Function Approximation Technique (FAT)-Based Nonlinear Control Strategies for Smart Thin Plates with Cubic Nonlinearities

Hayder F.N. Al-Shuka
Lecturer
University of Baghdad
Department of Aeronautical Engineering
Iraq

Ehab N. Abbas
Asst. Professor
Ministry of Higher Education and Scientific Research
Studies and Planning and Follow up Directorate
Iraq

The current work introduces three nonlinear control solutions for the regulation of a vibrating nonlinear plate considering model uncertainty. These solutions are feedback linearization control (FBL), virtual velocity error-based control (VVEC), and backstepping control (BSC). In the FBL control, a nonlinear control law is designed with linear closed-loop dynamics such that dynamic stability is ensured. Whereas, by the VVEC (or so-called passivity-based approach in the robotics community) the limitations of the feedback linearization are overcome. On the other side, the BSC selects virtual control variables with stabilized intermediate control laws based on Lyapunov theory. Systematic modeling for the target vibrating plate with piezo patches is described. In effect, considering the nonlinear influence makes the resulted mode shapes for the vibrating structure are highly coupled and careful control design is required. Using the Galerkin approach, the partial differential equation for the smart plate is transformed into definite ordinary differential equations; the multi-input multi-output model is established. The aforementioned control strategies are evaluated and investigated in detail. In essence, they are powerful tools for dealing with nonlinear dynamic systems, however, the VVEC could be considered superior in comparison with the FBL control and the BSC since the designed control structure does not include inertia inverse matrix and modal coordinate acceleration that could make computational problems. As a result, simulation experiments were focused on the VVEC strategy, and the latter was implemented on a simply supported thin plate structure with collocated piezo-patches. The results show the validity of the proposed control architecture.

Keywords: *Nonlinear vibrations, feedback linearization, passivity-based control, backstepping control, adaptive approximation control*

1. INTRODUCTION

Piezoelectric materials can transform electrical energy into mechanical energy or vice versa. Much attention is devoted to using them in vibration suppression of flexible structures such as strings, beams, plates, shells, etc. [1]. Integration of smart materials, e.g. piezoelectric patches, with flexible components (structures) makes them adaptively accommodate external disturbances and attenuate vibrational motions if exist. A suitable control law is always required to get perfect performance. Consequently, this paper is concerned with the function approximation technique (FAT)-based adaptive control approaches for vibration suppression of a piezo-electrically actuated/sensed thin plate considering nonlinear effects. For modeling of transverse vibration of a plate structure, the following points should be noticed [2]:

In a thick plate structure, both the shear deformation and bending deflection are considered. In a membrane structure, the thickness is thinner than a thin plate with a negligible bending stiffness while the bending loadings are resisted by the membrane action. The bending deflection is considered for a thin plate considering the effect of the axial stretching (loading) that complicates the dynamics and control tasks. The current work is interested in modeling and control of this category of plates.

The key cause of structural nonlinearity comes from in-plane stretching of the flexible structure, however, the nonlinear behavior for thin structures could originate from nonlinear geometry, inelastic behavior of the structure, the nonlinear applied excitation loadings, etc. These structural phenomena cannot be sufficiently investigated by linear theory since the deflection of the target structure is large in comparison with the thickness. Moreover, considering the effect of such a large deflection on the response of the flexible structure is essential [3,4]. In effect, considering the nonlinear influence makes the resulted mode shapes for the vibrating structure is highly coupled and careful control design is required. Despite there being different types of nonlinearity sources, see e.g. [5,6], this study is focused

Received: April 2021, Accepted: November 2021
Correspondence to: Dr Hayder Al-Shuka, University of Baghdad, Department of Aeronautical Engineering, Baghdad, Iraq
E-mail: hayder.al-shuka@rwth-aachen.de
doi: 10.5937/fme2201168A

on the nonlinearity resulting from axial stretching of plates. Due to the accompanying complexity of the nonlinear equation of motion for the vibrating plates, a little work reported in the literature has been focused on the analytical derivation of the nonlinear dynamics for the plates and the finite element method FEM is the potential tool for modeling and analysis. . Plates and plate-like structures play important roles in miscellaneous application such as floor and foundation slabs, lock-gates, offshore structures, the wing structure of an aircraft, and aircraft fuselage, see e.g. [7-9]. The reader is referred to [10-14] for more details on applications and modeling of plate-like structures. To model a thin plate structure with piezo-patches, a partial differential equation PDF is obtained based on the Newton-Euler formulation. It is an infinite-dimensional system that has infinite degrees-of freedom DOFs. This complicates the control problem since it may theoretically require several distributed piezo-actuators to fully actuate and regulate the vibrating plate. However, the first mode shapes are considered since they are dominant in the low-frequency region such that high-frequency amplitudes would be neglected. Galerkin's approach is used to transform the infinite-dimensional PDF of the target plate into finite N-ordinary differential equations ODEs. The resulting nonlinear ODEs for the vibration plate are represented as a multi-input multi-output dynamic system. Cubic nonlinear stiffness terms are resulted due to the coupling of the bending-tension effect.

In effect, two essential strategies that are possible to damp vibration of structures are passive vibration damping [15,16] and active vibration damping. This study is focused on adaptive vibration damping of plate-type structures considering disturbance and complex nonlinearities. To suppress the nonlinear vibrations of adaptive plate-like structures, piezo-patches are bonded on the surface of the plates that can generate counteract moments and sense the resulting deflection for stabilization of the target vibrating plates. As a result, the objective of the controller is to regulate the vibrational motion of the target system under parameter uncertainty and disturbances, see e.g., [17-23]. However, if the dynamic system undergoes uncertain complex nonlinear terms, the linear control architectures e.g. PID family controllers are no longer useful and hence the best solution tools are nonlinear control strategies. In effect, there are two substantial techniques for dealing with the abovementioned problem: adaptive control and sliding mode control [24]. Adaptive control is a powerful tool to control dynamic systems with unknown constant parameters [25]. The Lyapunov theory is used as a basis for guaranteeing the stability of the proposed control law and the corresponding updating laws associated with unknown parameters. For unknown time-varying parameters, adaptive approximation control is used instead to comprise model-free techniques or so-called intelligent control [25-34]. On the other hand, robust control requires knowledge of limits of uncertainty. One of the well-known robust techniques used is the sliding mode control SMC that uses a signum function associated with a suitable sliding surface. However, a signum-based term can be easily integrated with adaptive control algorithms for compensating modeling error if exists.

The current work introduces three nonlinear control solutions for the regulation of the vibrating nonlinear plate considering the model uncertainty. These solutions are feedback linearization control FBL, virtual velocity error-based control VVEC, and backstepping control BSC. These control algorithms are strong tools to control complex dynamic systems such as the miniature robot described in [35]. In FBL control, a nonlinear control law is designed with linear closed-loop dynamics such that dynamic stability is ensured [36,37]. Whereas, by the VVEC (or so-called passivity-based approach in the robotics community) the limitations of the feedback linearization are resolved [25,26]. On the other side, the BSC assumes that virtual control variables with stabilized intermediate control laws based on Lyapunov theory [24]. The above-mentioned control algorithms are reformulated based on FAT in connection with a robust sliding term for compensating modeling/approximation errors if exist. Systematic modeling for the target vibrating plate with piezo patches is described. In effect, considering the nonlinear influence makes the resulted mode shapes for the vibrating structure are highly coupled and careful control design is required. Using the Galerkin approach, the partial differential equation for the smart plate is transformed into definite ordinary differential equations; the multi-input multi-output model is established. The aforementioned control strategies are evaluated and investigated in detail. In essence, they are powerful tools for dealing with nonlinear dynamic systems, however, the VVEC could be considered superior in comparison with the FBL control and the BSC since the designed control structure does not include inertia inverse matrix and modal coordinate acceleration that could make computational problems. As a result, simulation experiments were focused on the VVEC strategy, and the latter was implemented on a simply supported thin plate structure with collocated piezo-patches. The results show the validity of the proposed control architecture. The contributions of the current paper are:

1. Considering the axial stretching in the plate model complicates the dynamic behavior and control task since coupled cubic nonlinearity terms occur. To our knowledge, this point has been a little considered in previous work.
2. Introducing three powerful nonlinear control strategies and describing features and shortcomings of every technique in detail.
3. Regressor-free techniques have been presented as a basis for control laws and the associated closed-loop dynamics.

The remainder of the paper is organized as follows. Section 2 introduces the dynamic modeling of the smart thin plate while the nonlinear control structures are described in section 3. Section 4 introduces simulation results and discussions. Section 5 concludes.

Remark 1. In effect, we avoid using the terminology ‘’passivity-based control" for the VVEC since the passivity property may not hold for the current modeling of the vibrating plate. The idea of passivity-based control is to exploit the skew-symmetric matrix property such that (M – 2C, where M is the inertia matrix and C is the damping matrix). Fortunately, the target plate has an inertia matrix with constant-value elements while the elements of the damping matrix are assumed to be positive (c_{ij}) and these ease the proof of the stability for the VVEC away from the passivity concept. In effect, the damping term will be integrated into the complex stiffness terms and hence it will not make an obstacle for proving the dynamic stability for the investigated system.

2. DYNAMIC MODELLING OF THE SMART PLATE

This section is focused on modeling a vibrating thin plate with surface bonded piezo patches. The effect of the piezo-patches can be added as moment loadings. To this end, consider a thin plate with m collocated piezo-patches, see Fig. 1. The following assumptions are considered [1]:

1. The plate is thin with a uniform thickness.
2. The dynamics of the piezo-patches are neglected.
3. The plate undergoes large deflection that imposes axial stretching on it.

$$D\nabla^4 w = \frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2\frac{\partial^2 F}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y} - m\frac{\partial^2 w}{\partial t^2} + p + \frac{\partial^2 M_{px}}{\partial x^2} + \frac{\partial^2 M_{py}}{\partial y^2} \tag{1a}$$

$$\frac{\nabla^4 F}{Eh} = \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \tag{1b}$$

with

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}, D = \frac{Eh^3}{12(1-v^2)}$$

where F is the Airy stress function, m is the mass of the plate, ∇^4 is the biharmonic operator, h refers to the plate thickness, p is the external transverse load per unit area, $M_p(.)$ is the piezo-actuator external moment per unit length, D is the flexural rigidity, E is Young's modulus, and v is the Poisson's ratio.

Now it is necessary to transform the resulting PDFs of (1) into ODEs using the Galerkin technique, hence expanding w and F in terms of mode shape function and modal amplitudes

$$w(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\psi_{mn}(x,y)q_{mn}(t) \tag{2a}$$

$$F(x,y,t) = \sum_{r=1}^{\infty}\sum_{s=1}^{\infty}\phi_{rs}(x,y)a_{rs}(t) \tag{2b}$$

By substituting (2) into (1), multiplying (1a) and (1b) by $\psi_{ij}(x,y)$ and $\square_{tu}$ respectively and integrating the equations across the area of the target plate to obtain

$$M_{mnij}\ddot{q}_{mn} + \sum_{mn} q_{mn}K_{mnij} + p_{ij} - \sum_{m,n,r,s} a_{rs}q_{mn}L_{rsmnij} = \int_0^a\int_0^b\left(\frac{\partial^2 M_{px}}{\partial x^2} + \frac{\partial^2 M_{py}}{\partial y^2}\right)\psi_{ij}(x,y)dxdy \tag{3a}$$

$$\sum_{r,s} a_{rs}A_{rstu} = \sum_{m,n,k,l} q_{kl}q_{mn}B_{mnkltu} \tag{3b}$$

where,

$M_{mnij} = \iint m\psi_{mn}\psi_{ij}dxdy = m_{ij}$

(Orthogonality property)

$K_{mnij} = \iint \nabla^4\psi_{mn}\psi_{ij}dxdy = k_{ij}$

(Orthogonality property)

$p_{ij} = \iint p\psi_{ij}dxdy$

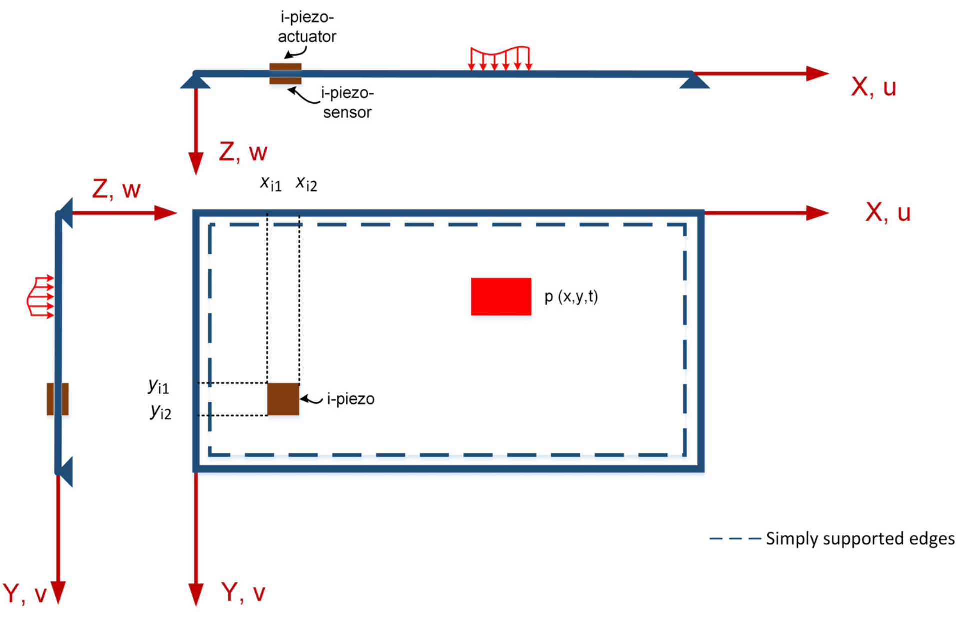


Fig.1: A simply supported thin plate with surface bonded piezo-transducers

$$A_{rstu} = \iint \frac{\nabla^4 \phi_{rs}}{Eh} \phi_{tu} dxdy$$

$$B_{mnkltu} = \iint \left(\frac{\partial^2 \psi_{mn}}{\partial x \partial y} \frac{\partial^2 \psi_{kl}}{\partial x \partial y} - \frac{\partial^2 \psi_{mn}}{\partial x^2} \frac{\partial^2 \psi_{kl}}{\partial y^2} \right) \phi_{tu} dxdy$$

$$L_{rsmnij} = \iint \left(\begin{array}{l} \frac{\partial^2 \phi_{rs}}{\partial y^2} \frac{\partial^2 \psi_{mn}}{\partial x^2} + \frac{\partial^2 \phi_{rs}}{\partial x^2} \frac{\partial^2 \psi_{mn}}{\partial y^2} - \\ 2 \frac{\partial^2 \phi_{rs}}{\partial x \partial y} \frac{\partial^2 \psi_{mn}}{\partial x \partial y} \end{array} \right) \psi_{ij} dxdy$$

By substituting (3b) into (3a), complex cubic nonlinear terms appear

$$\begin{aligned} & m_{ij} \ddot{q}_{ij} + k_{ij} q_{ij} + \sigma_{ij} (\phi_{(.)}, \psi_{(.)}, q_{(.)} q_{(.)} q_{(.)}) + p_{ij} = \\ & \int_0^a \int_0^b \left(\frac{\partial^2 M_{px}}{\partial x^2} + \frac{\partial^2 M_{py}}{\partial y^2} \right) \psi_{ij}(x, y) dxdy \end{aligned} \tag{4}$$

As we see, (4) includes a cross-coupling term represented by $\sigma_{(.)}$ and this complicates the controller task; hence advanced nonlinear control techniques are required to regulate the vibration motion of the plate. Now, let us expand the right-hand side of (4) related to piezo moments. Assuming equal piezoelectric charge constants in x and y directions, then

$$M_{px} = M_{py} = \lambda [H(x - x_1)][H(y - y_1) - H(y - y_2)] v_a(t) \tag{5}$$

where λ is a constant that depends on the physical parameters of the regular plate and the piezo-actuators, see [1] for more details. $H(.)$ is a Heaviside unit step function and $v_a(t)$ is the piezo-actuator voltage. Taking the second derivatives for M_{px} and M_{py} and summing the result with further simplifications, we can get

$$\frac{\partial^2 M_{px}}{\partial x^2} + \frac{\partial^2 M_{py}}{\partial y^2} = \sum_{k=1}^{N_a} \alpha_k(x, y) v_{ak}(t) \tag{6}$$

with

$$\begin{aligned} \alpha_k(x, y) = \lambda_k \Bigg[& \left(\frac{\partial \delta(x - x_{1k})}{\partial x} - \frac{\partial \delta(x - x_{2k})}{\partial x} \right) \big(H(y - y_{1k}) \\ & - H(y - y_{2k}) \big) + \left(\frac{\partial \delta(y - y_{1k})}{\partial y} - \frac{\partial \delta(y - y_{2k})}{\partial y} \right) \big(H(x - \\ & x_{1k}) - H(x - x_{2k}) \big) \Bigg] \end{aligned}$$

and N_a refers to the number of piezo-actuators used that is equal to the number of piezo-sensors. So the right-hand side of (4) can be reformulated as

$$\begin{aligned} & \int_0^a \int_0^b \left(\frac{\partial^2 M_{px}}{\partial x^2} + \frac{\partial^2 M_{py}}{\partial y^2} \right) \psi_{ij}(x, y) dydx = \\ & \int_0^a \int_0^b \sum_{k=1}^{N_a} \alpha_k(x, y) v_{ak}(t) \psi_{ij}(x, y) dydx = \\ & \sum_{k=1}^{N_a} \left(\int_0^a \int_0^b \alpha_k(x, y) \psi_{ij}(x, y) dydx \right) v_{ak}(t) \end{aligned} \tag{7}$$

Using the following Dirac-Delta function property

$$\int_{-\infty}^{\infty} \frac{d^n \delta(x - x_0)}{dx^n} \sigma(x) dx = (-1)^n \left. \frac{d^n (\sigma(x))}{dx^n} \right|_{x = x_0}$$

Then (7) becomes

$$\begin{aligned} & \sum_{k=1}^{N_a} \left(\int_0^a \int_0^b \alpha_k(x, y) \psi_{ij}(x, y) dydx \right) v_{ak}(t) = \\ & \sum_{k=1}^{N_a} \lambda_k \mu_{ijk} v_{ak}(t) \end{aligned} \tag{8}$$

where

$$\begin{aligned} \mu_{ijk} = & \left(\int_{y_{ik}}^{y_{2k}} \frac{\partial \psi_{ij}(x_{2k}, y)}{\partial x} dy - \int_{y_{ik}}^{y_{2k}} \frac{\partial \psi_{ij}(x_{1k}, y)}{\partial x} dy \right) + \\ & \left(\int_{x_{ik}}^{x_{2k}} \frac{\partial \psi_{ij}(x\ \ , y_{2k})}{\partial x} dx - \int_{y_{ik}}^{y_{2k}} \frac{\partial \psi_{ij}(x\ \ , y_{1k})}{\partial x} dx \right) \end{aligned}$$

Equation (4) can be reformulated as

$$\begin{aligned} & m_{ij} \ddot{q}_{ij} + k_{ij} q_{ij} + \sigma_{ij} + d_{ij} = u_{ij}, \\ & i = 1,2,3,..., N \\ & j = 1,2,3,..., M. \end{aligned} \tag{9}$$

where

$$d_{ij} = -\int_0^a \int_0^b p(t, x, y) \psi_{ij}(x, y) dydx,$$

$$u_{ij} = \sum_{k=1}^{N_a} \lambda_k \mu_{ijk} v_{ak}(t)$$

Writing (9) in matrix form to obtain

$$M\ddot{q} + Kq + \sigma + d = u \tag{10}$$

where

$$M = \begin{bmatrix} m_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & m_{NM} \end{bmatrix}, K = \begin{bmatrix} k_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & k_{NM} \end{bmatrix}, \sigma = \begin{bmatrix} \sigma_{11} \\ \vdots \\ \sigma_{NM} \end{bmatrix},$$

$$d = \begin{bmatrix} d_{11} \\ \vdots \\ d_{NM} \end{bmatrix}, u = \begin{bmatrix} u_{11} \\ \vdots \\ u_{NM} \end{bmatrix}.$$

At this stage, it is suitable to consider the damping effect and a viscous damping term is used

$$M\ddot{q} + C\dot{q} + Kq + \sigma + d = u \tag{11a}$$

with

$$C = \begin{bmatrix} c_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & c_{NM} \end{bmatrix}$$

For control purposes, (11a) can be reformulated as

$$M\ddot{q}+\eta(q,\dot{q},t)=u,\\ \eta(q,\dot{q},t)=C\dot{q}+Kq+\sigma+d \tag{11b}$$

Remark 2. In effect, the damping term can help suppress the model vibration; however, it should have positive values for each element of the damping matrix (*C*). Therefore, to avoid this constraint, it is integrated within the nonlinear term $\eta(q,\dot{q},t)$ such that the damping matrix does not appear while proving the dynamic stability of the investigated plate.

Remark 3. For detailed derivation for dynamics of a simply supported plate considering axial stretching due to large deflection, the reader is referred to [3,4].

Remark 4. For simplicity, we will refer to the dimension of the dynamic matrices in terms of the number of the mode shapes (*l*). Therefore, the dimension of the dynamic matrices can be represented as

$$M=\begin{bmatrix} m_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & m_l \end{bmatrix}, K=\begin{bmatrix} k_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & k_l \end{bmatrix}, \sigma=\begin{bmatrix} \sigma_1 \\ \vdots \\ \sigma_l \end{bmatrix},\\ d=\begin{bmatrix} d_1 \\ \vdots \\ d_l \end{bmatrix}, u=\begin{bmatrix} u_1 \\ \vdots \\ u_l \end{bmatrix}.$$

3. CONTROL STRUCTURE

The proposed controller aims at suppressing the vibrational motion of the target smart plate. The input control is provided by the piezo-actuator while the beam deflection (or equivalently modal amplitude) is sensed indirectly via the sensor voltages. This section introduces three powerful nonlinear control strategies considering the model uncertainty and the nonlinear effect due to large deflection. These strategies are 1) feedback linearization FBL, 2) virtual velocity error-based control VVEC, and 3) backstepping control BSC. The key idea of FBL is to cancel the nonlinearity embedded in the dynamic system by selecting a nonlinear input control such that we obtain closed-loop dynamics. However, the problem associated with FBL is the presence of an inertia inverse matrix in the control structure that could complicate the solution if an adaptive control algorithm is used. Therefore, VVEC is used alternatively to overcome these drawbacks by introducing filtered errors. The last approach includes applying intermediate control law in a recursive style until we obtain the actual control law and this scheme is called the BSC.

3.1 FAT-based FBL

As aforementioned, the FBL strategy includes designing a nonlinear control law such that the closed-loop dynamics is linear, and hence the intuitive controller can be expressed as [26]

$$u=\hat{M}\left[\ddot{q}_d-K_d(\dot{q}-\dot{q}_d)-K_p(q-q_d)\right]+\hat{\eta}-\kappa\,\mathrm{sgn}(B^T P^T x) \tag{12a}$$

where the symbol $(\hat{\cdot})$ refers to the estimation, $q_d \in R^l$ is the desired value for the modal vector, $K_d \in R^{l\times l}$ and $K_p \in R^{l\times l}$ are both diagonal positive definite matrices, $\kappa \in R^{l\times l}$ is a robust sliding gain, $B=\begin{bmatrix} 0 \\ I_l \end{bmatrix} \in R^{2l\times l}$ $x=\begin{bmatrix} e^T & \dot{e}^T \end{bmatrix}^T \in R^{2l}$, $P=P^T \in R^{2l\times 2l}$ is a symmetric positive definite matrix satisfying the Lyapunov equation

$$A^T P+PA=-\Theta \tag{12b}$$

with

$$A=\begin{bmatrix} 0 & I_l \\ -K_p & -K_d \end{bmatrix} \in R^{2l\times 2l}$$

and $\Theta=\Theta^T \in R^{2l\times 2l}$ is also a symmetric positive definite matrix, and $e=q-q_d$. Substituting (12a) into (11) results in the following closed-loop dynamics

$$\ddot{e}+K_d\dot{e}+K_p e+\kappa\,\mathrm{sgn}(B^T P^T x)=\\ -\hat{M}^{-1}(\tilde{M}\ddot{q}+\tilde{\eta})+\varepsilon, e=q-q_d \tag{13}$$

where $(\tilde{\cdot})=(.)-(\hat{\cdot})$ $\varepsilon \in R^l$ **and** represents the modeling/approximation error. Equation (13) is a linear closed-loop dynamics (if it is without the robust sliding term), however, due to the presence of the robust term $\kappa\,\mathrm{sgn}(B^T P^T x)$ the system is no longer linear. Using the FAT, the inertia and nonlinear matrices/vectors can be represented as

$$M=W_M^T\varphi_M+\varepsilon_M \tag{14a}$$

$$\eta=W_\eta^T\varphi_\eta+\varepsilon_\eta \tag{14b}$$

where $W_M \in R^{l\beta\times l}$, and $W_\eta \in R^{l\beta\times l}$ are weighting matrices, $\varphi_M \in R^{l\beta\times l}$ and $\varphi_\eta \in R^{l\beta}$ are matrices of basis function, with β referring to the number of basis function terms. Using the same set of basis functions, the corresponding estimates can be represented as

$$\hat{M}=\hat{W}_M^T\varphi_M \tag{15a}$$

$$\hat{\eta}=\hat{W}_\eta^T\varphi_\eta \tag{15b}$$

Thus, the controller (12a) becomes

$$u=\hat{W}_M^T\varphi_M\left[\ddot{q}_d-K_d(\dot{q}-\dot{q}_d)-K_p(q-q_d)+\hat{W}_\eta^T\varphi_\eta-\right.\\ \left.\kappa\,\mathrm{sgn}(B^T P^T x)\right] \tag{16}$$

and the closed-loop dynamics (13) becomes

$$\ddot{e}+K_d\dot{e}+K_p e+\kappa\,\mathrm{sgn}(B^T P^T x)=\\ -\hat{M}^{-1}(\tilde{W}_M^T\varphi_M\ddot{q}+\tilde{W}_\eta^T\varphi_\eta)+\varepsilon \tag{17}$$

Representing (17) in a state-space form

$$\dot{x} = Ax - B\Big[\hat{M}^{-1}\Big(\widetilde{W}_M^T \varphi_M \ddot{q} + \widetilde{W}_\eta^T \varphi_\eta\Big) - \varepsilon + \kappa\,\mathrm{sgn}(B^T P^T x)\Big] \tag{18}$$

Let us select the following updated laws

$$\dot{\hat{W}}_M = -Q_M \varphi_M \ddot{q}(x^T PB\hat{M}^{-1}) \tag{19a}$$

$$\dot{\hat{W}}_\eta = -Q_\eta \varphi_\eta (x^T PB\hat{M}^{-1}) \tag{19b}$$

where $Q_{(.)} \in R^{l\beta \times l\beta}$ is a positive definite adaptation matrix?

Theorem 1. The dynamics of the vibrating plate modeled in (11) with the control law, closed-loop dynamics, and the associated updating laws described in (16)-(19) are stable in the sense of the Lyapunov theory.

Proof.

Let us select the following Lyapunov-like function along with the closed-loop dynamics (18)

$$V = \frac{1}{2}x^T Px + \frac{1}{2}tr\Big(\widetilde{W}_M^T Q_M^{-1}\widetilde{W}_M + \widetilde{W}_\eta^T Q_\eta^{-1}\widetilde{W}_\eta\Big) \tag{20}$$

Taking the time-derivative of (20) and substituting (18) leads to

$$\dot{V} = -\frac{1}{2}x^T \Theta x - x^T P\Big[B\hat{M}^{-1}\Big(\widetilde{W}_M^T \varphi_M \ddot{q} + \widetilde{W}_\eta^T \varphi_\eta\Big) - B\varepsilon + B\kappa\,\mathrm{sgn}(B^T P^T x)\Big] - tr\Big(\widetilde{W}_M^T Q_M^{-1}\dot{\hat{W}}_M\Big) - tr\Big(\widetilde{W}_\eta^T Q_\eta^{-1}\dot{\hat{W}}_\eta\Big) \tag{21}$$

Equation (21) can be re-written as

$$\dot{V} = -\frac{1}{2}x^T \Theta x - tr\Big(\widetilde{W}_M^T\Big(\varphi_M \ddot{q}\big(x^T PB\hat{M}^{-1}\big) + \Phi_M^{-1}\dot{\hat{W}}_M\Big)\Big) - tr\Big(\widetilde{W}_\eta^T\Big(\varphi_\eta\big(x^T PB\hat{M}^{-1}\big) + \Phi_\eta^{-1}\dot{\hat{W}}_\eta\Big)\Big) - x^T PB(-\varepsilon + \kappa\,\mathrm{sgn}(B^T P^T x)) \tag{22}$$

Substituting (19) into the above equation to get

$$\dot{V} = -\frac{1}{2}x^T \Theta x - x^T PB(-\varepsilon + \kappa\,\mathrm{sgn}(B^T P^T x)) = -\frac{1}{2}x^T \Theta x + \chi^T \varepsilon - \sum_i \kappa_i |\chi_i| \tag{23}$$

where $\chi = B^T P^T x$. Selecting the components κ_i such that

$$\kappa_i \geq |\varepsilon_i| + \delta_i \tag{24}$$

where δ_i is a positive constant and hence (23) becomes

$$\dot{V} = -\frac{1}{2}x^T \Theta x - \sum_1^l \delta_i |\chi_i| \tag{25}$$

Equation (25) is stable in the sense of Lyapunov theory.

Remark 5. A good way for selecting the positive definite matrices *P* and *Q* in (12b) is choosing *Q* first than solving (12b) to determine *P* for a given *A*. If the produced *P* is positive definite, then the system stability is ensured.

Remark 6. Two disadvantages can be noted for the adaptive FBL technique which is: (1) the regressor matrix depends on angular link acceleration, and (2) the update adaptive law of parameter vector requires the inversion of inertia matrix which could be subject to singularity and computationally intense. These shortcomings will be resolved by using the control strategy mentioned in the next subsection.

3.2 FAT-based VVEC

Although this approach does not linearize the motion equation of the target system, it has two distinct advantages: (1) it does not require measurement /estimation of manipulator acceleration, and (2) calculation of inertia matrix inverse is not requested [3826]. In effect, this technique has been invented by Slotine and Li [37] and it depends on the sliding mode concept. Consequently, the following control law can be selected as

$$u = \hat{M}\dot{r} + \hat{\eta} - K_{ds}s - \kappa\,\mathrm{sgn}(s) \tag{26}$$

where

$$\begin{aligned} s &= \dot{q} - r = \dot{e} + \Lambda e, \\ e &= q - q_d, \\ r &= \dot{q}_d - \Lambda e. \end{aligned} \tag{27}$$

where $K_{ds} \in R^{l\times l}$ is a positive definite feedback gain matrix, $s \in R^l$ is the virtual velocity error, $r \in R^l$ is the required velocity and is equal to the desired velocity minus the scaled position error, and $\Lambda \in R^{l\times l}$ is a diagonal positive definite matrix that is related to the time constant of the closed-loop dynamics. Substituting (27) and (26) into Eq. (11) to obtain

$$M\dot{s} + K_{ds}s + \kappa\,\mathrm{sgn}(s) = \widetilde{M}\dot{r} + \widetilde{\eta} + \varepsilon \tag{28}$$

Using the FAT, we can get the following closed-loop dynamics

$$M\dot{s} + K_{ds}s + \kappa\,\mathrm{sgn}(s) = \widetilde{W}_M^T \varphi_M \dot{r} + \widetilde{W}_\eta^T \varphi_\eta + \varepsilon \tag{29}$$

As noted in Eq. (29), the inertia matrix inverse and the modal acceleration disappear from the right-hand side. Let us select the following update laws of the unknown parameter vector that can ensure convergence of tracking errors.

$$\begin{aligned} \dot{\hat{W}}_M &= Q_M \varphi_M \dot{r} s^T \\ \dot{\hat{W}}_\eta &= Q_\eta \varphi_\eta s^T \end{aligned} \tag{30}$$

where $Q_{(.)} \in R^{l\beta\times l\beta}$ $W_M \in R^{l\beta\times l}$, and $W_\eta \in R^{l\beta\times l}$ are weighting matrices, $\varphi_M \in R^{l\beta\times l}$ and $\varphi_\eta \in R^{l\beta}$ are matrices of basis function, with β referring to the number of basis function terms.

Theorem 2. The dynamics of the vibrating plate described in Eq. (11) with the control law, closed-loop dynamics, and the associated updating laws for the weighting matrices introduced in (26)-(30) respectively is stable the sense of Lyapunov theory.

Proof.

$$V = \frac{1}{2}s^T M s + tr(\widetilde{W}_M Q_M^{-1}\widetilde{W}_M + \widetilde{W}_\eta Q_\eta^{-1}\widetilde{W}_\eta) \tag{31}$$

Taking the time-derivative of (31) to obtain

$$\dot{V} = s^T M\dot{s} + \frac{1}{2}s^T \dot{M}s - tr(\widetilde{W}_M^T Q_M^{-1}\dot{\hat{W}}_M + \widetilde{W}_\eta^T Q_\eta^{-1}\dot{\hat{W}}_\eta) \tag{32}$$

Knowing that $\dot{M} = 0$ and Substituting (29) into the above equation to get

$$\begin{aligned}\dot{V} &= -s^T K_{ds}s + s^T\varepsilon + s^T\kappa\,\mathrm{sgn}(s) - \\ & tr\left(\widetilde{W}_M^T\left(-\varphi_M \dot{r}s^T + Q_M^{-1}\dot{\hat{W}}_M\right)\right) - \\ & tr\left(\widetilde{W}_\eta^T\left(-\varphi_\eta r s^T + Q_\eta^{-1}\dot{\hat{W}}_\eta\right)\right)\end{aligned} \tag{33}$$

Substituting (30) in (33) leads to

$$\dot{V} = -s^T K_{ds}s - s^T(\varepsilon - \sum_{i=1}^{l}\kappa_i|s_i|) \tag{34}$$

Selecting the components κ_i such that

$$\kappa_i \geq |\varepsilon_i| + \delta_i \tag{35}$$

where δ_i is a positive constant and hence (34) becomes

$$\dot{V} \leq -s^T K_{ds}s - \sum_{i=1}^{l}\delta_i|s_i| \tag{36}$$

Equation (34) is stable in the sense of Lyapunov theory.

3.3 FAT-based BSC

The key idea of backstepping control is to design a control law recursively by considering some of the state variables as virtual control variables and designing intermediate control laws in a backstepping manner. To this end, let us design a systematically backstepping control framework for the target plate dynamics described in (11) considering the following state variables

$$\begin{aligned} x_1 &= q \\ x_2 &= \dot{q} \end{aligned} \tag{37}$$

The dynamics can be reformulated in terms of state-space form as follows.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= M^{-1}(u+\eta) = f + gu \end{aligned} \tag{38}$$

where, $f = M^{-1}\eta, g = M^{-1}$

Two steps are required to design the target control architecture: 1) reformulating (38) in terms of error dynamics, and 2) capturing the control law and the associated updating laws based on a Lyapunov function.

Step 1. Let us denote the error dynamics $\widetilde{x}$ for x_1 as

$$\widetilde{x}_1 = x_1 - y_d \tag{39}$$

where $y_d = q_d$. Hence the associated dynamics with (39) that ensure stability are

$$\dot{\widetilde{x}}_1 = -k_1\widetilde{x}_1 + \widetilde{x}_2 \tag{40a}$$

where

$$\widetilde{x}_2 = x_2 - \rho, \dot{\rho} = -k_1(x_1 - y_d) + \dot{y}_d \tag{40b}$$

with $\rho \in R^l$ referring to the virtual input control variable that stabilizes (40a), and $k_1 \in R^{l\times l}$ is a positive definite gain matrix. Now, let us describe the dynamics $\widetilde{x}_2$ as

$$\dot{\widetilde{x}}_2 = \hat{f} + \hat{G}u + \widetilde{W}_f^T\varphi_f + \widetilde{W}_g^T\varphi_g u - \dot{\rho} + \varepsilon \tag{41}$$

where $W_{(.)} \in R^{l\beta\times l}$ is a weighting matrix

and $\varphi_f \in R^{l\beta}$ $\varphi_g \in R^{l\beta\times l}$ are basis function matrices.

Step 2. Now define the following Lyapunov function for the modified dynamics described in (40a) and (41)

$$V = \frac{1}{2}\left[\widetilde{x}_1^T\widetilde{x}_1 + \widetilde{x}_2^T\widetilde{x}_2 + Tr(\widetilde{W}_f^T Q_f^{-1}\widetilde{W}_f^T + \widetilde{W}_g^T Q_g^{-1}\widetilde{W}_g^T\right] \tag{42}$$

where $Q_{(.)} \in R^{l\beta\times l\beta}$ is an adaptation matrix. Taking the time-derivative of (42) and substituting (40a) and (41) to obtain

$$\begin{aligned}\dot{V} &= -\widetilde{x}_1^T k_1\widetilde{x}_1 + \widetilde{x}_2\left[\widetilde{x}_1 + \hat{f} + \hat{G}u + \widetilde{W}_f^T\varphi_f + \widetilde{W}_g^T\varphi_g u - \dot{\rho}\right] - \\ & Tr\left(\widetilde{W}_f^T Q_f^{-1}\dot{\hat{W}}_f^T + \widetilde{W}_g^T Q_g^{-1}\dot{\hat{W}}_g^T\right)\end{aligned} \tag{43}$$

Expanding (43) with some manipulations to get

$$\begin{aligned}\dot{V} &= -\widetilde{x}_1^T k_1\widetilde{x}_1 - \widetilde{x}_2^T k_2\widetilde{x}_2 + \widetilde{x}_2\left[\widetilde{x}_1 + k_2\widetilde{x}_2 + \hat{f} + \hat{G}u - \dot{\rho} + \varepsilon\right] + \\ & Tr\left(\widetilde{W}_f^T\left(\varphi_f\widetilde{x}_2^T - Q_f^{-1}\dot{\hat{W}}_f^T\right) + \widetilde{W}_g^T\left(\varphi_g u\widetilde{x}_2^T - Q_g^{-1}\dot{\hat{W}}_g^T\right)\right)\end{aligned} \tag{44}$$

Choosing the following control law with associated updating laws

$$\begin{aligned} u &= \hat{G}^{-1}\left[-\hat{f} - (\widetilde{x}_1 + k_2\widetilde{x}_2) + \dot{\rho} - \kappa\,\mathrm{sgn}(\widetilde{x}_2)\right] \\ \dot{\hat{W}}_f^T &= Q_f\varphi_f\widetilde{x}_2 \\ \dot{\hat{W}}_g^T &= Q_g\varphi_g u\widetilde{x}_2 \end{aligned} \tag{45}$$

Then we can obtain

$$\begin{aligned}\dot{V} &= -\widetilde{x}_1 k_1\widetilde{x}_1 - \widetilde{x}_2 k_2\widetilde{x}_2 + \widetilde{x}_2^T\varepsilon - \widetilde{x}_2^T\kappa\,\mathrm{sgn}(\widetilde{x}_2) \\ \dot{V} &= -\widetilde{x}_1 k_1\widetilde{x}_1 - \widetilde{x}_2 k_2\widetilde{x}_2 + \widetilde{x}_2^T\varepsilon - \sum_i \kappa_i|\widetilde{x}_{2i}|\end{aligned} \tag{46}$$

We see that choosing the component κ_i such that $\kappa_i \geq |\varepsilon_i| + \delta_i$, where δ_i is strictly positive leads to

$$\dot{V} \leq -\tilde{x}_1 k_1 \tilde{x}_1 - \tilde{x}_2 k_2 \tilde{x}_2 - \sum_i \delta_i |\tilde{x}_{2i}| \tag{47}$$

Table I makes a detailed comparison for the aforementioned three control algorithms.

Table 1. A Comparison between the FBL, the VVEC, and the BSC strategies

FAT-FBL	• Control law: Eq. (12). • Closed-loop dynamics: Eq. (13). • Updating laws: Eq. (19). • Although the control law is nonlinear, the closed-loop dynamics are linear. • The closed-loop dynamics and the updating laws include acceleration variable and inertia inverse matrix that make computational problems such as singularity. • One of the preferred solutions to the above is to assume that the inertia matrix is known (identification procedure is required). • It does not need satisfaction of passivity conditions to ensure stability.
FAT-VVEC	• Control law: Eq. (26). • Closed-loop dynamics: Eq. (29). • Updating laws: Eq. (30). • Both the control law and the closed-loop dynamics are nonlinear. • The closed-loop dynamics and the associated updating laws do not include inertia inverse matrix or modal acceleration. • However, there is a constraint associated with the design of this controller is that the passivity condition should be satisfied to ensure stability. This includes $s^T(\dot{M} - 2C)s = 0$. However, for our plate $\dot{M} = 0$ while $s^T Cs \geq 0$ and hence the stability can be still guaranteed. In effect, the damping term is integrated with the nonlinear term to avoid the constraint $s^T Cs \geq 0$.
FAT-BSC	• Control law: Eq. (45a). • Closed-loop dynamics: Eqs. (40a) and (41). • Updating laws: Eq. (45b). • Two disadvantages associated with control law and the related updating laws are 1) presence of $\hat{g}^{-1}$ in the control law that would make singularity problem, and 2) presence of the input control in the updating law associated with the unknown term $\hat{g}$. • Due to the recursive technique required, for a dynamic system having higher orders, the computations would be tedious and complex expressions would result. • The above-mentioned problems can be solved if we assume that $\hat{g}$ is known (system identification is required to estimate $\hat{g}$). • It does not need satisfaction of passivity conditions to ensure stability.

4. SIMULATION RESULTS AND DISCUSSIONS

A flexible plate (shown in Fig. 1) with all edges that are simply supported is simulated using MATLAB /Simulink package for verification of the proposed control structure, see Table II for more details on physical parameters used in simulation experiments. Two collocated piezo patches are bonded on the surface of the plate. A pulse force of value 10 N during 1 sec is applied at a location ($a/2, b/2$) to excite the vibration of the beam. Since the dominant modes locate at the low-frequency region of the frequency response, the first two mode shapes are considered in our simulation experiments. Before going into implementation of control architecture, let us recall (11a) but with $d = 0$.

$$M\ddot{q} + C\dot{q} + Kq + \sigma = u \tag{48}$$

From Eq. (48), we can see that the dynamic equation includes the convention stiffness term (the fourth term) and a nonlinearly coupled cubic stiffness term that complicates the frequency response. By dividing (49) by M , the coefficient $(M^{-1}K)$ represents the linearized natural frequency. However, the following points should be noted [36]:

Table 2. Physical parameters and feedback gains used in simulation experiments

Plate	$a = 400[mm], b = 350[mm],$ $h = 3[mm], E = 210[GPa],$ $\rho = 7800[kg/m^3], \nu = 0.3.$
Piezo-materials	$a_p = 25[mm], b_p = 25[mm],$ $h_p = 0.25[mm], E_p = 6.3\times10^{10}[N/m^2]$
Control gains	$\Lambda = 20I_2, Q_M = 0.01I_{22},$ $Q_\eta = 75I_{22}, K_{ds} = 300I_2,$

1. If the excitation force term is added, peak resonances in natural frequencies may occur.
2. Due to the presence of nonlinear cubic stiffness term, the natural frequency may not be the same as the

linearized natural frequency and hence nonlinear resonances may occur. In effect, if we consider only one mode shape for (49), then the equation is similar to Duffing's equation where the system stiffness varies as a function of the modal coordinate. Here there will be a jump in the frequency spectrum and as well as the type of nonlinear stiffness is determined based on the coefficient of cubic modal coordinate, see page 48 of [39] for more details.

3. The coefficient $M^{-1}C$ represents the damping term that can enhance vibration suppression of the target dynamic system. It should be noted that this work assumes that the damping coefficient is constant while the plate has constant mass distribution and hence the damping is mass proportional, i.e. there is no dynamic coupling associated with damping term, see page 14 of [40] for more details.

4. There are different methods for modal analysis of nonlinear vibration, however, this topic is beyond the scope of this paper and the reader is referred to [41-43] for more details.

Now let us come back to the implementation of control algorithms for the target vibrating plate. As mentioned previously in Table II, the FAT-based VVEC has two significant features: 1) the control structure does not include the inertia inverse matrix, and also 2) it does not require the calculation of modal acceleration and hence this technique is a powerful tool comparing with FAT-based FBL and FAT-based BSC. Because of the above, the simulation experiments are focused on FAT-VVEC under the conditions that the system parameters are fully unknown. One intrinsic disadvantage for this technique is that the term $s^T(\dot{M}-2C)s$ should be equal or less than zero to ensure stability.

This constraint is equivalent to satisfying the passivity condition for the target dynamic system. Fortunately for our plate system, the time-derivative for the inertia matrix equals zero $\dot{M}=0$, besides, the damping term of the plate system is integrated with the non–linear term η and hence the stability of the vibrating plate can be proved to overcome the condition of satisfaction of passivity property. The control structure for FAT-based VVEC includes feedforward terms that are updated based on FAT and a PD feedback term plus a sliding term for compensating any modeling error if exists. The orthogonal Chebyshev polynomials are used as approximators for the FAT. The number of basis functions used is 15 and the initial values for the weighting coefficient matrix are assumed zero. For simplicity, it is assumed that modeling error is neglected and hence the robust sliding term can also be neglected.

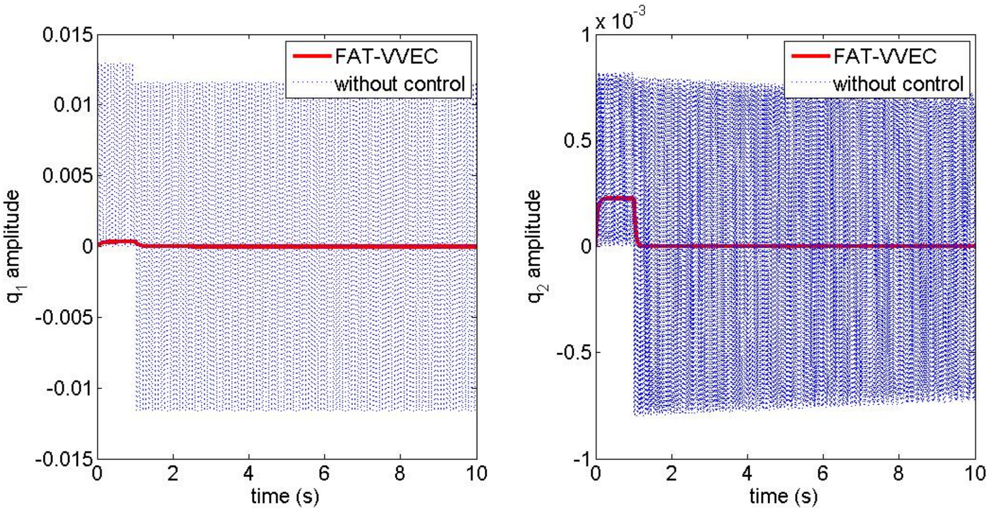


Fig. 2: Modal amplitude response

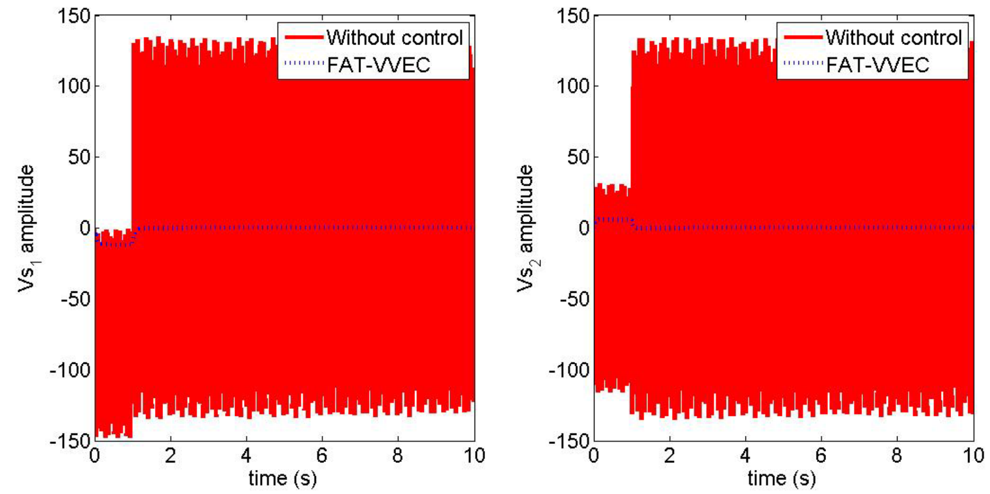
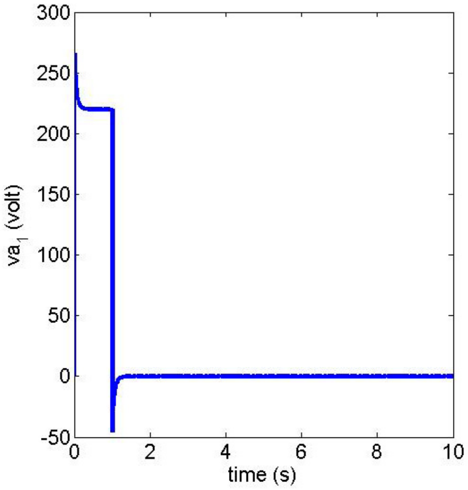


Fig. 3: Sensor voltage response



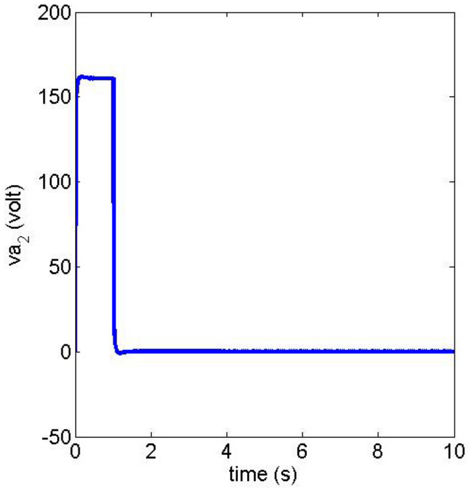


Fig. 4: Input control piezo-voltages

Figures 2 and 3 show the responses of modal amplitudes and sensor voltages for the vibrating smart plate considering the first two-mode shapes. As expected, for an open-loop system the modal coordinates undergo oscillation due to the application of the impulse force. Whereas, the proposed VVEC strategy would suppress the modal vibrations precisely despite the presence of model uncertainty. The feedback and adaptation gains used in the simulation are listed in Table II. The input control response is shown in Fig. 4. The Pseudo-inverse matrix plays an important role in the calculation of optimal input voltage if the number of piezo-actuators is not equal to the investigated number of mode shapes, see [19] for details.

5. CONCLUSIONS

This work makes a comprehensive study on modeling and vibration control of smart thin plate-like structures considering cubic nonlinearities related to axial stretching loads. Three control strategies are investigated in detail considering features and shortcomings. The function approximation techniques are used as a basis for control structures that make the control task easily dealing with complex robotic systems. The VVEC shows superior benefits in comparison with others; the inertia inverse matrix and the modal acceleration are avoided in the controller structure. Therefore, the simulation experiments here are focused on this control structure. In effect, the VVEC is called passivity-based control in the robotic community and it is a strong tool for stabilization and tracking control of complex dynamic systems. An important point should be mentioned that the passivity property associated with the VVEC strategy is guaranteed here since the inertia matrix is assumed constant in modeling and the Coriolis components are negative. However, integration of piezoelectric materials inertias with plate inertia matrix makes the system is coupled, besides complex motions of plate structure may include variant-time inertia terms that complicate the proof of passivity constraint. How–ever, future work could be focused on the following points:

1. Modeling and control of 3D plate-like structure considering different smart actuation /sensing scenarios.
2. Modeling and control of floating base robots with integrated flexible structures.
3. Extending the current control algorithms for vibration suppression of smart shells; here the nonlinear restoring force term is very complex.
4. Integration of adaptive approximation observer with the control architecture is important for flexible structures interacting with external environments such as aero-elastic nonlinear structures.

REFERENCES

[1] Moheimani, R., Fleming, A. J., Halim, D.: Spatial control of vibration: theory and experiments, World Scientific Publishing Company, 2003.

[2] Vepa, R.: Dynamics of smart structures, John Wiley and Sons, Ltd, 2010.

[3] Wagg, D.: Control and exploitation of nonlinearity in smart structures. In: D.J. Wagg, L. Virgin (eds) Exploiting Nonlinear Behavior in Structural Dynamics. CISM Courses and Lectures, vol 536. Springer, Vienna, 2012.

[4] Teng, T., Liang, C., Liao, C.: Nonlinear forced vibration analysis of the rectangular plates by the Fourier series method. Computational Mechanics 23, 1–7, 1999.

[5] Bajović, M., Zivanović, M., Rašuo, B., Stojaković, P.: A New General Approach to Airplane Rotation, Transactions of the Japan Society for Aeronautical & Space Sciences, Vol.53, No.180, August 2010, pp. 130-137, doi: 10.2322/tjsass.53.130.

[6] Ilic, Z., Rašuo, B., Jovanovic, M., Jankovic, D.: Impact of changing quality of air/fuel mixture during flight of a piston engine aircraft with respect to vibration low frequency spectrum, FME Transactions, 41 (1), 25-32, 2013

[7] Szilard, R.: Theories and applications of plate analysis: classical, numerical and engineering methods, Johna Wiley & Sons, Inc. 2004.

[8] Garinis, D., Dinulovic, M., Rašuo, B.: Dynamic analysis of modified composite helicopter blade, FME Transactions, Vol. 40, No. 2, pp. 63-68, 2012.

[9] Dinulović, M., Rašuo, B., Trninić, MR., Adžić, V.M.: Numerical modeling of nomex honeycomb core composite plates at meso scale level, FME Transactions Vol. 48, No. 4, pp. 874-881, 2020.

[10] Dowell, E.H.: Aeroelasticity of plates and shells, Springer Netherlands, 1975.

[11] Amabili, M.: Nonlinear Vibrations and Stability of Shells and Plates, Cambridge University Press, Cambridge, 2008.

[12] Awrejcewicz, J., Krysko, V.A., Krysko, A.V.: Spatio-temporal chaos and solitons exhibited by von Kármán model, Int. J. Bifurc Chaos. 12, 1465–1513, 2002.

[13] Gordnier, R.E., Visbal, M.R.: Development of a three-dimensional viscous aeroelastic solver for nonlinear panel flutter, J. Fluids Struct. 16, 497–527, 2002.

[14] Sathyamoorthy, M.: Nonlinear vibrations of plates: an update of recent research developments, Appl. Mech. Rev. 49, S55–S62, 1996.

[15] Ilic, Z., Rašuo, B., Jovanovic, M., Jovicic, S., Tomic, L., Jankovic, M. and Petrašinovic, D.: The efficiency of passive vibration damping on the pilot seat of piston propeller aircraft, Measurement, Vol. 95, pp. 21-32, 2017. doi: 10.1016/j.measurement.2016.09.042

[16] Ilic, Z., Rašuo, B., Jovanovic, M., Pekmezovic, S., Bengin, A., Dinulovic, M.: Potential connections of cockpit floor - seat on passive vibration reduction at a piston propelled airplane, Technical Gazette, Vol.21 No.3, June 2014, ISSN: 1330-3651, (pp. 471-478).

[17] Lee, Y.K.: Active vibration control of a piezoelectric laminate plate using spatial control approach, MSc Thesis, Department of Mechanical Engineering, University of Adelaide, 2005.

[18] Gupta, V., Sharma, M., Thakur, N., Singh, S.P.: Active vibration control of a smart plate using a piezoelectric sensor–actuator pair at elevated temperatures, Smart Materials, and Structures, Volume 20, Number 10, 2011.

[19] Yaman, Y., Calıskan, T., Nalbantoglu, V., Prasad, E., Waechter, D., Yan, B.: Active Vibration Control of a Smart Beam, Canada−US CanSmart Workshop on Smart Materials and Structures (Canada Proceedings, Montreal), pp. 137–147, 2001.

[20] Dimitriadis, E.K., Fuller, C.R., Rogers, C.A.: Piezoelectric actuators for distributed vibration excitation of thin plates, journal of vibration and acoustics, vol. 113, pp. 100-107, 1991.

[21] Abdullah, E.J., Majid, D.L., Romli, F.I., et al.: Active control of strain in a composite plate using shape memory alloy actuators, Int J Mech Mater Des 11, 25–39, 2015.

[22] Tairidis, G., Foutsitzi, G., Koutsianitis, P., Stavroulakis, G.E.: Fine tuning of a fuzzy controller for vibration suppression of smart plates using genetic algorithms, Advances in Engineering Software Volume 101, Pages 123-135, 2016.

[23] Vinyas, M.: Vibration control of skew magneto-electro-elastic plates using active constrained layer damping, Composite Structures Volume 208, Pages 600-617, 2019.

[24] Farrell, J:A., Polycarpou, M.M.: Adaptive approximation based control: unifying neural, fuzzy and traditional adaptive approximation approaches, John Wiley and Sons, Ltd, 2006.

[25] Al-Shuka, H.F.N.: On local approximation-based adaptive control with applications to robotic manipulators and biped robots, Int. J. Dynam. Control 6, 339–353, 2018.

[26] Huang, A.-C., Chien, M.-C.: Adaptive control of robot manipulators: a unified regressor-free approach, World Scientific, 2010.

[27] Al-Shuka, H.F.N., Abas, E.N.: Regressor-free adaptive vibration control of constrained smart beams with axial stretching, SN Appl. Sci. 2, 2146 (2020). https://doi.org/10.1007/s42452-020-03859-9.

[28] Al-Shuka, H.F.N.: FAT-Based Adaptive Backstepping Control of an Electromechanical System with an Unknown Input Coefficient, FME Transactions 49(1), 113-120, 2021.

[29] Al-Shuka, H.F.N.: Proxy-Based Sliding Mode Vibration Control with an Adaptive Approximation Compensator for Euler-Bernoulli Smart Beams, Journal Européen des Systèmes Automatisés 53(6), 825-834, 2021.

[30] Al-Shuka, H.F.N., Corves, B., Zhu, W.-H.: Function approximation technique-based adaptive virtual decomposition control for a serial-chain manipulator, Robotica 32(3), 375-399, 2014.

[31] Al-Shuka, H.F.N, Song, R.: Hybrid Regressor and Approximation-Based Adaptive Control of Robotic Manipulators with Contact-Free Motion, 2nd IEEE Advanced Information Management, Communicates, Electronic and Automation Control Conference(IMCEC 2018), 325-329, 2018.

[32] Kaleel, A., Al-Shuka, H.F.N., Hussein, O.: Adaptive Approximation-Based Feedback Linearization Control for a Nonlinear Smart Thin Plate, International Journal of Mechanical Engineering and Robotics Research 10(8), 458-463, 2021.

[33] Al-hamadani, A.N.S., Kadhim, K.J., Al-Shuka, H.F.N.: Proportional-derivative pd vibration control with adaptive approximation compensator for a nonlinear smart thin beam interacting with fluid, International Journal of Mechanical Engineering and Robotics Research Vol. 10, No. 10, 566-571, 2021

[34] Al-Shuka, H.F.N., Song, R.: Decentralized adaptive partitioned approximation control of high degrees-of-freedom robotic manipulators considering three actuator control modes, International Journal of Dynamics and Control 7(2), 744-757, 2019.

[35] Stevanović, I., Rašuo, B.: Development of a Miniature Robot Based on Experience Inspired by

Nature, FME Transactions, Vol. 45, No. 1, pp. 189-197, 2017.

[36] Wagg, D., Neild, S.: Nonlinear vibration with control, for flexible and adaptive structures, Springer Cham, 2010.

[37] Slotine, J.J., Li, W.: Applied nonlinear control, Pearson, 1991.

[38] Spong, M.W., Hutchinson, S., Vidyasagar, M.: Robot modelling and control, Wiley, 2005.

[39] Jauregui, J.C.: Parameter identification and monitoring of mechanical systems under nonlinear vibration, Woodhead Publishing, 2015.

[40] Thomsen, J.J.: Vibrations and stability: advanced theory, analysis, and tools, Springer, 2003.

[41] Nayfeh, A.H., Lacarbonara, W., Chin, C.-M.: Nonlinear normal modes of buckled beams: Three-to-one and one-to-one internal resonances. Nonlinear Dynamics, 18, 253–273, 1999.

[42] Vakakis, A.F., Manevitch, L.I., Mikhlin, Y.V., Pilipchuk, V.N., Zevin, A.A.: Normal modes and localization in nonlinear systems, Wiley: New York, 1996.

[43] Shaw, S.W., Pierre, C.: Normal modes for non-linear vibratory systems. Journal of Sound and Vibration, 164 (1), 85–124, 1993.

NOMENCLATURE

$a_{rs}(t)$	Time function associated with Airy stress function
D	Flexural rigidity
E	Young's modulus
F	Airy stress function
h	Plate thickness
$H(.)$	Heaviside step function
I_n	Identity matrix of *(n,n)* dimension
K_d	A positive definite feedback gain matrix associated with derivative-control term, $\in R^{l\times l}$
K_p	A positive definite feedback gain matrix associated with proportional-control term, $\in R^{l\times l}$
K_{ds}	A positive definite feedback gain matrix, $\in R^{l\times l}$
m	Mass of the plate
N_a	Number of piezo-actuators
$M_p(.)$	Actuator external moment per unit length
p	External transverse load per unit area
P	A symmetric positive definite matrix satisfying the Lyapunov equation, $\in R^{2l\times 2l}$
$q_{mn}(t)$	Modal amplitude
q_d	desired value for the modal vector
$Q_{(.)}$	An adaptation matrix
V	A Lyapunov-like function
$w(x,y,t)$	Deflection of plate
$W_{(.)}$	Weighting matrices

Greek symbols

∇^4	Biharmonic operator
$\psi_{mn}(x,y)$	Mode shape of the deflection function
$\phi_{rs}(x,y)$	Space functions associated with Airy stress function
κ	A robust sliding gain, $\in R^{l\times l}$
$\varphi_{(.)}$	Basis function matrices
β	Number of basis function
λ	A constant depending on the physical parameters of the plate and the piezo-actuators
ε	The modelling/approximation error, $\in R^l$
$v_{ak}(t)$	Input piezo-voltage
v	Poisson's ratio

НЕЛИНЕАРНА СТРАТЕГИЈА КОНТРОЛЕ ПАМЕТНИХ ТАНКИХ ПЛОЧА СА КУБНОМ НЕЛИНЕАРНОСТИМА ПРИМЕНОМ ТЕХНИКЕ АПРОКСИМАЦИЈЕ ФУНКЦИЈАМА (ТАФ)

Х.Ф.Н. Ал-Шука, Е.Н. Абас

У овом раду су уведена три нелинеарна управљачка решења за регулацију вибрирајуће нелинеарне плоче с обзиром на несигурност модела. Ова решења су контрола линеаризације повратне спреге (ФБЛ), контрола заснована на грешци виртуелне брзине (ВВЕЦ) и контрола повратног корака (БСЦ). У ФБЛ контроли, нелинеарни закон управљања је ди–зајниран са линеарном динамиком затворене петље тако да је осигурана динамичка стабилност. Док су ВВЕЦ (или такозвани приступ заснован на пасив–ности у заједници роботике) превазиђена огра–ничења линеаризације повратне спреге. Са друге стране, БСЦ бира виртуелне контролне промењљиве са стабилизованим средњим законима управљања заснованим на теорији Љапунова. Описано је сис–тематско моделирање циљне вибрационе плоче са пиезо-закрпама. У ствари, с обзиром да нелинеарни утицај чини да су резултовани облици модова за вибрирајућу структуру веома повезани и потребан је пажљив дизајн управљања. Користећи Галеркин приступ, парцијална диференцијална једначина за паметну плочу се трансформише у одређене обичне диференцијалне једначине; успостављен је модел са више улаза и више излаза. Наведене стратегије контроле се детаљно процењују и истражују. У суштини, они су моћни алати за рад са нелинеарним динамичким системима, међутим, ВВЕЦ би се могао сматрати супериорнијим у поређењу са ФБЛ контролом и БСЦ пошто пројектована контролна структура не укључује инерциону инверзну матрицу и модално координатно убрзање које би могло да учини рачунарским проблеме. Као резултат,

експерименти симулације су били фокусирани на ВВЕЦ стратегију, а ова друга је имплементирана на једноставно подупрту структуру танких плоча са заједничким пиезо-закрпама. Резултати показују валидност предложене архитектуре управљања.