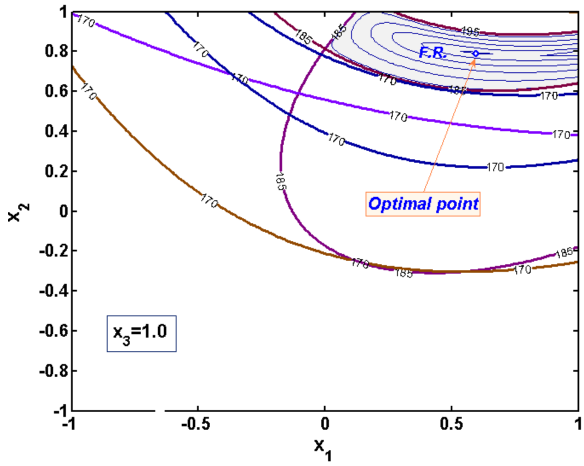

Figure 3. Overlaid contour plot for three design factors and six response constraints with x3: the heater block tempe–rature at optimal level (F.R. is the abbreviation for the feasible region)

4.2 Example 3

Mangili et al. [37] presented a paper in which they studied and optimized the ultrasonic devulcanization of a ground tire in a co-rotating twin-screw extruder.

In this work a twenty-eight, fully randomized, central composite face-centered design was used to investigate the chosen experimental space.

Table 7 shows the levels for each chosen design factor, including the ultrasonic amplitude (x_1), screw speed (x_2), slow rate (x_3), and temperature (x_4).

The complex viscosity (y_1), cross-link density (y_2), gel fraction (y_3), modulus at 100% of elongation (y_4), tensile strength (y_5), and elongation at break (y_6) were chosen as responses.

Table 8 depicts the response types as well as bound values for all the observed responses.

Table 7. Factors and their levels

Design factors		Units	Factor levels		
			Lower (-1)	Middle (0)	Upper (+1)
x_1	Amplitude	μm	5	8.5	12
x_2	Screw speed	rpm	150	200	250
x_3	Flow rate	g/min	4	6	8
x_4	Temperature	^{0}C	130	170	210

Table 8. Type of response and their specifications

Response	Type	y_j^{min}	y_j^{max}	r_j [a]	t_j [b]
y_1	STB	0.80	4.16	1	-1
y_2	STB	0.018	0.044	1	-1
y_3	STB	73.8	85.3	1	-1
y_4	LTB	2.52	3.67	3	-3
y_5	LTB	3.69	6.46	3	-3
y_6	LTB	102	194	1	-1

[a] Shape factors for Derringer-Suich desirability approach

[b] Exponents for Kim-Lin desirability approach

Also, in this table are given the corresponding va–lues of the shape factors (r_j) and exponents (t_j) used for the two different desirability approaches. The DM has (arbitrarily) assigned larger values of these parameters to the responses y_4 and y_5.

The results of this extensive experiment were given in the original paper [37].

A second-order polynomial model was fitted to each of the six responses and then reduced to the best subset model as follows:

$$\hat{y}_1(\mathbf{x}) = 2.45 - 0.74x_1 - 0.36x_2 + 0.18x_3 - 0.36x_4 + 0.40x_1^2 - 0.47x_4^2 - 0.0000002x_1x_2 + 0.21x_1x_4 - 0.19x_2x_4 + 0.15x_1x_2x_4$$

$$\hat{y}_2(\mathbf{x}) = 0.0295 - 0.0047x_1 - 0.0038x_2 + 0.0002x_3 - 0.0037x_4 - 0.0004x_1x_2 + 0.0009x_1x_3 + 0.0021x_1x_4 - 0.0012x_2x_3 + 0.0003x_3x_4 + 0.0016x_1x_2x_3 - 0.0012x_1x_3x_4$$

$$\hat{y}_3(\mathbf{x}) = 79.0 - 3.2x_1 - 0.9x_2 + 0.4x_3 - 0.9x_4 + 1.1x_1^2 - 1.1x_3^2 + 0.02x_1x_2 + 0.3x_1x_3 + 0.8x_1x_4 - 0.4x_2x_3 - 0.7x_2x_4 + 0.4x_1x_2x_3 + 0.4x_1x_2x_4$$

$$\hat{y}_4(\mathbf{x}) = 3.05 - 0.28x_1 - 0.03x_2 + 0.02x_3 - 0.10x_4 + 0.13x_1^2 - 0.22x_3^2 - 0.01x_1x_2 + 0.06x_1x_4 - 0.07x_2x_3 + 0.03x_2x_4 - 0.06x_3x_4 + 0.10x_1x_2x_4$$

$$\hat{y}_5(\mathbf{x}) = 5.3 + 0.51x_1 + 0.59x_2 + 0.20x_4 - 0.3x_1^2 + 0.08x_1x_2 - 0.02x_1x_4 + 0.37x_2x_4 - 0.32x_1x_2x_4$$

$$\hat{y}_6(\mathbf{x}) = 153 + 20x_1 + 12x_2 - 2x_3 + 6x_4 - 14x_1^2 + 12x_4^2 + 3x_1x_2 - 4x_1x_4 + 5x_2x_3 + 6x_2x_4 - 9x_1x_2x_4$$

In the present paper, the same data set and conditions were used as in the original work [37]. For the same reason, it was deceded to assign different values to the shape factors ($r_1^t = r_2^t = r_3^t = r_6^s = 2.5$; $r_4^s = r_5^s = 3.5$).

After optimization procedure, the optimal solution at the point $\mathbf{X}^* = \{12.0, 250, 5.44, 210\}$, with the corres–ponding estimated response values $\hat{\mathbf{y}}^* = \{1.04, 0.019, 75.3, 2.96, 6.141, 184\}$, and the overall desirability $D^* = 0.75$ was found.

All the estimated response values satisfy the limita–tions given in Table 8. In this case, the improvement procedure was not necessary.

4.4 Example 4

Manohar et al. [38] investigated and optimized the turning of difficult-to-cut annealed superalloy Inconel 718 by using coated carbide tool inserts.

For experimentation, the Taguchi's orthogonal array $L_{27}(3^{13})$ was employed, which is in this case a three-level full factorial design (Table 9).

The design factors (process parameters) of interest as cutting speed (X_1), feed (X_2), and depth of cut (X_3) were taken (Table 10).

The responses as feed ('axial') cutting force (y_1), thrust ('radial') cutting force (y_2), main ('tangential') cutting force (y_3), surface roughness (y_4), and material removal rate (y_5) were selected. These responses were varied in different ranges (Table 11).

Table 9. Design matrix and experimental results

	Design factors			Responses				
N	X_1	X_2	X_3	y_1	y_2	y_3	y_4	y_5
1	40	0.20	1.0	168	41	66	3.12	1568
2	40	0.20	1.5	178	45	70	3.15	2352
3	40	0.20	2.0	192	52	75	3.22	3136
4	40	0.25	1.0	179	47	69	3.24	1960
5	40	0.25	1.5	190	53	74	3.37	2940
6	40	0.25	2.0	201	58	80	3.42	3920
7	40	0.30	1.0	213	65	85	3.60	2352
8	40	0.30	1.5	222	71	89	3.71	3528
9	40	0.30	2.0	231	78	95	3.76	4704
10	50	0.20	1.0	160	36	59	2.98	2450
11	50	0.20	1.5	171	42	64	3.09	3675
12	50	0.20	2.0	180	48	68	3.13	4900
13	50	0.25	1.0	174	44	62	3.20	3063
14	50	0.25	1.5	182	48	69	3.25	4594
15	50	0.25	2.0	191	54	73	3.32	6125
16	50	0.30	1.0	204	60	79	3.56	3675
17	50	0.30	1.5	211	68	86	3.69	5513
18	50	0.30	2.0	220	75	91	3.75	7350
19	60	0.20	1.0	152	33	55	3.08	3528
20	60	0.20	1.5	160	39	60	3.01	5292
21	60	0.20	2.0	169	46	64	3.07	7056
22	60	0.25	1.0	170	40	58	3.15	4410
23	60	0.25	1.5	177	44	63	3.20	6615
24	60	0.25	2.0	185	49	67	3.28	8820
25	60	0.30	1.0	191	52	65	3.49	5292
26	60	0.30	1.5	199	57	71	3.60	7938
27	60	0.30	2.0	209	63	78	3.71	10584

Table 10. Factors and their levels

Design factors		Units	Factor levels		
			Lower (-1)	Midlle (0)	Upper (+1)
X_1	Cutting speed	m/min	40	50	60
X_2	Feed	mm/rev	0.20	0.25	0.30
X_3	Depth of cut	mm	1.0	1.5	2.0

Table 11. Type of response and their specifications

Response	Type	y_j^{min}	y_j^{max}	r_j^s or r_j^t
y_1	STB	152	231	1
y_2	STB	33	78	1
y_3	STB	55	95	1
y_4	STB	2.98	3.88	1
y_5	LTB	1.568	10.584	1

As can be seen from Table 9 and Table 11 the extreme values of the responses within the entire investigated experimental region were considered as the basic constraints for the optimization procedure (see Eqs. (5) and (6)).

The full second-order mathematical model was utilized to find appropriate approximion for the functional relationship between design factors and the response as follows:

$$\hat{y}_1(\mathbf{x}) = 235.53 - 0.13x_1 - 927.22x_2 + 31.72x_3 - 0.0x_1^2 + 2933.33x_2^2 + 2.0x_3^2 - 1.67x_1x_2 - 30x_2x_3 - 0.23x_1x_3$$

$$\hat{y}_2(\mathbf{x}) = 68.12 + 1.43x_1 - 679.44x_2 + 6.17x_3 - 0.01x_1^2 + 2155.56x_2^2 + 1.56x_3^2 - 3.67x_1x_2 + 10x_2x_3 - 0.03x_1x_3$$

$\hat{y}_3(\mathbf{x}) = 105.44 + 1.26x_1 - 672.78x_2 + 4.83x_3 - 0.01x_1^2 +$
$2000x_2^2 - 0.65x_3^2 - 3.83x_1x_2 + 26.67x_2x_3 - 0.017x_1x_3$

$\hat{y}4(\mathbf{x}) = 5.24 - 0.019x_1 - 17.24x_2 - 0.014x_3 + 0.00013x_1^2 +$
$41.33x_2^2 - 0.013x_3^2 + 0.01x_1x_2 + 1.10x_2x_3 - 0.0017x_1x_3$

$\hat{y}_5(\mathbf{x}) = 13654 - 367.4x_1 - 36738x_2 - 6125x_3 + 1.836x_1^2 -$
$22.22x_2^2 - 0.22x_3^2 + 735x_1x_2 + 12576.67x_2x_3 + 122.5x_1x_3$

In the mentioned work, the individual desirability functions were calculated for the considered responses and subsequently the overall desirability value was obtained. The optimal set of process parameters, under given conditions, corresponding to the maximal value of the overall desirability was identified (run number 20-Table 9).

Numerous investigations have confirmed that responses in machining processes do not deviate much from linear functions [8,16,22,23,45]. For instance, in the turning process the material removal rate (MRR) is a linear function of process parameters.

For this reason, by employing the novel desirability function the following shape parameters $r_1^t = r_2^t = r_3^t =$ $= r_4^t = 1.5$ (LTB) and $r_5^s = 0.5$ (STB) were selected.

4.5 Example 5

Gunaraj and Murugan [39,40] reported results of an analysis and optimization into the submerged arc welding (SAW), which is one of the major processing technology in metal industry.

The chosen design matrix was a five-level, four-factor central composite rotatable design, consisting of 31 combinations of design factor levels.

Details of the experimental design, procedures, and design matrix are given in the original paper [39].

The process factor levels with their units and notations are given in Table 12.

Table 12. Factors and their levels

Design factors		Unit	Factor levels				
			-2	-1	0	+1	+2
x_1	Welding voltage	V	24	6	28	30	32
x_2	Wire feed rate	$\frac{m}{min}$	0.7	0.93	1.16	1.39	1.62
x_3	Welding speed	$\frac{m}{min}$	0.43	0.51	0.59	0.67	0.75
x_4	Nozzle-to-plate distance	mm	30.0	32.5	35.0	37.5	40.0

The responses as penetration (y_1), reinforcement (y_2), width of the bead (y_3), area of penetration (y_4), area of reinforcement (y_5), dilution of the bead (y_6), and total weld bead volume (y_7) were selected.

The responses representing the weld bead quality parameters were expressed in the form of full second-order polynomial.

The design factors are given in their coded form. The coded design factors for intermediate values were calculated from the following relationship:

$$x_i = 2\frac{2X_i - \left(X_i^{\max} + X_i^{\min}\right)}{X_i^{\max} - X_i^{\min}} \quad (13)$$

where X_i is any value of the process factor (natural factor), $X_i^{\min}$ is the lower level of the process factor, and $X_i^{\max}$ is the upper level of the process factor.

Gunaraj and Murugan [40] employed the quasi-Newton method to optimize this multi-response problem (which belongs to one of the most popular methods from a class of nonlinear constrained optimization techniques).

After calculating the unknown coefficients in the mathematical model for predicting the weld bead geometry, the following regression equations were obtained:

$\hat{y}_1(\mathbf{x}) = 3.5714 - 0.1125x_1 + 0.3333x_2 - 0.2167x_3 +$
$0.0484x_1^2 + 0.0959x_2^2 + 0.0334x_3^2 - 0.0079x_4^2 -$
$0.0450x_1x_2 + 0.0400x_1x_3 + 0.0037x_1x_4 -$
$0.0112x_2x_3 - 0.0075x_2x_4 + 0.0825x_3x_4$

$\hat{y}_2(\mathbf{x}) = 1.27 - 0.0704x_1 + 0.1587x_2 - 0.1829x_3 -$
$0.0338x_4 + 0.0715x_1^2 + 0.0852x_2^2 + 0.1527x_3^2 +$
$0.0127x_4^2 + 0.0169x_1x_2 + 0.0319x_1x_3 - 0.0144x_1x_4 -$
$0.0031x_2x_3 - 0.0194x_2x_4 + 0.0256x_3x_4$

$\hat{y}_3(\mathbf{x}) = 10.7543 + 1.1883x_1 + 0.4533x_2 - 1.9042x_3 +$
$0.2333x_4 + 0.4110x_1^2 - 0.1727x_2^2 + 0.2898x_3^2 +$
$0.1160x_4^2 - 0.0463x_1x_2 - 0.6425x_1x_3 - 0.1500x_1x_4 -$
$0.3462x_2x_3 + 0.0913x_2x_4 - 0.2900x_3x_4$

$\hat{y}_4(\mathbf{x}) = 21.5571 + 1.0458x_1 + 1.8542x_2 - 1.6042x_3 -$
$0.2125x_4 + 0.0409x_1^2 + 0.2909x_2^2 - 0.0966x_3^2 +$
$0.1534x_4^2 + 0.1437\,x_1x_2 - 0.2063x_1x_3 + 0.0562x_1x_4 -$
$0.2437x_2x_3 - 0.1562x_2x_4 - 0.1562x_3x_4$

$\hat{y}_5(\mathbf{x}) = 21.4429 + 0.4429x_1 + 0.1871x_2 - 1.7559x_3 +$
$2.1079x_4 + 1.3886x_1^2 - 0.3889x_2^2 + 1.2236x_3^2 +$
$0.6148x_4^2 + 0.4056x_1x_2 - 0.0469x_1x_3 + 0.1406x_1x_4 -$
$0.9431x_2x3 + 0.7694x_2x_4 - 0.3256x_3x_4$

$\hat{y}_6(\mathbf{x}) = 47.2857 + 0.7375x_1 + 2.5042x_2 - 0.2542x_3 -$
$2.2542x_4 - 1.3516x_1^2 - 0.7141x_2^2 - 1.3141x_3^2 -$
$0.4516x_4^2 - 0.0937x_1x_2 - 0.2562x_1x_3 - 0.3187x_1x_4 +$
$0.4313x_2x_3 - 0.9062x_2x_4 + 0.1563x_3x_4$

$\hat{y}_7(\mathbf{x}) = 45.8857 + 1.5792x_1 + 2.2042x_2 - 3.5042x_3 +$
$2.0208x_4 + 1.6692\,{}_1^2 + 0.1692x_2^2 + 1.3692x_3^2 +$
$0.9692x_4^2 + 0.2813x_1x_2 - 0.2062x_1x_3 + 0.4813x_1x_4 -$
$0.8687x_2x_3 + 0.6438x_2x_4 - 0.7687x_3x_4$

The total weld bead volume was adopted as the primary response, while the remaining weld bead quality parameters were treated as constraint functions (priority-based optimization approach) [40].

Datta et al. [41] used the same experimental data and regression equations from previous researches [39], [40], with the aim of presenting the features of a desirability function approach, coupled with RSM, to solve multi-response optimization problems in SAW.

Table 13 depicts the response types as well as bound values for all the responses. In this example, all the observed responses are of one-sided type and, consequently, one of the given bounds merely serves to determine the corresponding individual desirability function.

The individual desirability functions, for each of the responses, have been selected in such a way that their target values are the same as those obtained after optimization done by Gunaraj and Murugan [40].

In this paper, in order to compare the results of previous researches [39], [40], [41], and those obtained by using the novel desirability method [29], the original experiment and all the same conditions were utilized.

For the same reason, optimal response values (for each of the optimal design factor settings) were calculated by the same equations.

Table 13. Type of response and their specifications

Response	Type	y_j^{min}	y_j^{max}	$r_j^s = r_j^t$
y_1	LTB	3.00	3.07	1
y_2	STB	1.28	1.80	1
y_3	STB	8.33	15.00	1
y_4	STB	18.30	20.00	1
y_5	STB	20.21	22.00	1
y_6	STB	38.00	50.00	1
y_7	STB	41.33	15.00	1

In this example, the "default'' shape parameters were selected for each of the individual desirability functions $r_j^s = r_j^t = 2.5$.

The initial solution at the optimal point $\mathbf{x}^* = \{-0.257,\ -2.000,\ 0.604,\ -0.075\}$, with the corresponding estimated response values $\hat{\mathbf{y}}^* = \{3.182, 1.267, 8.334, 18.144, 20.210, 37.997, 41.462\}$ and the overall desirability $D^* = 0.999$ was found.

4.6 Example 6

Aggarwal et al. [42] studied the CNC turning of AISI P 20 tool steel by using the TiN coated tungsten carbide cutting inserts and liquid nitrogen as a coolant.

Table 14 shows the levels for each chosen input parameter, including cutting speed (X_1), feed (X_2), depth of cut (X_3), and nose radius (X_4).

Table 14. Factors and their levels

Design factors		Unit	Factor levels		
			Lower (-1)	Middle (0)	Upper (+1)
X_1	Cutting speed	m/min	120	160	200
X_2	Feed	mm/rev	0.10	0.12	0.14
X_3	Depth of cut	mm	0.20	0.35	0.50
X_4	Nose radius	mm	0.40	0.80	1.20

As responses surface roughness (y_1), tool life (y_2), cutting force (y_3), and power consuption (y_4) were chosen. All the observed responses were varied in different ranges (Table 15).

Table 15. Type of response and their specifications

Response	Type	y_j^{min}	y_j^{max}	r_j^s or r_j^t
y_1	STB	0.17	0.99	1
y_2	LTB	34.00	55.50	1
y_3	STB	92.15	249.94	1
y_4	STB	660	1780	1

The turning process was studied according to three-level full factorial CCD design. Thirty independent trials were performed under the previously described cutting conditions. Each trial was repeated twice.

For more details, readers are refering to the original paper [42].

For each response the entire experimental region was employed to fit the second-order regression model as follows:

$$\hat{y}_1(\mathbf{x}) = 6.975 - 0.0044x_1 - 101.621x_2 - 1.087x_3 - 0.211x_4 + 0.00729x_1x_3 + 4.583x_2x_3 - 2.031x_2x_4 + 446.774x_2^2 + 0.054x_4^2$$

$$\hat{y}_2(\mathbf{x}) = 82.205 - 0.198x_1 + 72.021x_2 - 13.886x_3 + 1.163x_4 + 0.078x_1x_2 + 0.002x_1x_4 - 3.906x_2x_4 - 0.521x_3x_4 - 926.535x_2^2 + 16.862x_3^2 - 0.754x_4^2$$

$$\hat{y}_3(\mathbf{x}) = -223.496 - 5.186x_1 + 9559.856x_2 + 468.183x_3 - 19.802x_4 + 4.411x_1x_2 + 0.507x_1x_3 + 0.020x_1x_4 - 1483.542x_2x_3 - 64.922x_2x_4 + 3.427x_3x_4 + 0.015x_1^2 - 34885.526x_2^2 - 172.632x_3^2 + 17.349x_4^2$$

$$\hat{y}_4(\mathbf{x}) = 261.784 + 3.559x_1 - 6127.924x_2 - 408.382x_3 + 153.655x_4 - 6.250x_1x_2 + 8.333x_1x_3 + 0.156x_1x_4 + 833.333x_2x_3 - 0.00110x_1^2 + 45614.035x_2^2 + 366.472x_3^2 - 0.965x_4^2$$

Very high values of coefficients of determination ($R_1^2 = 0.983,\ R_2^2 = R_3^2 = R_4^2 = 0.999$) indicate that the presented regression equations quite adequately exp–lained the variability in the turning process.

In such circumstances, it has been shown that the desirability functions may be less convex i.e. more concave. Generally, an estimated response with poorer prediction should have less impact on optimization [6]. Therefore, for the first (initial) solution by employing the novel desirability function the shape parameters $r_2^s = r_3^t = r_4^t = 2.0$ and $r_1^t = 0.20$ were chosen.

After optimization procedure, the initial optimal solution at the point $\mathbf{X}^* = \{120,\ 0.10,\ 0.20,\ 0.95\}$, with the corresponding estimated response values $\hat{\mathbf{y}}^* = \{0.46,\ 55.40,\ 93.08,\ 753.90\}$, and the overall desirability $D^* = 0.990$ was found.

In this case, the improvement procedure was not necessary.

5. DISCUSSION

Looking at all examples, it is observed that there is no run in experimental matrices that provides the optimal factor levels combination for simultaneous optimization of all considered responses.

Tables 16-21 compare the results obtained for all considered experiments by the proposed method with the results reported by other researchers.

As Table 16 shows, the initial solution of Jeong and Kim [26] is not satisfactory. After the third iteration (using shape and bound mode) these authors obtained a quite satisfactory solution with an optimal point $\mathbf{x}^* = \{-0.157, 1.219, -0.604\}$ and the corresponding responses $\hat{\mathbf{y}}^* = \{139.82, 1239.1, 446.51, 73.93\}$, as well as overall desirability $D^* = 0.626$.

However, once again, this solution is no better than the improved solution obtained by the method proposed in this paper (see Section 4.1.).

In this paper, in Example 5, the focus was on an alternative method. Namely, the results of the method proposed are summarized and compared with those of Gunaraj and Murugan [40] who applied the classical nonlinear constrained method. At the same time, it is worth noting that the approaches by Gunaraj and Murugan [40] and Datta et al. [41] produce almost identical results.

Table 16. Comparison of results for example 1: Methods from literature and approach proposed

Method	Deringer-Suich	Park S.-Park J.	Jeong-Kim	Lee et al.	Proposed method
x_1	-0.050	-0.158	-0.499	0.18	0.126
x_2	0.145	0.437	1.029	0.51	0.406
x_3	-0.868	-0.879	1.156	-1.06	-1.241
y_1	129.5	130.38	157.79	**131.62**	127.1
y_2	1300.0	1300.02	1689.5	1408.1	**1443.0**
y_3	465.7	**471.0**	346.6*	449.42	469.3
y_4	**68.0**	69.62	76.43*	68.8	**68.0**
D	0.583	-	-	0.481	**0.702**

The values in bold indicate the best solution

* Response outside of acceptable values

Table 17. Comparison of results for example 2: Methods from literature and approach proposed

Method	Del Castillo et al.	Ortiz et al.	Ch'ng et al.	Proposed method
X_1	84.16	74.55	78.26	103.64
X_2	450.00	472.90	450.00	424.30
X_3	329.87	332.75	336.54	350.00
y_1	186.0	187.0	185.0	**190.1**
y_2	174.5	176.7	174.6	**178.4**
y_3	172.1	173.8	172.3	**180.7**
y_4	192.6	192.9	**190.0**	191.5
y_5	173.1	174.2	172.5	**174.7**
y_6	**185.0**	186.2	185.4	189.6
D	0.306	0.408	0.108	**0.589**

The values in bold indicate the best solution

Example 6 was also discussed by Noorossana et al. [43]. In their work, an ANN approach is presented which utilizes a process capability index (PCI) to combine multiple responses into a single function. In Table 21, their optimal (iterative) solutions are compared to those obtained from dissimilar approaches.

As already established, the first (initial) solutions obtained by the proposed method simultaneously satisfy all specific requirements and gives good balance between the responses.

Obviously, a better and more reliable solution can be achieved using the improvement procedure described above, based on the optimal setting of shape parameters.

This approach can also be used to generate an initial solution. However, because it is a tedious and time-consuming work, choosing the 'default' shape para–me–ters for individual desirability functions remains the first option.

Generally speaking, the pair-wise comparisons clearly show that different approaches give similar or comparable results. However, looking at all the results concurrently, it is evident that the proposed method outperforms the others to some extent. Similar observations were made in the paper [29].

The performance supremacy of the proposed method over some known approaches should be verified by testing the larger number of complex optimization problems. The truth is, even in such a case where above qualitative description is acceptable, it does not mean that the proposed method is the best. Namely, according to the no free lunch (NFL) theorem [46], no such method exists. In other words, none of the optimization methods can be claimed to be superior to others for each specific optimization problem.

Table 18. Comparison of results for example 3: Methods from literature and approach proposed

Method	Derringer-Suich approach[a]	Kim-Lin approach[a]	Proposed method
X_1	7.20	5.00	12.00
X_2	250	250	250
X_3	5.50	5.60	5.44
X_4	210	202	210
y_1	1.22	2.16	**1.04**
y_2	0.023	0.027	**0.019**
y_3	77.4	80.2	**75.3**
y_4	3.03	**3.27**	2.96
y_5	6.39	5.87	**6.41**
y_6	183	155	**184**
D	0.71	0.48	**0.75**

The values in bold indicate the best solution

[a] Calculated by Mangili et al. [37]

Table 19. Comparison of results for example 4: Methods from literature and approach proposed

Method	Manohar et al.	Proposed method
X_1	60	60
X_2	0.2	0.215
X_3	1.5	1.970
y_1	**160**	165.91
y_2	39	**38.87**
y_3	60	**57.36**
y_4	**3.01**	3.05
y_5	5292	**6129**
D	0.771	**0.955**

The values in bold indicate the best solution

Table 20. Comparison of results for example 5: Methods from literature and approach proposed

Method	Gunaraj -Murugan [a]	Datta et al.[b]	Proposed method
x_1	0	0	-0.257
x_2	-2.000	-2.000	-2.000
x_3	0.625	1.325	0.604
x_4	-0.160	0	-0.075
y_1	3.169	3.090	**3.182**
y_2	**1.240**	1.243	1.267
y_3	8.537	8.513	**8.334**
y_4	18.280	18.277	**18.144**
y_5	**20.030**	20.073	20.210
y_6	38.253	38.210	**37.997**
y_7	41.569	41.585	**41.462**
D	-	0.972	**0.999**

The values in bold indicate the best solution

[a] Nonlinear constrained optimization method

[b] Standard Derringer's desirability optimization method

Table 21. Comparison of results for example 6: Methods from literature and approach proposed

Method	Noorossana et al.[a]		Aggarwal et al.	Proposed method
	1st iteration	2nd iteration		
X_1	-	-	120	120
X_2	-	-	0.10	0.10
X_3	-	-	0.20	0.20
X_4	-	-	1.20	0.95
y_1	0.31	**0.17**	0.38	0.46
y_2	50.64	40.50	55.23	**55.40**
y_3	156.12	134.34	95.80	**93.08**
y_4	1012.9	871.15	781.66	**753.90**
D	-	-	0.890	**0.990**

The values in bold indicate the best solution

[a] ANN-based PCI optimization approach

6. CONCLUSION

During the last two decades, the desirability function has attracted worldwide attention and has been recognized as a useful and powerful tool in multi-response optimization methodologies due to its novelty and remarkable performance.

This paper deals with the application of a newly recommended desirability function in multi-response optimization of higher-dimensional problems (in terms of the number of design factors and the number of observed responses).

The efficiency and accuracy of the proposed method has been successfully tested by using six practical examples with specific characteristics taken from previously published articles.

The optimal solutions obtained in all examples by applying the novel desirability function contain the highest overall desirability in comparison with the existing desirability functions, implying the best balance between the considered quantitatiive responses.

In general, the presented results suggest the superiority of the proposed method over the other approaches known in the literature. Therefore, it is concluded that this method represents an excellent approach at solving a wide area of multi-response optimization problems.

In addition, an exact method was suggested (deve–loped under the RSM framework) for determining the favorable shape parameters in desirability functions. Consequently, due to the proposed procedure, it does not require any (arbitrarily) specification of the shape parameters before the optimization procedure. This method was successfully applied in the first example.

Finally, like many other desirability function app–roaches, the method proposed can be easily understood and implemented by researchers and practitioners with little mathematical or statistical knowledge.

APPENDIX

The table below lists some of the factorial design (saturated or near-saturated), with three or more factor levels, which are suitable for the analysis shown in this paper. The recommended designs (denoted by asterisk) were selected according to the number of runs and their performances. It is known that using a proper transformation one can get some of the hybrid ("parsimonious") designs. Among these designs, some of them with many factors have good space-filling properties, but this can be a limiting feature in their application. It is natural to be expected that designs with equal sample sizes have different performance. Also, it should be expected that larger designs have better performance than smaller designs, but it is not always the case. The formulation "incomplete mathematical model" means that the given model takes into account only the main effects (without interactions).

Today, computers can help researchers in building a (approximately) optimal experimental design ("best" with respect to some optimality criterion). Computer-generated design can be an alternatively option, but the researcher in the RSM frame should primarily find out an appropriate standard design from the available collection.

Design factors (k)	Alternative designs	Factor levels (n)	Runs (N)	Model parameters (p)
2	Box-Draper design (BDD)*	3	6	6
	Rehtschaffner design*	3	6	
	Doehlert design*	7	7	
	Hartley design*	3	7	
	Taguchi design (L_9)	3	9	
	Central composite design (CCD)	3	10	
	Taguchi design (L_9)*	3	9	
	Box-Draper design (BDD)*	4	10	

3	Rehtschaffner design*	3	10	10
	Hoke design (D_2)*	3	10	
	Koshal design*	3	10	
	Notz design*	3	10	
	Draper-Lin design (DLD)*	3	10	
	Roquemore hybrid design (D_{311B})*	9	11	
	Hartley design*	3	11	
	Small composite design (SCD)	5	11	
	Pesocinsky design	3	13	
	Doehlert design	13	13	
4	Taguchi design (L_9)*	3	9	15
	Jones-Nachtsheim design*	3	9	
	Box-Draper design (BDD)*	4	15	
	Rehtschaffner design*	3	15	
	Notz design*	3	15	
	Hoke design(D_2)*	3	15	
	Roquemore hybrid design (D_{416C})*	8	16	
	Taguchi design (L'_{16})*	4	16	
	Draper-Lin design (DLD)*	3	16	
	Hartley design	3	17	
5	Jones-Nachtsheim design*	3	11	21
	Taguchi design (L'_{16})*	4	16	
	Draper-Lin design (DLD)*	3	21	
	Box-Draper design (BDD)*	4	21	
	Rehtschaffner design*	3	21	
	Taguchi design (L_{25})	5	25	
6	Jones-Nachtsheim design*	3	13	28
	Taguchi design (L_{25})*	5	25	
	Taguchi design (L_{27})*	3	27	
	Box-Draper design (BDD)*	4	28	
	Rehtschaffner design*	3	28	
	Draper-Lin design (DLD)*	3	28	

Notes: The designs that allow fitting only an incomplete second-order mathematical model are bolded

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НЕКЕ ПРИМЕНЕ НОВЕ ФУНКЦИЈЕ ПОЖЕЉНОСТИ У СИМУЛТАНОЈ ВИШЕКРИТЕРИЈУМСКОЈ ОПТИМИЗАЦИЈИ

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У оквиру техника вишекритеријумске оптимизације, методологија оптимизације заснована на функцији пожељности једна је од најпопуларнијих и најчешће коришћених методологија код истраживача и практичара из области инжењерства, хемије, технологије и многих других области науке и технике. Бројне функције пожељности уведене су ради побољшања перформанси ове методологије оптимизације. Недавно је предложена нова функција пожељности за вишекритеријумску оптимизацију, која је глатка, континуирана и диференцијабилна и тиме погоднија за примену неких ефикаснијих метода оптимизације на бази градијента. Овај рад процењује перформансе предложене методе на шест примера из праксе. После упоредне анализе резултата, показало се да предложена метода у одређеној мери надмашује остале конкурентне методе оптимизације.