

An Empirical Approach to a Comprehensive Damage-Equivalent Stress Function for Fatigue

This paper develops an empirical damage-equivalent stress function for fatigue. Classical analysis methods are used to 'fit' an equation to a number of S-N curves for various grades of carbon steel. The resulting equivalent-damage stress function is applicable to steels subjected to a wide range of heat treatments, from normalised up to hardened and tempered to 1900MPa. It is also applicable to a wide range of stress concentrations, unnotched up to $K_t = 5.0$ and typical of screw threads. A range of stress ratios and mean stresses are also considered. The function overcomes some of the limitations of existing methods of 'correcting' for mean stress. Existing methods are limited in that, while they may give good results over a range of conditions, there are some circumstances where the results are highly inaccurate. The damage-equivalent stress function is suitable for use in automated calculation procedures such as spreadsheets, MathCAD ©, and SMATHStudio ©.

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1. INTRODUCTION

Central to any fatigue analysis is the S-N curve, a plot of characteristic stress (S) against the number of life cycles (N). Ideally a stress engineer would like a multitude of S-N curves applicable to a range of geometries and local configurations for each material. However, the experimental procedures to produce S-N curves are complex, require specialist tensile test equipment capable of applying cyclic loading at controlled strain rates, and are time-consuming. Hence, there is a limited amount of fatigue data for materials readily available in real life.

Typically, available data will comprise one S-N curve for alternating stress with zero mean stress, produced using unnotched, polished specimens. S-N curves may supplement this for a small selection of stress concentration factors obtained from notched specimens and/or S-N curves for alternating stress with a given mean stress or stress ratio $R = \text{minimum stress} / \text{maximum stress}$. In some instances, curve fitting techniques are applied to the data for each S-N curve, or a group of S-N curves, enabling them to be expressed as an equation.

Fatigue is predominantly dominated by the alternating stress range but is also influenced by other factors such as the mean stress, surface finish, porosity, and the geometry of the mechanical components that can result in an elastic stress concentration. It is known that positive mean stress acts to reduce the number of life cycles a component can achieve, whereas negative mean stress acts to increase its life. However, obtaining

S-N curves for a wide range of mean stresses or stress ratios is not practical or economical. Hence, several methods of 'correcting' for mean stress by determining a 'damage-equivalent stress' for fatigue have been developed. The damage-equivalent stress is the alternating stress under fully reversal load conditions that would produce an amount of damage equivalent to that caused by the combination of both an alternating and non-zero mean stress. In essence, the known or calculated alternating stress is factored prior to using it with the S-N curve produced using zero mean stress.

Several methods have been developed to determine damage-equivalent stress, most notably the modified Goodman or Goodman-Haig diagram and other methods proposed by Gerber, Soderberg, and Smith-Watson-Topper. However, each of these methods has limitations in their application to fatigue analyses. The modified Goodman diagram is conservative for ductile materials and optimistic for mean compressive stresses [1]. Gerber is better than Goodman for high mean stress levels but is not applicable to mean compressive stresses [1]. Soderberg is more conservative than the modified Goodman method [1]. Smith-Watson-Topper is better than Goodman for low mean stress levels [1].

The work presented here aims to define a method of determining the "damage-equivalent stress" for fatigue suitable for use in the fatigue analysis of preloaded bolted assemblies. Preloaded bolts have high mean stress, typically 60% to 80% of proof stress, and a relatively small alternating stress range.

The log-linear nature of S-N curves means that the calculated fatigue life is very sensitive to stress. Hence, it is important to determine applied stresses with high accuracy. Some of the detail analysis techniques described in references [2 to 5] are particularly suited to this purpose. Similarly, the method of calculating the fatigue damage-equivalent stress also needs to introduce the minimum of error. The high mean stresses asso-

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ciated with preloaded bolts result in 'corrections' for mean stress having to be made over a large increment. The method of determining the damage-equivalent stress has to be able to deal with these large increments. This requirement virtually rules out the use of both the Goodman and Soderberg methods.

2. MATERIALS

The S-N curves referenced by this analysis were obtained from 'Metallic Materials Properties Development and Standardization (MMPDS),' reference [6]. The materials and the elastic stress concentration for the curves used in the analysis are presented in Table 1.

Table 1. Materials considered in the analyses

Material	Condition: Tensile Strength	Stress Concentration	Product form
AISI 4130	Normalised (<i>Ftu</i> 117ksi) (807MPa)	Unnotched $K_t = 1.5$ $K_t = 2.0$ $K_t = 4.0$ $K_t = 5.0$	Sheet 0.075 inch (1.905mm) thick
	<i>Ftu</i> 180ksi (1241MPa)	Unnotched $K_t = 2.0$ $K_t = 4.0$	Sheet 0.075 inch (1.905mm) thick
AISI 4340	<i>Ftu</i> 200ksi (1379MPa)	Unnotched $K_t = 3.3$	Rolled bar 1.125 inches (28.575mm) diameter
300M	<i>Ftu</i> 280ksi (1931MPa)	Unnotched $K_t = 2.0$ $K_t = 3.0$ $K_t = 5.0$	Die forged
Note: 300M can be regarded as a modified AISI 4340			

The tensile strength and yield/proof stress of normalised AISI 4130 are assumed to be the tensile strength *TUS* and the yield/proof stress *TYS* quoted in MMPDS-03 Figure 2.3.1.2.8(a) for unnotched specimens. It should be noted that the tensile test used to produce these values would have been carried out at the strain rate used for the fatigue tests. Hence they could be slightly higher than those produced under a quasi-static tensile test, as would usually be performed to determine material properties.

Additional data used to test the final damage-equivalent stress equation is presented in Table 2. This data was not used in the analysis because there was an insufficient number of S-N curves for each material condition to provide 'points' for equation fitting.

Table 2. Materials used as validation cases

Material	Condition: Tensile Strength	Stress Concentration	Product form
AISI 4340	<i>Ftu</i> 125ksi (862MPa)	Unnotched $K_t = 3.3$	Rolled bar 1.125 inches (28.575mm) diameter
	<i>Ftu</i> 150ksi (1034MPa)	Unnotched $K_t = 3.3$	Rolled bar 1.12 inch (28.448mm) diameter

3. METHODOLOGY

It was assumed that an equation for the damage-equivalent stress for fatigue would take the form:

$$\sigma_{equ} = \sigma_{alt} \cdot f_{n1} \quad (1)$$

where f_{n1} is a function of the material properties, particularly the tensile strength *Ftu* and/or the yield or proof stress *Fty*, the stress ratio *R*, mean stress σ_{mean} , and the stress concentration K_t .

Equation (1) would be used to calculate the damage-equivalent stress for fatigue applicable to a curve with a specific elastic stress concentration factor. It would not be appropriate to use equation (1) to calculate the damage equivalent stress for a curve with a different elastic stress concentration factor. Hence, the part of the function involving K_t would be a scaling factor, constant for the curve being used.

The part of the function involving the tensile strength, or proof strength, of the bolt material, *Ftu* and *Fty*, respectively, would ideally need to be a dimensionless stress function. The modified Goodman or Haig equation and the Gerber equation use a dimensionless ratio:

$$\frac{\sigma_{mean}}{Ftu}$$

The stress ratio *R* is the ratio of minimum and maximum cyclic stresses:

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

Hence, the part of the function involving the stress ratio *R* is also a function of stresses due to loading.

The part of the function involving the mean stress σ_{mean} has to result in $s_{equ} = s_{alt}$ when $\sigma_{mean} = 0$; hence $f_{n1} = 1$ when $\sigma_{mean} = 0$. Therefore, two possible shapes for the function f_{n1} were considered:

$$f_{n1} = 1 + f_{n2} \quad (2a)$$

and

$$f_{n1} = \frac{1}{1 - f_{n2}} \quad (2b)$$

The function f_{n2} in equation (2a) is not necessarily of the same form as the function f_{n2} in equation (2b) and would not have the same numerical value.

Equation (2b) takes a similar form to those for the modified Goodman/Haig or the Gerber equations for damage-equivalent stress for fatigue.

Both equation (2a) and equation (2b) were considered during the analysis. It was found that equation (2a) provided the more accurate prediction of damage-equivalent stress for fatigue. Hence, only the work involving equation (2a) is presented in this paper.

4. EFFECT OF STRESS RATIO ON THE DAMAGE-EQUIVALENT STRESS FOR FATIGUE EQUATION

The first step in studying the effect of the stress ratio was to identify groups of S-N curves for each of the

material specifications being considered in the analysis. An equation relating stress ratio R to the damage-equivalent stress for fatigue σ_{equ} was assumed, and each group of S-N curves was then used to find constants used in defining the assumed equation.

4.1 S-N Curves for AISI 4340 Carbon Steel

The first group of S-N curves was for AISI 4340 carbon steel, heat treated to give a tensile strength of 1379MPa (200ksi). Three different S-N curves were each defined as a set of data points. The criteria for selecting the S-N curves were that they were for the same material, had the same stress concentration of $K_t = 3.3$, and shared a common range of mean stresses σ_{mean} . The variables between each of the curves were the stress ratios R . The stress ratios of the chosen curves were $R = 0.43$, $R = 0.60$ and $R = 0.74$, reference Figure 2.3.1.3.8(1) of MMPDS-03 [6]. Figure 1 shows plots of the alternating stress against life for each of the data sets and the data points used in the analysis.

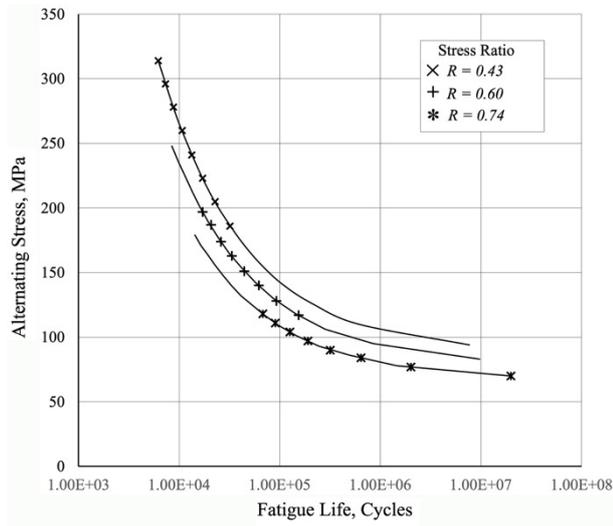


Figure 1. AISI 4340 $F_{tu} = 1397\text{MPa}$, $K_t = 3.3$

The first step in the analysis was to determine numerical values for function f_{n2} . Hence, from equations (1) and (2a) ;

$$\sigma_{equ} = \sigma_{alt} \cdot (1 + f_{n2}) \quad (3)$$

Rearranging

$$f_{n2} = \frac{\sigma_{equ}}{\sigma_{alt}} - 1 \quad (4)$$

Equation (4) was then used to calculate values of f_{n2} for each of the three sets of data considered. Data sets 1, 2 and 3 related to stress ratios of $R = 0.43$, $R = 0.60$ and $R = 0.74$ respectively. The values of function f_{n2} for data set 1 were calculated to be within the range of 0.551 to 0.555, representing a variation of 0.4% from the mean value. Similarly, the values function f_{n2} for data set 2 were calculated to be within the range of 0.757 to 0.765, representing a variation of 0.5% from the mean value. Finally, the values function f_{n2} for data set 3 were calculated to be within the range of 1.067 to 1.105, representing a variation of 1.2% from the mean value.

It was concluded that the mean stress σ_{mean} had a negligible effect on the value of the function f_{n2} . Hence, since data sets 1, 2, and 3 were for the same material and stress concentration K_t , it was also concluded that the numerical values for f_{n2} were a function of the material tensile strength or yield/proof stress the stress ratios R and the stress concentration K_t .

Assuming the form of an equation to describe the function f_{n2} in terms of the stress ratio R that also meets the criteria $f_{n1} = 1$ when $\sigma_{mean} = 0$;

$$f_{n2} = f_{n3} \cdot (1 + R)^{a_1} \quad (5)$$

The terms for f_{n3} and a_1 will be functions of the material properties and stress concentration K_t . Rearranging equation (5);

$$f_{n3} = \frac{f_{n2}}{(1 + R)^{a_1}} \quad (6)$$

Since data sets 1, 2, and 3 were all for the same material, having the same tensile strength, and for the same stress concentration K_t and shared a common range of mean stresses, it was possible to assume that the numerical value of function f_{n3} would be the same for each of the data sets being considered. That is;

$$f_{n3} \text{ for data set 1} = f_{n3} \text{ for data set 2} = f_{n3} \text{ for data set 3}$$

Using this assumption, it was possible to adopt an iterative approach to calculate an optimum value for the constant a_1 that gave the minimum amount of variation in the values of the function f_{n3} for any of the data points of data sets 1, 2, and 3.

This iterative solution showed that a value of $a_1 = 3.349$ gave the optimum condition. The values of function f_{n3} were calculated to be within the range of 0.157 to 0.167; hence, the overall variation for the values of function f_{n3} was 3.3%. Individually, the values of function f_{n3} were within the range of 0.166 to 0.167 for data set 1, 0.157 to 0.158 for data set 2, and 0.163 to 0.167 for data set 3. The individual variations in the values of the functions f_{n3} were the same as those for the values of f_{n2} , namely 0.2%, 0.3%, and 2.8% for data sets 1, 2, and 3, respectively.

The overall variation in the values of f_{n2} across data sets 1, 2, and 3 was 33.5%, which is significantly larger than the variation in the values of f_{n3} . Hence, the variation in f_{n3} could be taken to imply the methodology had an accuracy of around 3.3%

4.2 S-N Curves for 300M Carbon Steel

Two different S-N curves were defined as sets of data points. The same criteria for selecting the first group of S-N curves were used, although in this instance, the S-N curves were for unnotched ($K_t = 1.0$) 300M. The second group of S-N curves was for 300M carbon steel, heat treated to give a tensile strength of 1931MPa (280ksi). Again, each set of data shared a common range of mean stresses σ_{mean} . The variables between each of the curves were the stress ratios R . In this case, the stress ratios of the chosen curves were $R = 0.1$ and $R = 0.2$, reference Figure 2.3.1.4.8(a) of MMPDS-03 [6]. Figure 2 shows

plots of the alternating stress against life for each data set and the data points used in the analysis.

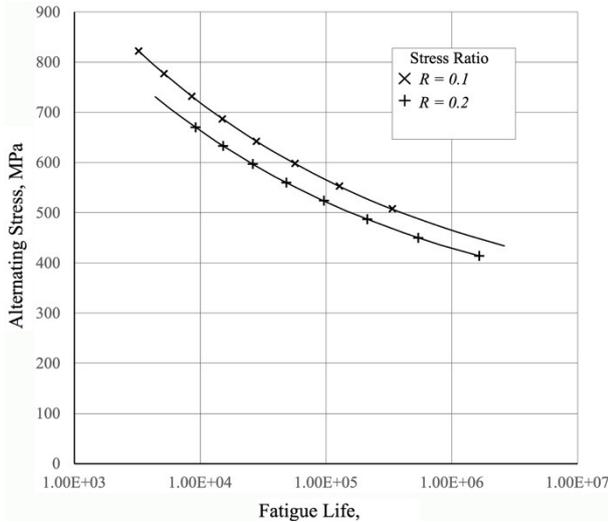


Figure 2. 300M $F_{tu} = 1931\text{MPa}$, unnotched ($K_t = 1.0$)

Equation (4) was again used to calculate values of f_{n2} for each of the data set. Data sets 4 and 5 related to stress ratios of $R = 0.1$ and $R = 0.2$ respectively. The values of function f_{n2} for data set 4 were calculated to be within the range 0.587 to 0.588 for each of the data points considered, representing a variation of 0.1% from the mean value. Similarly, the values of function f_{n2} for data set 5 were calculated to be within the range of 0.720 to 0.725, representing a variation of 0.3% from the mean value.

This supported the previously made conclusion that the mean stress σ_{mean} had a negligible effect on the value of the function f_{n2} . Again, since data sets 4 and 5 were for a common material, and stress concentration K_t , the previous conclusion that function f_{n2} was a function of the stress ratios R and material properties was supported.

Using equation (6) in an iterative solution, similar to that used previously, it was shown that a value of $a_1 = 2.365$ gave the optimum condition for minimum variation in the values function f_{n3} . The values of function f_{n3} for data sets 4 and 5 were calculated to be within the range of 0.468 to 0.470; hence, the variations in the values of function f_{n3} were found to be 0.2% for both data sets.

4.3 S-N Curves for Normalised AISI 4130 Carbon Steel

As previously, the selected S-N curves were for a common material, in this instance, normalised AISI 4130 carbon steel, which was assumed to have a tensile strength of 807MPa (117ksi). In total, four different S-N curves were each defined as a series of data points. However, due to the available data, different criteria for selecting the S-N curves had to be used. These four sets of data were sub-divided into groups of two sets of data. The S-N curves that formed the first of these two groups were for a stress concentration of $K_t = 4.0$. The S-N curves that formed the second of the two groups were for a stress concentration of $K_t = 5.0$. The two S-N curves within each group were each for different mean

stress. One curve was for a mean stress of $\sigma_{mean} = 138\text{MPa}$ (20ksi) and the other for a mean stress of $\sigma_{mean} = 207\text{MPa}$ (30ksi), reference Figures 2.3.1.2.8(d) and 2.3.1.2.8(e) of MMPDS-03 [6]. Hence, each data set represented a range of stress ratios R .

The focus of the presented analysis was to develop a damage-equivalent stress equation that could be applied to high positive stress ratios, such as those typical of preloaded bolted joints. The positive stress ratios occurred at the low alternating stress / low maximum stress / high life cycles end of the S-N curves. Hence, the data points used for the analysis were restricted to only positive stress ratios.

Figures 3 and 4 show plots of the alternating stress against life for each of the data sets and the data points used in the analysis.

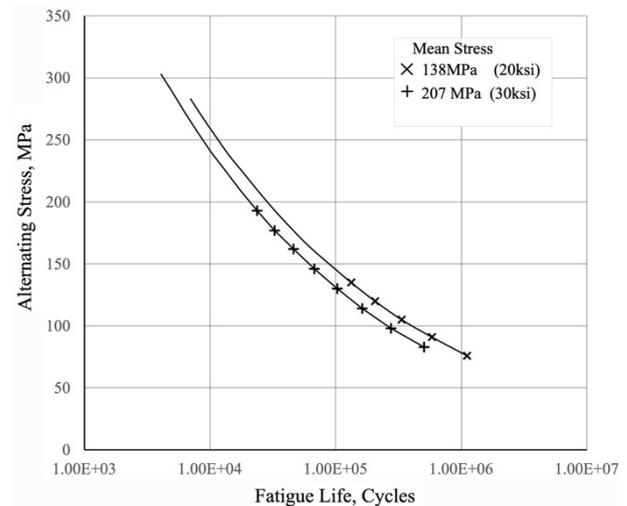


Figure 3. Normalised AISI 4130 ($F_{tu} = 807\text{MPa}$), $K_t = 4.0$

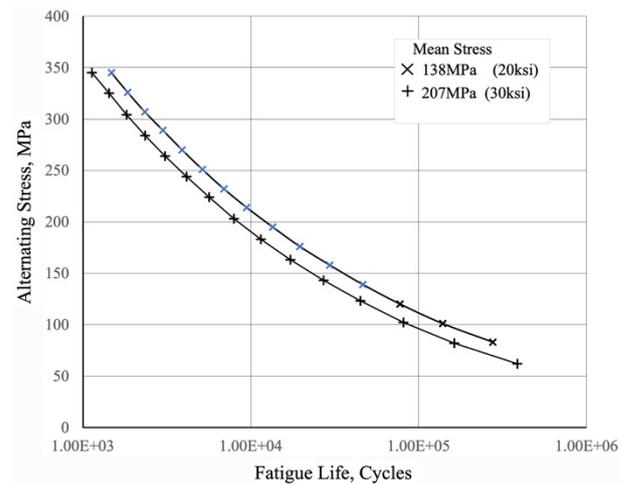


Figure 4. Normalised AISI 4130 ($F_{tu} = 807\text{MPa}$), $K_t = 5.0$

Equation (4) was again used to calculate values of f_{n2} for each of the four data sets. Data sets 6 and 7 were related to a stress concentration $K_t = 4.0$ and mean stresses of $\sigma_{mean} = 138\text{MPa}$ (20ksi) and $\sigma_{mean} = 207\text{MPa}$ (30ksi) respectively. Similarly, data sets 8 and 9 were related to a stress concentration $K_t = 5.0$ and mean stresses of $\sigma_{mean} = 138\text{MPa}$ and $\sigma_{mean} = 207\text{MPa}$, respectively. Since all four data sets were for constant mean stresses, and therefore over a range of stress ratios R , the resulting values for f_{n2} also covered a cor-

responding range. Hence, accuracy for the calculated values of f_{n2} could not be inferred from the results.

An iterative solution for equation (6), similar to that used previously, was applied to data sets 6 and 7. This iterative solution showed that for a stress concentration of $K_t = 4.0$, a value of $a_1 = 1.988$ gave the optimum condition for minimum variation in the values function f_{n3} . Similarly, applying the iterative solution of equation (6) to data sets 8 and 9 showed that for a stress concentration of $K_t = 5.0$, a value of $a_1 = 2.179$ gave the optimum condition.

The values of function f_{n3} for data sets 6 and 7 were calculated to be within the range of 0.282 to 0.292. Similarly, the values function f_{n3} for data sets 8 and 9 were calculated to be within the range of 0.332 to 0.356. The variations in the calculated values of the functions f_{n3} were 1.8% for data sets 6 and 7, and 3.5% for data sets 8 and 9.

4.4 S-N Curves for AISI 4130 Carbon Steel

Two individual S-N curves for AISI 4130 carbon steel, heat treated to give a tensile strength of 1241MPa (180ksi), were considered. The same criteria used for selecting the previous four sets of data, data sets 6, 7, 8, and 9, were used. Each of the S-N curves was defined as a series of data points. The first of these two S-N curves were for a stress concentration of $K_t = 2.0$. The second S-N curve was for a stress concentration of $K_t = 4.0$. Both S-N curves were for a mean stress of $\sigma_{mean} = 345MPa$ (50ksi), reference Figures 2.3.1.2.8(g) and 2.3.1.2.8(h) of MMPDS-03 [6]. Hence, each data set represented a range of stress ratios R . Again, the data points used for the analysis were restricted to only positive stress ratios. Figures 5 show plots of the alternating stress against life for each of the data sets and the data points used in the analysis.

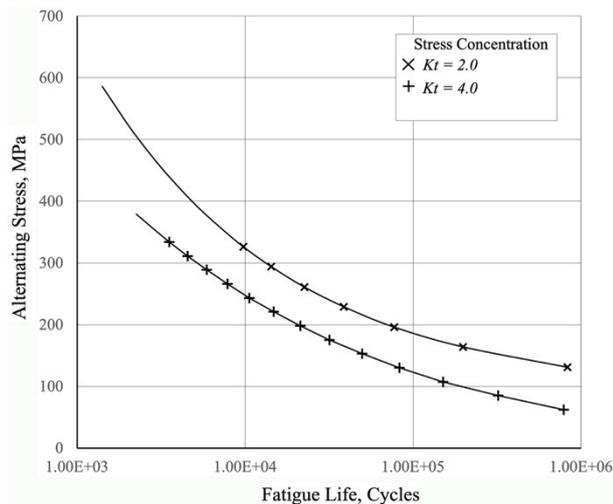


Figure 5. AISI 4130 $F_{tu} = 1241MPa$, $\sigma_{mean} = 345MPa$

Equation (4) was used to calculate values of f_{n2} for both data sets. Data sets 10 and 11 were related to stress concentrations $K_t = 2.0$ and $K_t = 4.0$ respectively. Both data sets were for a mean stress of $\sigma_{mean} = 345MPa$ (50ksi). Since both sets were for constant mean stresses, and therefore over a range of stress ratios R , the resulting values for f_{n2} also covered a corresponding

range. Hence, the results could not infer an accuracy for the calculated values of f_{n2} .

Again, the iterative solution for equation (6) was applied to data sets 10 and 11. Applying the iterative solution to data set 10 showed that for a stress concentration of $K_t = 2.0$, a value of $a_1 = 2.178$ gave the optimum condition for minimum variation in the values function f_{n3} . Similarly, applying the iterative solution to data set 11 showed that for a stress concentration of $K_t = 4.0$, a value of $a_1 = 2.417$ gave the optimum condition.

This supported the conclusion that the material constant a_1 was a function of the material properties and stress concentration K_t .

The values of function f_{n3} for data set 10 were calculated to be within the range of 0.431 to 0.449. Similarly, the values function f_{n3} for data set 11 was calculated to be within the range of 0.276 to 0.316. The variations in the calculated values of the functions f_{n3} were 2.1% for data set 10 and 6.9% for data set 11.

5. EFFECT OF STRESS CONCENTRATION AND MATERIAL PROPERTIES ON MATERIAL CONSTANT a_1

Values of the constant a_1 have been calculated for each material being considered. It has been observed that the value a_1 is not only dependent on the material properties but is also influenced by the stress concentration K_t of the specimens.

It was assumed that a_1 could be best described by a straight-line equation of the form:

$$a_1 = b_1 + b_2 \cdot \varepsilon_w \quad (7)$$

where b_1 and b_2 are constants and ε_w is a function of the material properties and the stress concentration K_t .

After trialling several plots of a_1 against various functions of the stress concentration K_t , tensile strength F_{tu} and the proof strength F_{ty} it was considered the best fit for the available data would be given by:

$$\varepsilon_w = K_t \cdot \left(\frac{F_{ty}}{E} \right)^{b_2} \quad (8)$$

The constants b_1 , b_2 , and b_3 were calculated using an iterative procedure. A range of initial values for b_3 was assumed, values for ε_w were calculated using equation (8), and then a linear regression was performed to determine values for b_1 and b_2 that gave the best/minimum RMS error fit for the line for each assumed value of b_3 . This iterative procedure was used to optimise for the value of b_3 that gave the minimum amount of error in the fit of calculated values for a_1 .

The optimum values for the constants were found to be:

$$\begin{aligned} b_1 &= 1.854 \\ b_2 &= 4.224 \times 10^6 \\ b_3 &= 3.260 \end{aligned}$$

The maximum and minimum errors in the calculated value for a_1 were 17.1% and -17.1%, respectively, with an RMS error of 10.8%.

The proof strain for the material is given by the term F_{ty}/E ; hence the function for ε_w defined by equation (8)

can be regarded as being related to the effect of strain, or work, hardening at the root of the notch or thread root. Work by McMillan and Jones (2020) shows that plastic deformation at the notch remains highly localised, reference [7]. Hence, it is possible to conclude that the strain at the notch or the thread root of a bolt is controlled by the nett section of the notched component or the core of the thread.

6. EFFECT OF STRESS CONCENTRATION ON THE DAMAGE-EQUIVALENT STRESS EQUATION

The approach to studying the stress concentration effect was again to identify groups of S-N curves for each of the materials being considered. In these cases, however, the groups of S-N curves were selected to have stress concentration as the variable.

6.1 Additional S-N Curves for 300M Carbon Steel

A group of S-N curves for 300M carbon steel, heat treated to give a tensile strength of 1931MPa (280ksi), was selected. Three different S-N curves were defined as a set of data points. The criteria for selecting the S-N curves were that they were for the same material, had the same stress ratio of $R = 0.33$, and shared a common range of mean stresses σ_{mean} . The variables between each of the curves were the stress concentration K_t . The stress concentrations of the chosen curves were $K_t = 2.0$, $K_t = 3.0$ and $K_t = 5.0$, reference Figures 2.3.1.4.8(b), 2.3.1.4.8(c) and 2.3.1.4.8(d) of MMPDS-03 [6]. Figure 6 shows plots of the alternating stress against life for each of the data sets.

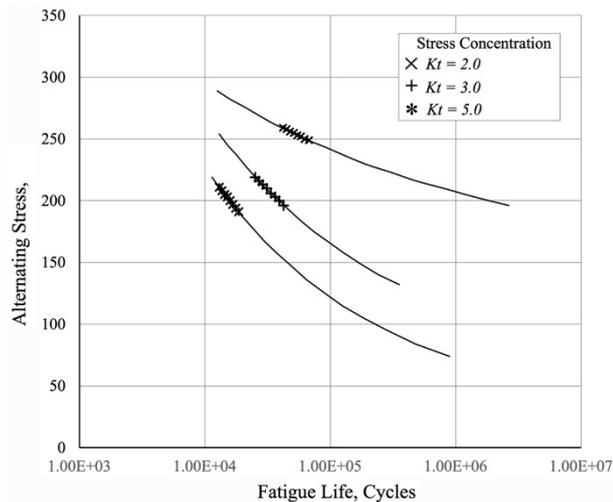


Figure 6. 300M $F_{tu} = 1931\text{MPa}$, $R = 0.33$

Equation (4) was again used to calculate values of f_{n2} for each of the data sets. Data sets 12, 13 and 14 related to stress concentrations of $K_t = 2.0$, $K_t = 3.0$ and $K_t = 5.0$ respectively. The values of function f_{n2} for data set 12 were calculated to be within the range 1.013 to 1.014 for each of the data points considered, representing a variation of 0.05% from the mean value. Similarly, the values function f_{n2} for data set 13 were calculated to be within the range of 0.710 to 0.722, representing a variation of 0.8% from the mean value. Finally, the values function f_{n2} for data set 14 were calculated to be

within the range of 0.690 to 0.692, representing a variation of 0.1% from the mean value. This was in line with the earlier conclusion that the mean stress σ_{mean} had a negligible effect on the value of the function f_{n2} .

Applying equations (7) and (8) using $K_t = 2.0$, assuming $F_{ty} = 0.83 \times F_{tu}$ (hence $F_{ty} = 1602\text{MPa}$) and $E = 200\text{MPa} \times 10^3$ the material constant was calculated as $a_1 = 3.094$. However, this value of the material constant a_1 is based on an estimated value for the proof stress F_{ty} based on the mean ratio of TYS/TUS given in MMPDS-03 [6] and a typical value for Young's modulus of elasticity E .

An alternative and possibly more accurate way of estimating the material constant a_1 would be to use a value for the strain hardening factor e_w based on the value of the more accurately known value of the material constant for unnotched 300M obtained using data sets 4 and 5. Hence, rearranging equation (7) to calculate the strain hardening factor for unnotched specimens, stress concentration $K_t = 1.0$:

$$\varepsilon_w = \frac{a_1 - b_1}{b_2} \quad (9)$$

By reference to equation (8), an estimate of the strain hardening factor can be made by the ratio of stress concentration factors:

$$\varepsilon'_w = \varepsilon_w \cdot \frac{K_t}{K_{t,datum}} \quad (10)$$

where, in this instance, $K_{t,datum} = 1.0$ is the datum stress concentration applicable to data sets 4 and 5.

Using equations (9) and (10) with the material constant of $a_1 = 2.365$ found from data sets 4 and 5 and the stress concentration of $K_t = 2.0$ for data set 12, the effective strain hardening factor was calculated as $\varepsilon'_w = 2.420 \times 10^{-7}$. Using this value for the effective strain hardening constant in equation (7) gave a value for the material constant of $a_1 = 2.867$ for data set 12.

Similarly, using equations (9) and (10) with a stress concentration of $K_t = 3.0$, the effective strain hardening factor was calculated as $\varepsilon'_w = 3.630 \times 10^{-7}$. Again, using this value for the effective strain hardening constant in equation (7) gave a value for the material constant of $a_1 = 3.387$ for data set 13.

Finally, using equations (9) and (10) with a stress concentration of $K_t = 5.0$, the effective strain hardening factor was calculated as $\varepsilon'_w = 6.049 \times 10^{-7}$. And using this value for the effective strain hardening constant in equation (7) gave a value for the material constant of $a_1 = 4.409$ for data set 14.

Using equation (6) with the values for the material constant and the appropriate range of values for f_{n2} presented earlier in this section, the value of function f_{n3} was calculated. The value of function f_{n3} for data set 12 was calculated to be within the range of 0.446 to 0.447. Similarly, the values function f_{n3} for data set 13 were calculated to be within the range of 0.270 to 0.275. Finally, the value function f_{n3} for data set 14 was calculated to be within the range of 0.196 to 0.197. The variations in the calculated values of the functions f_{n3} were the same as those for the calculated values of f_{n2} .

6.2 Additional S-N Curve for AISI 4340 Carbon Steel

A total of three S-N curves for AISI 4340 carbon steel, heat treated to give a tensile strength of 1379MPa (200ksi) were considered. Again, each S-N curve was defined as a set of data points.

The first of the S-N curves were for unnotched material, i.e., a stress concentration of $K_t = 1.0$, with a constant stress ratio of $R = 0.43$ and hence had a variable range of mean stresses σ_{mean} , reference Figure 2.3.1.3.8(k) of MMPDS-03 [6]. The other two S-N curves had a common stress ratio of $R = 0.0$ and also had a common range of mean stresses. One was for unnotched material, stress concentration $K_t = 1.0$, and the other was for a stress concentration of $K_t = 3.3$, reference Figures 2.3.1.3.8(k) and 2.3.1.3.8(l) of MMPDS-03 [6]. Figures 7 and 8 show plots of the alternating stress against life for each of the data sets.

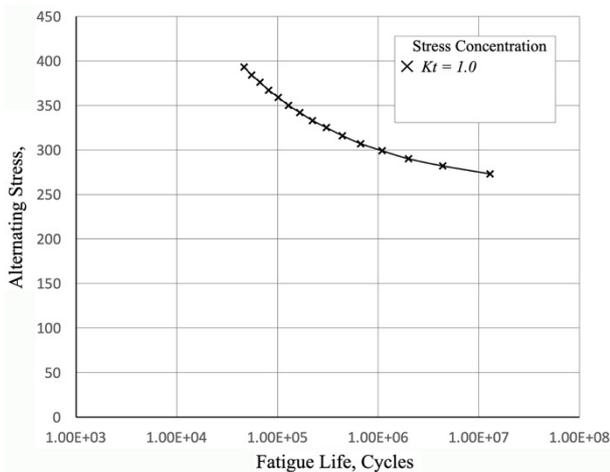


Figure 7. AISI 4340 $F_{tu} = 1379\text{MPa}$, $R = 0.43$

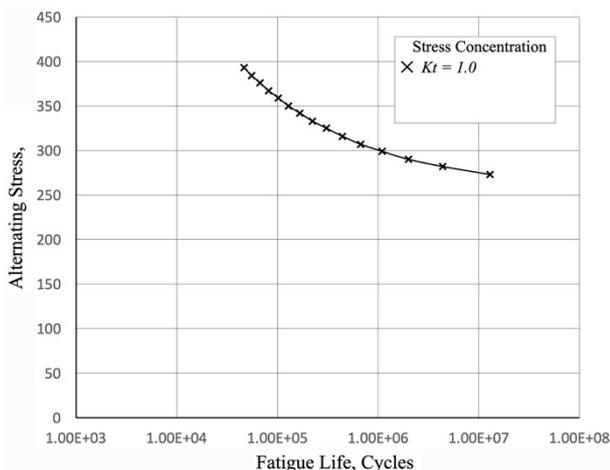


Figure 8. AISI 4340 $F_{tu} = 1379\text{MPa}$, $R = 0.0$

Equation (4) was used to calculate values of f_{n2} for each of the three sets of data considered. Data set 15 related to a stress ratio of $R = 0.43$ for a concentration $K_t = 1.0$. Data sets 16 and 17 related to stress ratios of $R = 0.0$ for stress concentrations of $K_t = 1.0$ and $K_t = 3.3$ respectively. The values of function f_{n2} for data set 15 were calculated to be within the range of 0.673 to 0.682, representing a variation of 0.6% from the mean value. Similarly, the values function f_{n2} for data set 16

were calculated to be within the range of 0.317 to 0.330, representing a variation of 2.2% from the mean value. Finally, the values function f_{n2} for data set 17 were calculated to be within the range of 0.274 to 0.275, representing a variation of 0.2% from the mean value.

Considering data set 15, equations (9) and (10) were used with a material constant of $a_1 = 3.349$, and a stress concentration of $K_{t,datum} = 3.3$ as the datum conditions, found from data sets 1, 2, and 3. Using the stress concentration of $K_t = 1.0$ for data set 15, the effective strain hardening factor was calculated as $\epsilon'_w = 6.613 \times 10^{-8}$. Equation (7) was then used to calculate a value of $a_1 = 2.134$ for the effective material constant.

Using equation (6) with the values for the material constant and the appropriate range of values for f_{n2} , the values of function f_{n3} were calculated. The variations in the calculated values of the functions f_{n3} were the same as those for the calculated values of f_{n2} . The values function f_{n3} for data set 15 were found to be within the range of 0.295 to 0.299.

When considering data sets 16 and 17, using equation (6) with a stress ratio of $R = 0.0$, the results showed that $f_{n3} = f_{n2}$ for both of these data sets. Note that any attempt to calculate a value for a_1 would result in a trivial solution.

7. ADDITIONAL S-N CURVES TO STUDY THE EFFECT OF STRESS CONCENTRATION

The objective of the work being presented was to derive a fatigue damage-equivalent stress function that could be applied to high, positive stress ratios typical of those found in preloaded threaded fasteners. Hence, up until this point in the analysis, only data for positive stress ratios had been considered. It was possible to continue to define a damage equivalent function with the data obtained for positive stress ratios only. However, this meant that some curve fitting had to be made through a very limited number of data points. This work was carried out, and it was found that when the derived equation was applied to loading conditions involving negative stress ratios, the results had a good correlation with actual S-N curves. This good correlation gave the confidence to use fatigue data collected for negative stress ratios in the derivation of the damage-equivalent stress for fatigue function without compromising its application to high, positive stress ratios.

7.1 Additional S-N Curves for Normalised AISI 4130 Carbon Steel

Two S-N curves for normalised AISI 4130 carbon steel, which was assumed to have a tensile strength of 807MPa (117ksi), were considered. Each curve was for a mean stress of $\sigma_{mean} = 207\text{MPa}$ (30ksi). Again, each S-N curve was defined as a set of data points. One was for a stress concentration of $K_t = 1.5$, and the other was for a stress concentration of $K_t = 2.0$, reference Figures 2.3.1.2.8(b) and 2.3.1.2.8(c) of MMPDS-03 [6]. Figure 9 shows plots of the alternating stress against life for the two data sets.

Equation (4) was used to calculate values of f_{n2} for both of the data sets. Data sets 18 and 19 related to stress

concentrations of $K_t = 1.5$ and $K_t = 2.0$ respectively. Since both data sets were for constant mean stresses, and therefore over a range of stress ratios R , the resulting values for f_{n2} also covered a corresponding range.

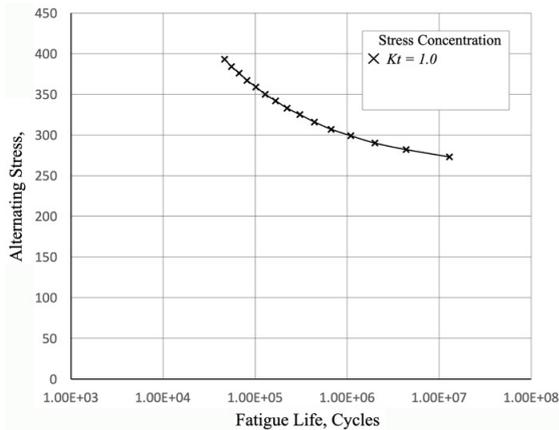


Figure 9. Normalised AISI 4130 ($F_{tu} = 807 \text{ MPa}$, $\sigma_{mean} = 207 \text{ MPa}$)

Using equation (6) in an iterative solution, similar to that used previously, it was shown that a value of $a_1 = 1.381$ gave the optimum condition for minimum variation in the values function f_{n3} . The values of function f_{n3} for data sets 18 and 19 were calculated to be within the range of 0.083 to 0.087; hence, the variations in the values of function f_{n3} was found to be 2.6%. Similarly, applying an iterative solution of equation (6) to data set 19 showed that for a stress concentration of $K_t = 2.0$, a value of $a_1 = 1.420$ gave the optimum condition. The values of function f_{n3} were calculated to be within the range of 0.102 to 0.104; hence, the variations in the values of function f_{n3} was found to be 1.0%.

7.2 Additional S-N Curve for AISI 4130 Carbon Steel

A single S-N curve for AISI 4130 carbon steel, heat treated to give a tensile strength of 1241MPa (180ksi), was considered. This S-N curve was defined as a series of data points. The curve was for unnotched material, stress concentration of $K_t = 1.0$, and a mean stress of $\sigma_{mean} = 345 \text{ MPa}$ (50ksi), reference Figure 2.3.1.2.8(f) of MMPDS-03 [6]. Figure 10 shows plots of the alternating stress against life.

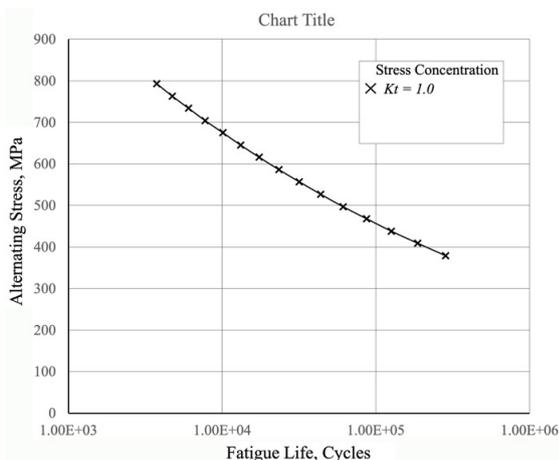


Figure 10. AISI 4130 $F_{tu} = 1241 \text{ MPa}$, $\sigma_{mean} = 345 \text{ MPa}$

8. EFFECT OF STRESS CONCENTRATION AND MATERIAL PROPERTIES ON FUNCTION f_{n3}

From the previous workings, it has been shown that f_{n3} is a function of the stress concentration K_t and the material properties, either the tensile strength F_{tu} or the yield/proof stress F_{ty} . Numerical values of the function f_{n3} have been calculated for each data case considered.

It was now assumed that the equation could describe the function f_{n3} for a straight line passing through the origin:

$$f_{n3} = a_2 \cdot f_{n,Kt} \quad (11)$$

where a_2 is the slope of the line and $f_{n,Kt}$ is the variable given by a function of the stress concentration K_t .

After trialling several plots of f_{n3} for each material against various functions of the stress concentration K_t it was considered that there were two potentially good fits for the available data. These were:

$$f_{n,Kt} = \frac{1}{(K_t + a)^{a_2}} \quad (12a)$$

or

$$f_{n,Kt} = \frac{1}{(K_t^2 + 1)^{a_2}} \quad (12b)$$

The constants a_2 and a_3 used in equations (11), (12a), and/or (12b) are both functions of the material properties.

Each of these equations for $f_{n,Kt}$ was considered, and it was found that equation (12a) provided the more accurate prediction of damage-equivalent stress for fatigue. Hence, only the work involving equation (12a) is presented in this paper.

7.3 Effect of Stress Concentration for 300M Carbon Steel

The calculated values for the function f_{n3} and the associated stress concentrations K_t for data sets 8, 9, 12, 13, and 14 were used to calculate the constants a_2 and a_3 for 300M Carbon steel with a tensile strength of 1931 MPa (280ksi). This group of data sets covered a range of stress concentrations, K_t equal to 1.0, 2.0, 3.0, and 5.0.

An iterative solution for the constants a_2 and a_3 was carried out, applying equations (11) and (12a) and using this data for 300M carbon steel. The constants a_2 and a_3 were calculated by first assuming a value for a_3 and then calculating values for $f_{n,Kt}$ using equation (12) before performing a linear regression to determine a value for a_2 that gave the best/minimum RMS error fit. By following this procedure, it was possible to use an iterative approach to calculate an optimum value for the constant a_3 that gave the minimum amount of error in the resulting values of f_{n3} . This iterative solution showed that a value of $a_3 = 0.675$ gave the optimum condition. This led to a value of $a_2 = 0.770$ and the worst error of -17.8% in the prediction of f_{n3} .

7.4 Effect of Stress Concentration for AISI 4340 Carbon Steel

The calculated values for the function f_{n3} and the associated stress concentrations K_t for data sets 1, 2, 3,

15, 16, and 17 were used to calculate the constants a_2 and a_3 for AISI 4340 Carbon steel with a tensile strength of 1379MPa (200ksi). This group of data sets covered a range of stress concentrations, K_t equal to 1.0 and 3.3.

The previously described iterative procedure was applied using this data for AISI 4340 carbon steel. This showed that a value of $a_3 = 0.525$ gave the optimum condition. This led to a value of $a_2 = 0.430$ and the worst error of -27.2% in the prediction of f_{n3} .

7.5 Effect of Stress Concentration for AISI 4130 Carbon Steel

The calculated values for the function f_{n3} and the associated stress concentrations K_t for data sets 10, 11, and 20 were used to calculate the constants a_2 and a_3 for AISI 4130 Carbon steel with a tensile strength of 1241 MPa (180ksi). This group of data sets covered a range of stress concentrations, K_t equal to 1.0, 2.0, and 4.0.

The previously described iterative procedure was applied using this data for AISI 4130 carbon steel. This showed that a value of $a_3 = 0.315$ gave the optimum condition. This led to a value of $a_2 = 0.544$ and the worst error of -12.5% in the prediction of f_{n3} .

7.6 Effect of Stress Concentration for Normalised AISI 4130 Carbon Steel

The calculated values for the function f_{n3} and the associated stress concentrations K_t for data sets 4, 5, 6, and 7 were used to calculate the constants a_2 and a_3 for Normalised AISI 4130 Carbon steel with a tensile strength of 807MPa (117ksi). This group of data sets covered a range of stress concentrations, K_t equal to 1.5, 2.0, 4.0, and 5.0.

The previously described iterative procedure was applied using this data for AISI 4340 carbon steel. This showed that a value of $a_3 = -1.705$ gave the optimum condition. This led to a value of $a_2 = 0.017$ and the worst error of -8.5% in the prediction of f_{n3} .

9. EFFECT OF MATERIAL PROPERTIES ON MATERIAL CONSTANTS a_2 AND a_3

Values for the constants a_2 and a_3 had been calculated for the individual materials being considered; hence it was assumed they could be described as functions of the material properties, tensile strength F_{tu} or yield/proof stress F_{ty} .

It was assumed that a_2 could be best described by a straight-line equation of the form:

$$a_2 = b_4 + b_5 \cdot f_{n,Fty} \quad (13)$$

where

$$f_{n,Fty} = \left(\frac{F_{ty}}{E} \right)^{b_6} \quad (14)$$

The constants b_4 , b_5 , and b_6 were calculated by assuming a value for b_6 , calculating values for $f_{n,Fty}$ using equation (14) and then performing a linear regression to determine values for b_4 and b_5 that gave the best/mini-

imum RMS error fit for the line. This procedure was used iteratively to optimise for the value of b_6 that give the minimum amount of error in the resulting values for a_2 .

The optimum values for the constants were found to be:

$$\begin{aligned} b_4 &= -1.015 \\ b_5 &= 38.120 \\ b_6 &= 0.635 \end{aligned}$$

The worst error in the calculated value for a_2 was 18.2%.

A similar procedure was applied to determine the function that describes the material constant a_3 . It was assumed that a_3 could be best described by:

$$a_3 = b_7 + b_8 \cdot \left(\frac{F_{ty}}{E} \right)^{b_9} \quad (15)$$

The optimum values for the constants were found to be:

$$\begin{aligned} b_7 &= 1.038 \\ b_8 &= -2.032 \times 10^{-6} \\ b_9 &= -2.485 \end{aligned}$$

The worst error in the calculated value for a_2 was -17.3%.

10. FATIGUE DAMAGE-EQUIVALENT STRESS FUNCTION

Equations (3), (5), (11), and (12) were combined to produce the final fatigue damage-equivalent stress function:

$$\sigma_{equ} = \sigma_{alt} \cdot \left(1 + \frac{a_2 \cdot (1+R)^{a_1}}{(K_t + 1)^{a_3}} \right) \quad (16)$$

where: a_1 is given by equations (7) and (8), a_2 is given by equations (13) and (14), and a_3 is given by equation (15).

The twenty S-N curves used in the derivation of equation (16) plus eighteen additional S-N curves were used to validate this function.

Two of the eighteen additional S-N curves were for AISI 4340 carbon steel, heat treated to give a tensile strength of 862MPa (125ksi), and were for stress concentrations of $K_t = 1.0$ and $K_t = 3.3$ with a stress ratio of $R = 0.0$, reference Figures 2.3.1.3.8(a) and 2.3.1.3.8(b) of MMPDS-03 [6]. Two further S-N curves were for the same grade of steel but heat treated to give a tensile strength of 1034 MPa (150ksi). These second two additional S-N curves were also for stress concentrations of $K_t = 1.0$ and $K_t = 3.3$ with a stress ratio of $R = 0.0$, reference Figures 2.3.1.3.8(c) and 2.3.1.3.8(d) of MMPDS-03 [6]. The remaining fourteen additional S-N curves were for the material grades used in the analysis, although these specific curves had not been used in the analysis. Nine were for normalised AISI 4130, and five were for 300m with a tensile strength of 1931MPa.

As part of the validation process, the damage-equivalent stress for fatigue calculated using equation (16) was compared with existing methods of calculating damage-equivalent stresses, reference ESDU 06009 [1]. The other methods used for comparison were:

Goodman-Haig:

$$\sigma_{equ} = \sigma_{alt} \cdot \left(1 + \frac{\sigma_{mean}}{\sigma_{alt}} \right) \quad (17)$$

Gerber:

$$\sigma_{equ} = \frac{\sigma_{alt}}{1 - \left(\frac{\sigma_{mean}}{F_{tu}} \right)^2} \quad (18)$$

Soderberg:

$$\sigma_{equ} = \frac{\sigma_{alt}}{1 - \frac{\sigma_{mean}}{F_{ty}}} \quad (19)$$

Smith-Watson-Topper:

$$\sigma_{equ} = \sigma_{alt} \cdot \left(1 + \frac{\sigma_{mean}}{\sigma_{alt}} \right) \quad (20)$$

It was found that the fatigue damage-equivalent stress function derived in this paper, equation (16), gave the most consistent results over the range of thirty-six S-N curves used for the validation process. Equation (16) provided the most accurate prediction of damage-equivalent stress for twenty-seven out of the thirty-six cases considered. In seventeen of the cases, equation (16) gave a very close correlation. The worst-case deviations of the fatigue damage-equivalent stress calculated by equation (16) from the datum S-N curve for the stress ratio $R = -1.0$ were +15.7% and -13.1%. Plots of results for the worst-case deviations are presented in Figures 11 and 12.

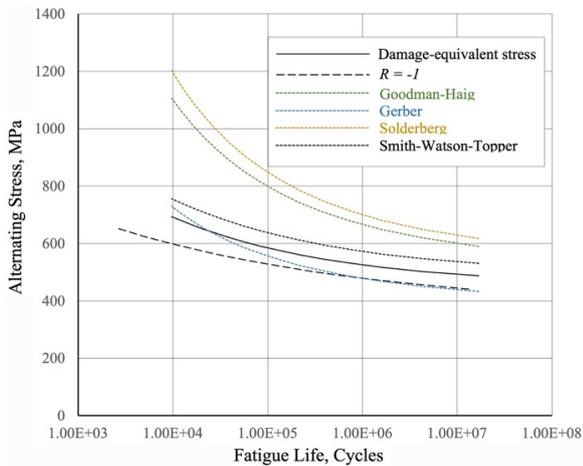


Figure 11. AISI 4340, $F_{tu} = 1034\text{MPa}$, $K_t = 1.0$, $R = 0.0$

Figure 11 shows the results of the validation case for unnotched AISI 4340 heat treated with a tensile strength of 1034MPa (150ksi) and a stress ratio of $R = 0.0$. The equation (16) results had a worst-case deviation of +15.7%. The results using Gerber's method, equation (18), gave the best fit.

Figure 12 shows the plots of results of data set 10, notched AISI 4130 heat treated to have a tensile strength of 1241MPa (180ksi) with a stress concentration of $K_t = 2.0$ and a mean stress of $\sigma_{mean} = 345\text{MPa}$ (50ksi). The results from equation (16) had a deviation of -13.1%. The results using the Smith-Watson-Topper method, equation (20), gave the best fit.

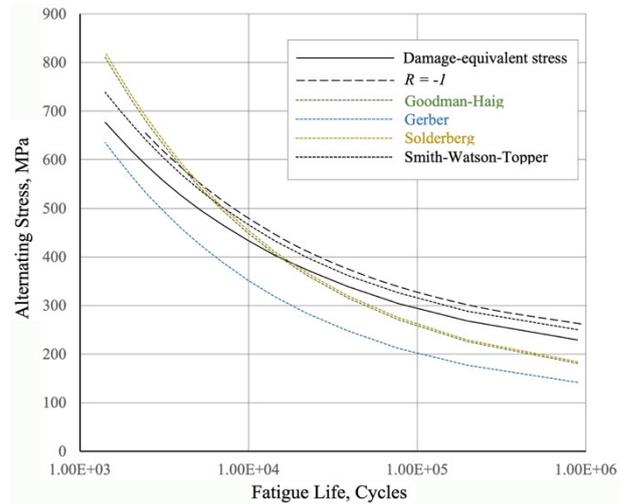


Figure 12. AISI 4130, $F_{tu} = 1241\text{MPa}$, $K_t = 2.0$, $\sigma_{mean} = 345\text{MPa}$

The stated objective was to determine a method of calculating a damage-equivalent stress for fatigue that could be applied to preloaded bolted assemblies. The most representative validation cases, in terms of tensile strength, stress concentration, and stress ratio or mean stress, are presented in Figures 13 to 16.

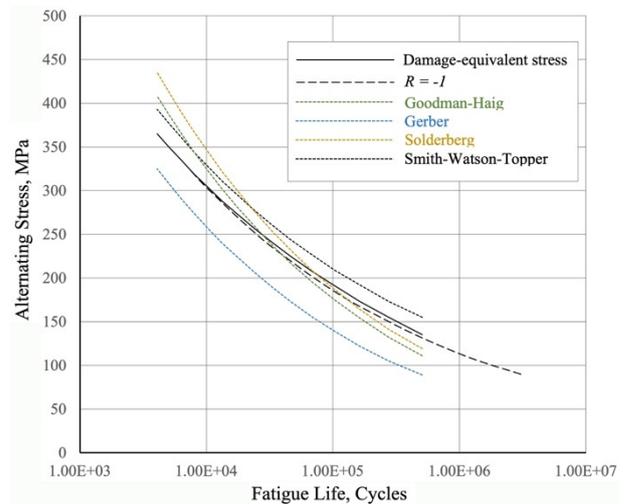


Figure 13. Normalised AISI 4130, $K_t = 4.0$, $\sigma_{mean} = 207\text{MPa}$

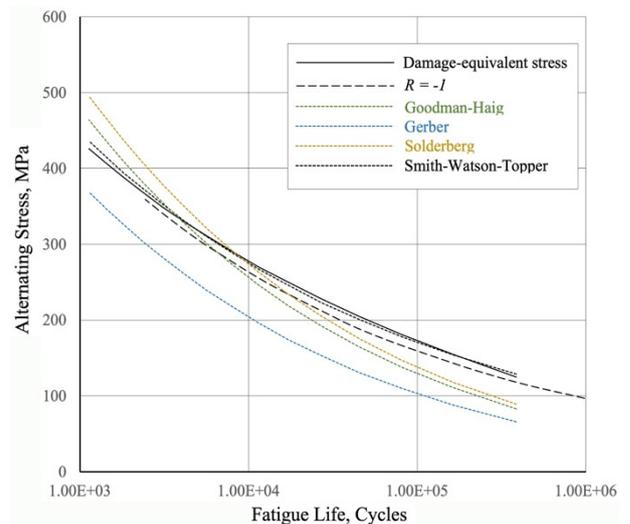


Figure 14. Normalised AISI 4130, $K_t = 5.0$, $\sigma_{mean} = 207\text{MPa}$

Figures 13 and 14 show the plots of results of data sets 7 and 9, respectively. Both of these figures are for normalised AISI 4130 that had an estimated tensile strength of 807MPa. This material specification could be regarded as typical of a Grade 8.8 bolt. Figure 13 presents plots for a stress concentration of $K_t = 4.0$ with a mean stress of $\sigma_{mean} = 207MPa$ (30ksi). Similarly, Figure 14 presents plots for a stress concentration of $K_t = 5.0$ with the same mean stress of $\sigma_{mean} = 207MPa$.

The errors in the calculated values for the damage-equivalent stress for data set 7 were all within +3.2%, with a RMS error of 2.0%. Similarly, the calculated values for the damage-equivalent stress for data set 9 were within the range +1.8% to +8.4%, with an RMS error of 5.4%.

A theoretical study of bolt thread elastic stress concentration factors has been conducted by Lehnoff et al. (2000) [8]. This work showed that M8 and M12 bolts, with the maximum metal condition, had stress concentration factors of 4.33 and 4.32, respectively. Hence, Figure 13 could be considered an approximate representation of these bolt sizes. Similarly, Figure 14 could be considered as representative of M16, M20, and M24, which Lehnoff (200) [8] shows had stress concentration factors of 4.67, 4.77, and 4.82, respectively for maximum metal conditions and stress concentration factors of 5.12, 5.17 and 5.22 respectively for minimum metal conditions.

The mean stress of $\sigma_{mean} = 207MPa$ could be considered as representing a bolt preload of 32% of proof stress. It is usual for high tensile bolts (Grade 8.8 and above) to be preloaded 60% to 80% of proof load, and in some circumstances, even higher; hence, the figure of 32% is a little low. However, the thread rolling of bolts during manufacture induces compressive residual stresses. This means that whilst a bolt may be preloaded to, say, 80% of proof load, only the core of the thread is subjected to that level of pre-stress, and the thread root would experience lower stress. Furukawa and Hagiwara's (2015) [9] work estimated that a thread's compressive residual stress, rolled after heat treatment, was 830MPa. Importantly, this was for a thread rolled after heat treatment; however, bolt manufactures prefer to heat treat after thread rolling. This minimises the power required to roll the thread, extends die life, and hence minimises production costs. It has been shown by Marcelo et al. (2011) [10] that threads rolled after heat treatment exhibited higher tensile strength than those rolled before being heat treated and tempered at the same temperature. Hence, it may be assumed that the mean stress of $\sigma_{mean} = 207MPa$ is representative of the mean stress at the thread root, where a crack would initiate, but is lower than would normally be expected in the core of the thread, which is the region the crack would grow into once initiated. Work by Leitner et al. (2000) [11] has shown that compressive residual stresses, in this case, induced by high-frequency peening, improve fatigue strength, particularly at stress concentrations.

Figure 15 shows the plots of results of data set 11 for notched AISI 4130 heat treated to give a tensile strength of 1241 MPa. This material specification could be regarded as typical of a Grade 12.9 bolt.

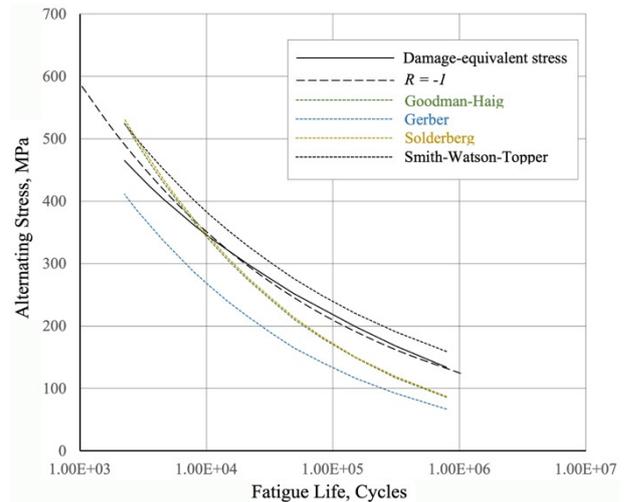


Figure 15. AISI 4130, $F_{tu} = 1241MPa$, $K_t = 4.0$, $\sigma_{mean} = 345MPa$

The errors in the calculated values for the damage-equivalent stress for data set 11 were within the range -5.4% to +4.4%, with an RMS error of 3.3%.

Again, the work by Lehnoff et al. (2000) [8] can be taken to show that the stress concentration of $K_t = 4.0$ could be considered as an approximate representation of M8 and M12 bolts.

In this case, the mean stress of $\sigma_{mean} = 207MPa$ could be considered as representing a bolt preload of 19% of proof stress. However, assuming that residual stresses combined with a preload of 60% of proof load could still represent the mean stress at the thread root, where a crack would initiate, is not unreasonable. Work by Leitner et al. (2000) [11] has shown that compressive residual stresses, in this case, induced by high-frequency peening, improve fatigue strength, particularly at stress concentrations.

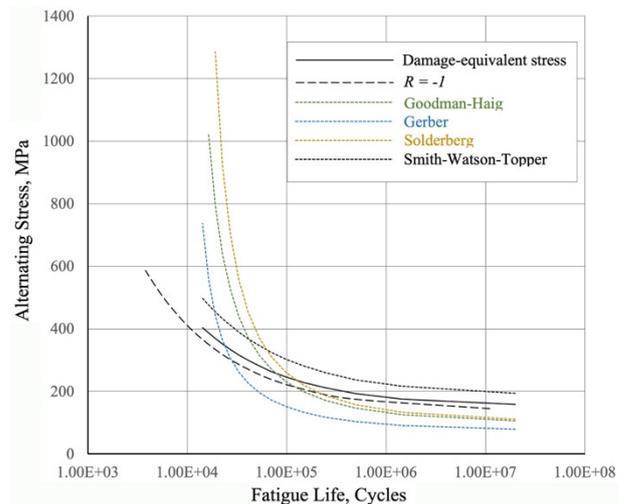


Figure 16. AISI 4340, $F_{tu} = 1379MPa$, $K_t = 3.3$, $R = 0.74$

Figure 16 shows the plots of results of data set 3 and relates to notched AISI 4130 heat treated to give a tensile strength of 1379MPa. This tensile strength is higher than the minimum requirement for Grade 12.9 bolts. However, statistically, approximately 50% of Grade 12.9 bolts could achieve this strength. Hence, the material can still be regarded as applicable to a Grade 12.9 bolt.

The errors in the calculated values for the damage-equivalent stress for data set 3 were within the range +9.1% to +13.5%, with an RMS error of 10.2%.

The work by Lehnoff et al. (2000) [8] shows that Figure 16 could be considered a representation of M16, M20, and M24 bolts.

When considering the results in Figure 16 as representing preloaded bolts, the tensile strength of 1379MPa is not relevant since bolt preloads based on the minimum material properties for the bolt grade. Hence, the arguments made regarding data set 11 for Figure 15 hold true.

11. CONCLUSIONS

A viable method of determining a damage-equivalent stress function for fatigue is presented. This empirical method was produced by developing an equation that fitted existing S-N curves.

The damage-equivalent stress function applies to carbon steels with heat treatments ranging from the normalised state to hardened and tempered for tensile strength of 1900MPa.

This new damage-equivalent stress function is consistently more accurate than existing methods of 'correcting' for mean stress.

Validation of this damage-equivalent stress function indicates that it has a maximum/minimum accuracy of +16% to -13%, with a root-mean-square error of 8%, for fatigue stress.

The damage-equivalent stress function presented is particularly suited to high positive stress ratios, particularly preloaded bolts. However, it also has a good correlation with results for negative stress ratios.

The work presented is based on available S-N available for carbon steel. The resulting damage-equivalent stress function is expected to be appropriate for other materials with similar hardness, such as stainless steel. Future work will be to test if this assumption is correct.

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NOMENCLATURE

a_1 to a_3	Constants, calculated from geometry and material properties
b_1 to b_9	Numerical constants
f_{n1} to f_{n3}	Function of stress and stress concentration
$f_{n.Fty}$	Function of the yield/proof stress
$f_{n.Kt}$	Function of the stress concentration
Ftu	Tensile strength of a material
Fty	Yield or proof stress of a material
K_t	Stress concentration factor
$K_{t,datum}$	A datum stress concentration factor
R	Stress ratio (s_{min} / s_{max})
TUS	Tensile strength of a material, at the fatigue test strain rate
TYS	Yield or proof stress of a material, at the fatigue test strain rate

Greek symbols

ε_w	Function, related to work hardening
ε'_w	Function, related to work hardening
σ_{alt}	Alternating stress
σ_{equ}	Damage-equivalent stress for fatigue
σ_{max}	Maximum stress
σ_{mean}	Mean stress
σ_{min}	Minimum stress

ЕМПИРИЈСКИ ПРИСТУП СВЕОБУХВАТНОЈ ФУНКЦИЈИ НАПОНА КОЈА ЈЕ ЕКВИВАЛЕНТНА ЗАМОРНОМ ОШТЕЋЕЊУ

M. Velc

Овај рад развија емпиријску функцију напона еквивалентну заморном оштећењу. Класичне методе

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анализе се користе за 'уклапање' једначине на бројне С-Н криве за различите врсте угљеничног челика. Резултирајућа функција напрезања еквивалентног оштећења применљива је на челике који су подвргнути широком спектру термичких обрада, од нормализованих до каљених и отпуштених до 1900 МПа. Такође је применљив на широк опсег концентрација напона, неурезан до $K_t = 5,0$ и типичан за навоје завртња. Разматра се и низ односа напона и

средњих напона. Функција превазилази нека од ограничења постојећих метода 'исправљања' за средњи стрес. Постојеће методе су ограничене у томе, иако могу дати добре резултате у низу услова, постоје неке околности у којима су резултати веома нетачни. Функција напрезања еквивалентна оштећењу је погодна за употребу у аутоматизованим прорачунским процедурама као што су табеле, MathCAD © и SMathStudio ©.